

# Contrast between angular and distance observables in geodetic inspection of deformation.

[Julio Manuel de Luis Ruiz, Felipe Piña García, Raúl Pereda García ]

**Abstract**— This work aims to establish a methodology to compare the displacements are obtained by performing geodetic auscultations by observable angular (theodolite) and electronic distancemeter (EDM), all with the object of choice "a priori" the best instruments to carry out such work.

To justify the contrast it have been proposed the methodology and instruments used in the resolution of auscultation of a dam called "La Cohilla", located in Cantabria (Spain). This one have been observed by classical methods, angles and distances, in two consecutive seasons. First of all, the displacements of a series of targets with both observables are obtained. After that, it is proposed a statistical procedure to validate the results obtained with both observable and thus be able to make the right decision about the appropriate choice of the observable and therefore what instrumental must be employed in geodesic auscultations.

**Keywords**— Microgeodesy, geodetic auscultation, angular and distance observables, statistical testing, bi-wide distribution.

## I. Introduction

Conventional structures can be characterised as a resilient whole that is deformed against an external or inside stress, transferring a tensor load to the set that must be kept under previously established values. The stress and strain are related, and analyzing the deformational behavior, definitive conclusions can be drawn, taking under control the strength of the assembly, allowing you to take the necessary preventive measures to correct the hypothetical anomalies detected in the control. It is usual to consider the requests are external (hydraulic pressure, earth pressure, etc.). And internal actions are driven by thermal or shrinkage phenomena.

The ultimate goal of a geodetic auscultation is the establishment of geodesic movement of a series of points within a structure, soil, etc., based on a series of survey observations performed with instruments and appropriate methods of observation. Depending on these movements the behavior of the structure can be revealed and, depending on the kind of structure (dam, wall, riprap, slope, etc..) the range of accuracy may vary, having found that depending on the observable the resolution an accuracy of the auscultation are conditioned.

## II. Material and methodology

### A. Participating elements.

Capturing observable not only determines the resolution method of the auscultation. Besides, it is conditioned by other

participating elements that are described next, regardless of the type of observable and resolution method used.

### Inspection pillars.

The pillars of auscultation are hardware where to locate the surveying instrument, and shall ensure that the point-season is always the same, therefore they have to have a robust and solid construction to withstand the weight of the instrument and also endure a throughout the development of the observing campaigns.

It is customary in the pillars has some centering mechanism also forced to place the pillars of auscultation to 100-200 meters from the element to observe in order to ensure the sought accuracy. It is desirable that the pillars are outside the catchment area of the hypothetical displacement, not in vain, before formalizing auscultation itself on the different points of the structure is subjected to the pillars to a stability control that helps ensure if coordinates of the pillars can be considered fixed.

Historically in this dam, there are four pillars of observation and, although from all them can not endorse all the points covered, this is no problem as there is sufficient redundancy of pillars as can be noted in figure 1.



Figure 1. Perspective of observation area.

### Targets.

They are the elements on which the measurement are made and, therefore, they are arranged evenly by the element to inspect, so that you can extrapolate the movement of these points to the whole structure, securing to the structure or the ground by different methods. Targets used in auscultations

with angular observables, the locus where the angular pointing perform is perfectly defined by millimetre-size indentation.

In case targets used in geodetic inspections using distance, it is necessary that the point surface is reflective, that is, it returns the signal emitted by the range finder that is to say carrier wave based on which runs the distance measurement. For this purpose conventional mini-prisms, usually made by the manufacturer of instruments are used, allowing performs measurement of distances up to 800-900 meters. To fasten the reflector prism on the dam, it is usually engage the wedge in the wall of the dam through a drill and appropriate resins

The coordinates of the target network is also identified with other networks and, given the minimal influence that have small variations that occur in their coordinates for the calculation of auscultation, are considered approximate but accurate more than sufficient to calculate displacement object of study. It should also be noted that the same coordinates that are given to the targets for angular reading, give also to the reflector prism in which makes observation of distances, since signage is arranged vertically one above the other, suggesting that the coordinates planarity of both elements are very similar.

The geometric characterization of the existing dam targets is that you can see in figure 2.

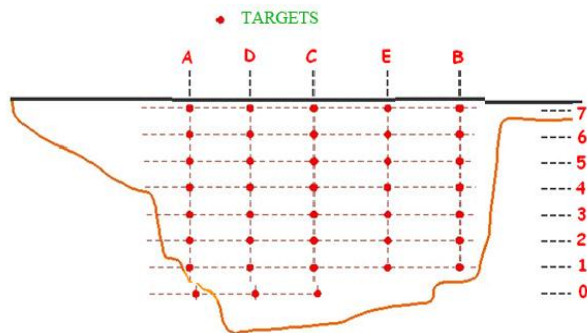


Figure 2. Targets of the Cohilla Dam.

### Security checkpoints.

The ultimate goal of these points is that the stability of the pillars of auscultation in the intervening period between observation campaigns, can be determined by also topographic methods since this will be a fact of departure for the resolution of auscultation.

The dam has a total of eight points of safety away between 100 and 300 metres from the pillars of auscultation, which in turn are far between 100 and 200 meters from the dam. This ensures that security checkpoints are totally away from the area of influence of the dam, which makes the total stability of security checkpoints and therefore, the perfect location for the control of stability of the pillars.

For distance observations the foundation is the same, except that when it projected the observation of distances were used to introduce four new security checkpoints, since the signalling to observe distances requires the prior placement of

reflective prisms. This means that security checkpoints for the observable distance may not be located in the same physical location that the pre-existing, even so, prisms were placed in the vicinity of the angular signals, for its easy location.

### Polars.

It is usually a point completely away from the area of influence, used in the angular geodesic auscultation in order to initialize the horizontal angle and subsequently obtain the angular differences between campaigns. If it also starts with the same measurement point, generates the advantage of working with similar angular values between campaigns, which reports the advantage of locate points of difficult location and eliminate errors in the limbo of the instrument, since always observed in the same area of limbo.

### Topographic instrumentation.

The object of this work is to compare different observable in the execution of geodetic inspection, in this sense the instrument used is a topographic high performance station Leica, TC2003 model, which is characterized by an electronic theodolite defined by having the following technical specifications:

- Precision in the measurement of horizontal and vertical angles. 0,5".
- Sensitivity level (electronic dual-axis compensator). 0,3".
- The telescope magnification. 30x.

This topographic station also includes a rangefinder with precision in the measurement of the distance of 1 mm 1 ppm.



Figure 3. Topographic Station TC2003.

## B. Solving methodology.

### Introducción

For processing field data regardless of obtaining the mean values of the different series made, usually four, two methodologies involved in the calculation of any auscultation:

- -Inverse intersections. This kind of intersections allow to obtain possible displacement of the pillars through

observations taken at security checkpoints. If you get a movement on any of the pillars, by the resolution of an eccentric angle gets to compare different observations with different positions of the pillar, ultimate goal of the work.

- Direct intersections. They allow to obtain the displacements that are the targets of aim, solving the different direct intersections that are formed from the pillars of auscultation

Therefore, after setting the coordinates of all the elements involved in the geodetic inspection, the first step to resolve the auscultation is check the stability of the pillars, so it is necessary to solve the set of inverse multiple intersections that are generated with the visual observation made between the pillars and safety points.

If everything is correct, both the pillars of inspection and safety points must not suffer movements for two important reasons: type of construction and location outside of the area of influence. If they do not suffer movements, angles observed throughout the different campaigns have to be the same. This condition is very easy to check and eliminates the need to start the procedure that allows to solve the set of inverse intersections, slow and laborious process.

Once it is tested the stability of the pillars of auscultation, it is necessary to solved the set of direct intersections that allows to detect the hypothetical movements of the targets. To solve this problem there are basically three methods clearly differentiated and with completely different connotations:

- Numerical method.
- Graphical method
- Method of variation of coordinates

**Method of variation of coordinates with angular measurements.**

The method of variation of coordinates with angular measurements is relatively easy to understand since it is known that for each variation in an angle evaluated from a fixed point, change the coordinates of the end point of the following form as it is shown in figure 4:

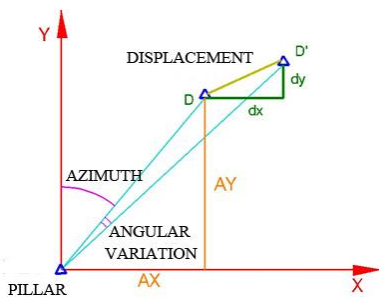


Figure 4. Variation of coordinates when a target has an angular movement

$$tg \theta_i^j = \frac{X_i - X_j}{Y_i - Y_j}; i = Pillar; j = Target$$

$$\frac{1}{Cos^2 \theta_i^j} d\theta_i^j = \frac{Y_i - Y_j}{(Y_i - Y_j)^2} dx - \frac{X_i - X_j}{(Y_i - Y_j)^2} dy$$

$$d\theta_i^j = \frac{1}{(D_i^j)^2} [\Delta Y_i^j \cdot dx - \Delta X_i^j \cdot dy]$$

Using this equation for each of the four pillars it is obtained a system of four equations with two unknown parameters that usually is resolved through a simple system of arrays:

$$\Delta \theta_I = \frac{1}{(D_I^j)^2} [\Delta Y_I^j \cdot dx - \Delta X_I^j \cdot dy]$$

$$\Delta \theta_{II} = \frac{1}{(D_{II}^j)^2} [\Delta Y_{II}^j \cdot dx - \Delta X_{II}^j \cdot dy]$$

$$\Delta \theta_{III} = \frac{1}{(D_{III}^j)^2} [\Delta Y_{III}^j \cdot dx - \Delta X_{III}^j \cdot dy]$$

$$\Delta \theta_{IV} = \frac{1}{(D_{IV}^j)^2} [\Delta Y_{IV}^j \cdot dx - \Delta X_{IV}^j \cdot dy]$$

With:

- $\Delta \theta$  .- Angular difference between campaigns.
- $D_i^j$  .- Distance between each pillar and each target.
- $\Delta X$ .- Movement in X-Axis between each pillar and each target.
- $\Delta Y$ .- Movement in Y-Axis between each pillar and each target.

Solving the following equation system, it is obtained:

$$[\Delta \theta] = [A] \cdot [X] + R$$

$$\Delta \theta = A \cdot X$$

$$A' \cdot \Delta \theta = A' \cdot A \cdot X$$

$$\left. \begin{aligned} A' \cdot \Delta\theta &= P \\ A' \cdot A &= N \end{aligned} \right\} P = N \cdot X \Rightarrow X = N^{-1} \cdot P$$

Because we usually observed more than two points of safety, there is a data redundancy which allows to set the deviations in a simple way:

$$\begin{aligned} \Delta\theta &= A \cdot X + R \\ R &= \Delta\theta - A \cdot X \end{aligned}$$

$$S^2 = \frac{R'R}{m-n}$$

with:

- m.- Number of equations = numbers of visuals from pillars
- n.- Number of unknown parameters= dx, dy

And the covariance of the unknown parameters is:

$$C = S^2 N^{-1} = \begin{pmatrix} \sigma_x^2 & \sigma_{xy} \\ \sigma_{xy} & \sigma_y^2 \end{pmatrix}$$

Applying the theory of eigenvalues and eigenvectors:

$$\sigma^2 = \frac{1}{2} \left[ \sigma_x^2 + \sigma_y^2 \pm \sqrt{(\sigma_x^2 - \sigma_y^2)^2 + 4\sigma_{xy}^2} \right] \Rightarrow \sigma \begin{cases} \sigma_{max} \\ \sigma_{min} \end{cases}$$

$$tg 2\theta = \frac{2\sigma_{xy}}{\sigma_y^2 - \sigma_x^2}$$

These parameters define the error ellipse, as can be seen in figure 5.

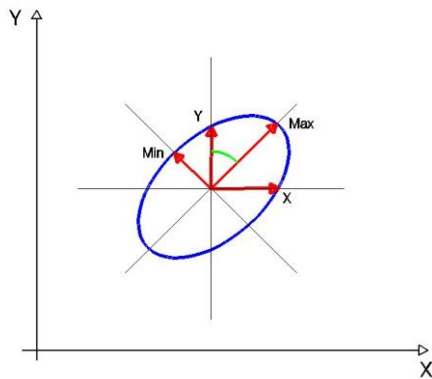


Figure 5. Error ellipse in variation of coordinates.

**Method of variation of coordinates with distance observable.**

The foundation that supports the method of variation of coordinate with distance observable is relatively easy to understand since it is known that by varying a distance

evaluated from a fixed point, change in a specific way the coordinates of the end point of the following form as it is shown in figure 6:

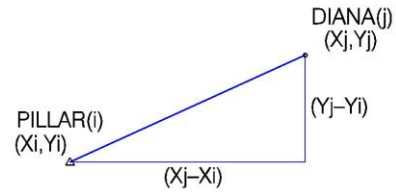


Figure 6. Position between pillar and TARGET.

The fundamental expression is:

$$D_i^j = \sqrt{(X_j - X_i)^2 + (Y_j - Y_i)^2}; i = Pillar; j = Target$$

Deriving this equation:

$$dD_i^j = \frac{(X_j - X_i) \cdot dx + (Y_j - Y_i) \cdot dy}{\sqrt{(X_j - X_i)^2 + (Y_j - Y_i)^2}}$$

Results:

$$dD_i^j = \frac{X_j - X_i}{D_i^j} dx + \frac{Y_j - Y_i}{D_i^j} dy$$

Once the topographic instrument is placed in the pillar we observed all prisms possible, making at least three or four sets of measurements, from which will be passed to a final average value of distance between pillar and target, for each campaign. After we established two distances over time, the way to operate is to obtain the variation in distance between two consecutive campaigns as a simple difference of values:

$$\Delta D = D_{PILLAR(n)}^{DIANA} = D_{PILLAR(n+1)}^{DIANA}$$

Applying the general expression of the method of variation of coordinate:

$$\Delta D_I = \frac{1}{D_I^D} [\Delta X_I^j \cdot dx + \Delta Y_I^j \cdot dy]$$

$$\Delta D_{II} = \frac{1}{D_{II}^D} [\Delta X_{II}^j \cdot dx + \Delta Y_{II}^j \cdot dy]$$

$$\Delta D_{III} = \frac{1}{D_{III}^D} [\Delta X_{III}^j \cdot dx + \Delta Y_{III}^j \cdot dy]$$



$$\Delta D_{IV} = \frac{1}{D_{IV}^D} [\Delta X_{IV}^j \cdot dx + \Delta Y_{IV}^j \cdot dy]$$

The resolution is based on a matrix system of type:

$$[\Delta D] = [A] \cdot [X] + R$$

This allows a resolution similar to the one used in the resolution with the measurement of angles.

### III. Results and discussion.

#### A. Introduction

To make the contrast proposed in this paper is necessary to take the observable angle and distance in two campaigns of geodetic inspection, numbers 76 and 77 respectively in the history of the “La Cohilla” Dam, and compare the results obtained with both observables.

The conditions in the dam in the moments in which the observation were made are mainly determined by the temperature at the time of observation and the height of water stored, being these parameters which basically defined the pressure that the dam is subjected and, therefore, the deformation of the dam between both campaign. The conditions of campaigns we used to contrast both observables are:

- CAMPAIGN 76: Observation dates: 23-24 May 2008. Average height of water stored: 58,50 m. Average Temperature: 14 °C.
- CAMPAIGN 77: Observation dates: 26 y 27 February 2009. Average height of water stored: 42,25 m. Average Temperature: 6 °C.

#### B. Auscultation results.

##### Results with angular observable.

Once the instruments, involved networks and methodologies of observation auscultation with observable angle calculation are defined we are going to explain the particular case we have studied in this article.

In table I are shown the displacements obtained for the aiming targets, these displacements are decomposed in the X and Y axis, polar displacement and data relating to the ellipses of error, major and minor axis and azimuth of the axis.

##### Results with distance as observable.

Operating similarly in the case of the observable distance, we obtained the following displacement as we can see in table II.

TABLE I. DISPLACEMENT AND ERROR ELLIPSES OF THE TARGETS WITH ANGULAR OBSERVABLE.

Dam:	Cohilla		Campaign:				77-76	
RESULTS	Observable:				Angle			
TARGETS OF THE COLUMN A-D								
POINT	Displacement				Error ellipses			
	dX (mm)	dY (mm)	Despl. (mm)	Azimuth (grad)	σ max (mm)	σ min (mm)	Azimuth (grad)	
7A	1,87	-2,13	2,83	154,0782	0,15	0,05	-18,6983	
6A	1,34	-1,67	2,14	156,9246	0,31	0,09	-0,1194	
5A	1,23	-1,66	2,07	159,3829	0,37	0,10	0,2920	
4A	1,15	-1,50	1,89	158,5570	0,28	0,08	1,3872	
3A	1,05	-1,32	1,68	157,2466	0,41	0,14	3,3666	
2A	-0,21	0,08	0,22	323,9650	--	--	--	
1A	--	--	--	--	--	--	--	
0A	--	--	--	--	--	--	--	
1D	0,25	-0,27	0,37	153,2470	0,01	0,00	13,9709	
2D	0,31	-0,80	0,86	176,1374	0,23	0,10	12,1930	
3D	0,60	-1,59	1,70	177,1398	0,23	0,09	12,7367	
4D	0,77	-1,88	2,03	175,3354	0,19	0,08	13,1171	
5D	0,88	-2,34	2,50	177,0011	0,26	0,10	13,4261	
6D	1,10	-2,80	3,00	176,1403	0,40	0,16	13,5604	
7D	1,21	-2,88	3,12	174,6834	0,53	0,21	13,3223	
TARGETS OF THE COLUMN C								
POINT	Displacement				Error ellipses			
	dX (mm)	dY (mm)	Despl. (mm)	Azimuth (grad)	σ max (mm)	σ min (mm)	Azimuth (grad)	
7C	-0,34	-3,20	3,22	206,8118	0,33	0,16	21,3628	
6C	0,23	-3,29	3,30	195,5209	--	--	--	
5C	-0,02	-2,42	2,42	200,6398	0,34	0,16	21,5946	
4C	0,07	-2,31	2,31	198,1366	0,36	0,20	16,7579	
3C	-0,06	-1,64	1,64	202,2697	0,36	0,17	20,7118	
2C	0,09	-1,21	1,21	195,2559	0,29	0,13	20,4899	
1C	0,03	-0,67	0,67	196,7835	0,30	0,14	20,9249	
0C	0,14	0,24	0,28	34,1702	0,16	0,08	23,3968	
TARGETS OF THE COLUMN B-E								
POINT	Displacement				Error ellipses			
	dX (mm)	dY (mm)	Despl. (mm)	Azimuth (grad)	σ max (mm)	σ min (mm)	Azimuth (grad)	
1E	-0,20	-0,32	0,38	235,7529	0,08	0,04	26,0778	
2E	-0,37	-0,99	1,06	222,5338	0,43	0,22	26,0066	
3E	-0,53	-1,32	1,43	224,3304	0,38	0,19	26,0119	
4E	-0,62	-1,64	1,75	222,9638	0,34	0,17	26,1013	
5E	-0,80	-2,24	2,38	221,7261	0,42	0,22	26,1020	
6E	-0,82	-2,49	2,62	220,1723	0,50	0,26	26,1051	
7E	-0,99	-2,84	3,01	221,3686	0,57	0,30	26,1958	
7B	-1,46	-2,24	2,67	236,7619	0,27	0,14	29,8637	
6B	-1,03	-1,79	2,07	233,2849	0,33	0,17	29,8331	
5B	-0,96	-1,47	1,76	236,6420	0,16	0,08	29,7073	
4B	-0,87	-1,15	1,44	241,1493	0,22	0,08	-49,2341	
3B	-0,71	-0,93	1,17	241,3725	0,36	0,13	-49,2452	
2B	-0,34	-0,43	0,55	242,6991	0,15	0,08	28,5743	
1B	-0,15	-0,24	0,28	235,3996	0,13	0,05	-49,1118	

#### C. Results of the contrast.

Once auscultation is resolved with different observables, method of observation and calculation, there is now need compare the different results obtained with the purpose of establishing whether the different observables and proposed methods solve the problem with the requirements of precision previously marked. In this line, we intend a contrast of results by two completely different techniques in terms of approach, resolution and type of results.

TABLE II. DISPLACEMENT AND ERROR ELLIPSES OF THE TARGETS WITH DISTANCE OBSERVABLE.

Dam: Cohilla		Campaign: 77-76					
RESULTS		Observable:				Angle	
TARGETS OF THE COLUMN A-D							
POINT	Displacement				Error ellipses		
	dX (mm)	dY (mm)	Despl. (mm)	Azimuth (grad)	$\sigma$ max (mm)	$\sigma$ min (mm)	Azimuth (grad)
7A	1,87	-2,13	2,83	154,0782	0,15	0,05	-18,6983
6A	1,34	-1,67	2,14	156,9246	0,31	0,09	-0,1194
5A	1,23	-1,66	2,07	159,3829	0,37	0,10	0,2920
4A	1,15	-1,50	1,89	158,5570	0,28	0,08	1,3872
3A	1,05	-1,32	1,68	157,2466	0,41	0,14	3,3666
2A	-0,21	0,08	0,22	323,9650	--	--	--
1A	--	--	--	--	--	--	--
0A	--	--	--	--	--	--	--
1D	0,25	-0,27	0,37	153,2470	0,01	0,00	13,9709
2D	0,31	-0,80	0,86	176,1374	0,23	0,10	12,1930
3D	0,60	-1,59	1,70	177,1398	0,23	0,09	12,7367
4D	0,77	-1,88	2,03	175,3354	0,19	0,08	13,1171
5D	0,88	-2,34	2,50	177,0011	0,26	0,10	13,4261
6D	1,10	-2,80	3,00	176,1403	0,40	0,16	13,5604
7D	1,21	-2,88	3,12	174,6834	0,53	0,21	13,3223
TARGETS OF THE COLUMN C							
POINT	Displacement				Error ellipses		
	dX (mm)	dY (mm)	Despl. (mm)	Azimuth (grad)	$\sigma$ max (mm)	$\sigma$ min (mm)	Azimuth (grad)
7C	-0,34	-3,20	3,22	206,8118	0,33	0,16	21,3628
6C	0,23	-3,29	3,30	195,5209	--	--	--
5C	-0,02	-2,42	2,42	200,6398	0,34	0,16	21,5946
4C	0,07	-2,31	2,31	198,1366	0,36	0,20	16,7579
3C	-0,06	-1,64	1,64	202,2697	0,36	0,17	20,7118
2C	0,09	-1,21	1,21	195,2559	0,29	0,13	20,4899
1C	0,03	-0,67	0,67	196,7835	0,30	0,14	20,9249
0C	0,14	0,24	0,28	34,1702	0,16	0,08	23,3968
TARGETS OF THE COLUMN B-E							
POINT	Displacement				Error ellipses		
	dX (mm)	dY (mm)	Despl. (mm)	Azimuth (grad)	$\sigma$ max (mm)	$\sigma$ min (mm)	Azimuth (grad)
1E	-0,20	-0,32	0,38	235,7529	0,08	0,04	26,0778
2E	-0,37	-0,99	1,06	222,5338	0,43	0,22	26,0066
3E	-0,53	-1,32	1,43	224,3304	0,38	0,19	26,0119
4E	-0,62	-1,64	1,75	222,9638	0,34	0,17	26,1013
5E	-0,80	-2,24	2,38	221,7261	0,42	0,22	26,1020
6E	-0,82	-2,49	2,62	220,1723	0,50	0,26	26,1051
7E	-0,99	-2,84	3,01	221,3686	0,57	0,30	26,1958
7B	-1,46	-2,24	2,67	236,7619	0,27	0,14	29,8637
6B	-1,03	-1,79	2,07	233,2849	0,33	0,17	29,8331
5B	-0,96	-1,47	1,76	236,6420	0,16	0,08	29,7073
4B	-0,87	-1,15	1,44	241,1493	0,22	0,08	-49,2341
3B	-0,71	-0,93	1,17	241,3725	0,36	0,13	-49,2452
2B	-0,34	-0,43	0,55	242,6991	0,15	0,08	28,5743
1B	-0,15	-0,24	0,28	235,3996	0,13	0,05	-49,1118

**Numerical and Graphical contrast of the results.**

The first contrast carried out consists of both numerical and graphical analysis of the results obtained for the different targets that have been observed by both techniques. This is not a statistical method, but it will see, in a first approach, the order of magnitude of the results and therefore establish visual considerations that depending on the magnitude of the differences, will enable initial conclusions that later can be contrasted with a quantitative method. The next table and graphics show by way of example the first numeric and then graphic contrast of row number 4.

TABLE III. DISPLACEMENTS AND ACCURACY (ROW 4).

TARGET 4A	Displacements				Error Ellipses		
	dX (mm)	dY (mm)	Despl. (mm)	Azimuth (grad)	$\sigma$ max (mm)	$\sigma$ min (mm)	Azimuth (grad)
Angles	1,15	-1,50	1,89	158,5570	0,28	0,08	1,3872
Distances	1,11	-1,57	1,93	160,8122	--	--	--
TARGET 4D	dX (mm)	dY (mm)	Despl. (mm)	Azimuth (grad)	$\sigma$ max (mm)	$\sigma$ min (mm)	Azimuth (grad)
Angles	0,77	-1,88	2,03	175,3354	0,19	0,08	13,1171
Distances	0,62	-1,76	1,87	178,3681	0,05	0,02	-13,5636
TARGET 4C	dX (mm)	dY (mm)	Despl. (mm)	Azimuth (grad)	$\sigma$ max (mm)	$\sigma$ min (mm)	Azimuth (grad)
Angles	0,07	-2,31	2,31	198,1366	0,36	0,20	16,7579
Distances	-0,18	-2,06	2,07	205,4814	0,39	0,19	10,9075
TARGET 4E	dX (mm)	dY (mm)	Despl. (mm)	Azimuth (grad)	$\sigma$ max (mm)	$\sigma$ min (mm)	Azimuth (grad)
Angles	-0,62	-1,64	1,75	222,9638	0,34	0,17	26,1013
Distances	-0,41	-1,42	1,48	217,7221	0,39	0,19	25,5984
TARGET 4B	dX (mm)	dY (mm)	Despl. (mm)	Azimuth (grad)	$\sigma$ max (mm)	$\sigma$ min (mm)	Azimuth (grad)
Angles	-0,87	-1,15	1,44	241,1493	0,22	0,08	-49,2341
Distances	-0,91	-1,29	1,58	239,1117	0,43	0,15	-48,3348

In Figure 7 is represented, in blue, displacements obtained with the angular observable and, in red, displacements with observable distance.

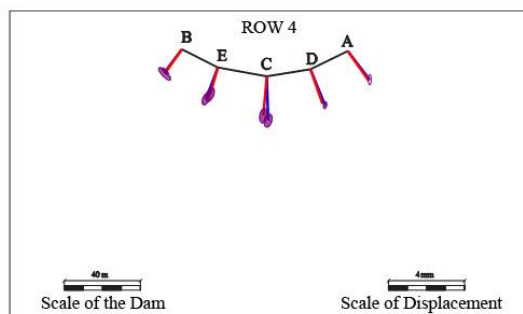


Figure 7. Displacements y Error Ellipses with both observables (Row 4).

In the same way that in the row grouping is also structured by columns in order to obtain a graphical representation that allows to be easily interpreted and the determination of systematic errors more easily.

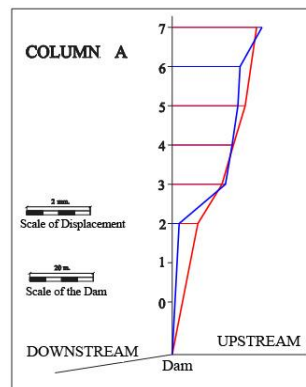


Figure 8. Displacements obtained with both observables (Column A).

TABLE IV. DISPLACEMENTS AND ACCURACY (COLUMN A).

TARGET 7A	Displacements				Error Ellipses		
	dX (mm)	dY (mm)	Despl. (mm)	Azimuth (grad)	σ max (mm)	σ min (mm)	Azimuth (grad)
Angles	1,87	-2,13	2,83	154,0782	0,15	0,05	-18,6983
Distances	1,65	-2,09	2,66	157,5303	--	--	--
TARGET 6A	dX (mm)	dY (mm)	Despl. (mm)	Azimuth (grad)	σ max (mm)	σ min (mm)	Azimuth (grad)
Angles	1,34	-1,67	2,14	156,9246	0,31	0,09	-0,1194
Distances	--	--	--	--	--	--	--
TARGET 5A	dX (mm)	dY (mm)	Despl. (mm)	Azimuth (grad)	σ max (mm)	σ min (mm)	Azimuth (grad)
Angles	1,23	-1,66	2,07	159,3829	0,37	0,10	0,2920
Distances	1,30	-1,90	2,30	161,7750	0,42	0,17	-18,9706
TARGET 4A	dX (mm)	dY (mm)	Despl. (mm)	Azimuth (grad)	σ max (mm)	σ min (mm)	Azimuth (grad)
Angles	1,15	-1,50	1,89	158,5570	0,28	0,08	1,3872
Distances	1,11	-1,57	1,93	160,8122	--	--	--
TARGET 3A	dX (mm)	dY (mm)	Despl. (mm)	Azimuth (grad)	σ max (mm)	σ min (mm)	Azimuth (grad)
Angles	1,05	-1,32	1,68	157,2466	0,41	0,14	3,3666
Distances	0,87	-1,31	1,57	162,8245	--	--	--
TARGET 2A	dX (mm)	dY (mm)	Despl. (mm)	Azimuth (grad)	σ max (mm)	σ min (mm)	Azimuth (grad)
Angles	-0,21	0,08	0,22	323,9650	--	--	--
Distances	0,56	-0,57	0,80	150,4797	--	--	--

**Statistical Contrast.**

For the implementation of the statistical contrast is required to define which of both observable generates more accurate results and, therefore, which of the two will be taken as a pattern in the contrast. In this sense, the criterion used to define what is the more accurate observable, relies on the determination of the error ellipses, taking how best observable one whose average areas of ellipses is smaller. To do this is necessary to calculate the areas of ellipses using the expression:

$$Area = \pi \cdot a \cdot b = \pi \cdot \sigma_{max} \cdot \sigma_{min}$$

obtaining the values that are shown in table 5:

Set out the different areas of the error ellipses for each technique, different estimators statistical indicators are obtained and allow to make the decision about which observable is more accurate results.

TABLE V. VALUE OF THE ERROR ELLIPSES AREAS FOR DIFFERENT OBSERVABLES

Dam: Cohilla	77-76			77-76		
	Angulars Results			Distance Results		
	ERROR ELLIPSE			ERROR ELLIPSE		
POINT	σ max (mm)	σ min (mm)	Ellipse Area (mm <sup>2</sup> )	σ max (mm)	σ min (mm)	Ellipse Area (mm <sup>2</sup> )
7A	0,15	0,05	0,0252	--	--	--
6A	0,31	0,09	0,0851	--	--	--
5A	0,37	0,10	0,1199	0,42	0,17	0,2239
4A	0,28	0,08	0,0718	--	--	--
3A	0,41	0,14	0,1769	--	--	--
2A	--	--	--	--	--	--
1A	--	--	--	--	--	--
0A	--	--	--	--	--	--
1D	0,01	0,00	0,0001	--	--	--
2D	0,23	0,10	0,0747	--	--	--
3D	0,23	0,09	0,0657	0,10	0,05	0,0155
4D	0,19	0,08	0,0453	0,05	0,02	0,0038
5D	0,26	0,10	0,0840	0,04	0,02	0,0031
6D	0,40	0,16	0,2031	0,09	0,04	0,0112
7D	0,53	0,21	0,3562	--	--	--
7C	0,33	0,16	0,1675	0,36	0,14	0,1593
6C	--	--	--	--	--	--
5C	0,34	0,16	0,1723	0,37	0,18	0,2032
4C	0,36	0,20	0,2250	0,39	0,19	0,2256
3C	0,36	0,17	0,1878	0,15	0,07	0,0332
2C	0,29	0,13	0,1217	0,08	0,04	0,0102
1C	0,30	0,14	0,1303	--	--	--
0C	0,16	0,08	0,0405	0,13	0,05	0,0224
1E	0,08	0,04	0,0114	0,31	0,15	0,1480
2E	0,43	0,22	0,3005	--	--	--
3E	0,38	0,19	0,2330	0,50	0,24	0,3858
4E	0,34	0,17	0,1821	0,39	0,19	0,2288
5E	0,42	0,22	0,2855	0,39	0,19	0,2400
6E	0,50	0,26	0,4083	0,45	0,22	0,3174
7E	0,57	0,30	0,5315	0,73	0,36	0,8215
7B	0,27	0,14	0,1150	--	--	--
6B	0,33	0,17	0,1750	--	--	--
5B	0,16	0,08	0,0387	0,20	0,10	0,0592
4B	0,22	0,08	0,0564	0,43	0,15	0,2073
3B	0,36	0,13	0,1510	--	--	--
2B	0,15	0,08	0,0397	--	--	--
1B	0,13	0,05	0,0217	0,05	0,02	0,0034

TABLE VI. CONTRAST BETWEEN THE RESULTS OBTAINED IN THE ERROR ELLIPSES AREAS.

	ELLIPSES AREA (mm <sup>2</sup> )	
	ANGULAR OBSERVABLE	DISTANCE OBSERVABLE
Sample Size	33	20
Sample Mean	0,149	0,166
Sample Variance	0,015	0,038
Standard Deviation	0,120	0,190
Sampling mean error	0,021	0,042

Analyzing the results obtained (Table 6), you can determine that all statistical indicators show that the observable more accurate, and which should be used as a pattern in the statistical contrast is therefore which is based on the classical angular observation.

The first requirement is to determine the difference between the resulting vectors with the observed angle and distance, so there is a difference between vectors that take into account module of the displacement and direction of this. For this reason, the difference between displacements is determined as the quadratic component of the difference of coordinate increases.

Table VII shows the results obtained for calculating the differences between vectors obtained from the angular displacement and displacement of distance.

Most of the phenomena that appear in nature involve different variables, and so can be said that many are related with several variables. If X and Y are random variables (discrete or continuous), distribution that shows the joint behavior of both variables is known as distribution bivariate, multi-variant distribution is called when you have more than two variables.

This distribution describes the joint behavior of two Gaussian variables, X and Y, which are defined by the expression:

$$f(x, y) = \frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-\rho^2}} \exp\left\{-\frac{1}{2(1-\rho^2)}\left[\left(\frac{x-\mu_x}{\sigma_x}\right)^2 + \left(\frac{y-\mu_y}{\sigma_y}\right)^2 - 2\rho\left(\frac{x-\mu_x}{\sigma_x}\right)\left(\frac{y-\mu_y}{\sigma_y}\right)\right]\right\}$$

This function defines a surface on the X-Y plane instead of a curve about the x-axis, as happens in the one-dimensional. In the bivariate distributions the probability corresponds geometrically with the volume under the surface, this is the condition for the bivariate distribution:

$$\iint_{x,y} f(x, y) dy dx = 1, \quad f(x, y) \geq 0$$

To solve the hypothesis testing is necessary to calculate, based on the results of the vector differences established, statistical values such as sample size, the sample mean, variance and standard deviation, whose results are shown in table VIII.

From the determination of these values can be graphically rebuilt the bivariate distribution, as shown in Figure 9:

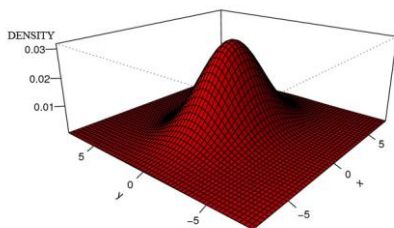


Figure 9. Bivariate distribution used in angle and distance contrast.

To set the accuracy to different levels of confidence, it is necessary to work with the variances. In this case, as in all bivariate distributions, there are two, one for the variable X and one for the variable Y. If the differences between the two are not important you can work with the arithmetic mean, and if you want to be strict, you can work with the larger of the two, being on the side of safety. Once established the parameters of the distribution for different levels of confidence and given the variances, the determination of the accuracy for this level of confidence is determined by the following expression:

$$r_{95\%} = a_{0,05} \cdot \sigma = 2,4477 \cdot \sqrt{0,058} = 0,59mm.$$

$$r_{99\%} = a_{0,01} \cdot \sigma = 3,0349 \cdot \sqrt{0,058} = 0,73mm.$$

allowing you to set, for 95% or 99%, the difference average between the vectors 0.59 and 0.73 mm, respectively, which fits perfectly into the accuracy established for this type of work and validates any of the two methods for the resolution of auscultation.

By reversing the problem, it is now set for precision geodetic auscultation marked, the procedure that allows to calculate the likelihood that the resulting value is lower.

$$a \cdot \sigma = 1 \text{ milímetro} \Rightarrow a = \frac{1}{\sigma} = \frac{1}{\sqrt{0,058}} = 4,1523$$

$$\alpha = \exp\left(\frac{-a^2}{2}\right) = \exp\left(\frac{-4,1523^2}{2}\right) = 0,0002 \Rightarrow 99,98\%$$

which means that there is only a 0.02% probability that the difference between the vectors is greater than 1 mm. This allows us to validate these methods for the implementation of auscultation with accuracy 1 mm.

## iv. Conclusions.

The first great aim of the study was to assess and compare the displacements obtained to perform a classical geodetic inspection, measuring angles first and distances after that, since both observables reported some displacements with similar precision. This is ratified with the numerical and graphic contrast that generates results of guidance, although highly illustrative, the results achieved with the two observables are extremely similar.

The statistical results obtained as a result of such a comparison can ensure that for 95% or 99% probability, the average between the vectors is 0.59 to 0.73 mm respectively, which fits perfectly into marked accuracy for this type of work and validates either of the two methods for the resolution of auscultation. By reversing the problem, you can set that there is only a 0.02% probability that the difference between the vectors is greater than 1 mm, which again confirms the



validation of these methods for the implementation of auscultation.

TABLE VII. DIFFERENCES BETWEEN DISPLACEMENTS RETRIEVED WITH ANGULAR AND DISTANCE OBSERVABLE.

Dam:	Cohilla		CONTRAST: ANGULAR AND DISTANCE OBSERVABLE				Campaign:	77-76	
RESULTS									
TARGET AIMS									
POINT	ANGLES			DISTANCES			DIFERENCES		
	dX (mm)	dY (mm)	Despl. (mm)	dX (mm)	dY (mm)	Despl. (mm)	Difference Coord_X	Difference Coord_Y	Difference TOTAL
7A	1,87	-2,13	2,83	1,65	-2,09	2,66	0,23	-0,04	0,23
6A	1,34	-1,67	2,14	--	--	--	--	--	--
5A	1,23	-1,66	2,07	1,30	-1,90	2,30	-0,07	0,24	0,25
4A	1,15	-1,50	1,89	1,11	-1,57	1,93	0,03	0,07	0,08
3A	1,05	-1,32	1,68	0,87	-1,31	1,57	0,18	-0,01	0,18
2A	-0,21	0,08	0,22	0,56	-0,57	0,80	-0,77	0,65	1,01
1A	--	--	--	--	--	--	--	--	--
0A	--	--	--	--	--	--	--	--	--
1D	0,25	-0,27	0,37	--	--	--	--	--	--
2D	0,31	-0,80	0,86	--	--	--	--	--	--
3D	0,60	-1,59	1,70	0,42	-1,19	1,26	0,18	-0,40	0,44
4D	0,77	-1,88	2,03	0,62	-1,76	1,87	0,15	-0,12	0,19
5D	0,88	-2,34	2,50	0,95	-2,21	2,41	-0,07	-0,13	0,14
6D	1,10	-2,80	3,00	0,98	-2,37	2,57	0,12	-0,42	0,44
7D	1,21	-2,88	3,12	1,12	-2,75	2,96	0,09	-0,13	0,16
7C	-0,34	-3,20	3,22	-0,04	-3,15	3,15	-0,30	-0,04	0,31
6C	0,23	-3,29	3,30	0,22	-3,04	3,05	0,01	-0,25	0,25
5C	-0,02	-2,42	2,42	-0,23	-2,39	2,40	0,20	-0,03	0,21
4C	0,07	-2,31	2,31	-0,18	-2,06	2,07	0,25	-0,25	0,35
3C	-0,06	-1,64	1,64	-0,05	-1,41	1,41	-0,01	-0,23	0,23
2C	0,09	-1,21	1,21	-0,07	-1,06	1,06	0,16	-0,15	0,22
1C	0,03	-0,67	0,67	0,11	-0,80	0,81	-0,08	0,13	0,15
0C	0,14	0,24	0,28	-0,12	-0,23	0,26	0,27	0,47	0,54
1E	-0,20	-0,32	0,38	-0,10	-0,48	0,49	-0,10	0,16	0,19
2E	-0,37	-0,99	1,06	-0,25	-0,69	0,73	-0,12	-0,30	0,32
3E	-0,53	-1,32	1,43	-0,41	-1,31	1,38	-0,12	-0,01	0,12
4E	-0,62	-1,64	1,75	-0,41	-1,42	1,48	-0,21	-0,21	0,30
5E	-0,80	-2,24	2,38	-0,60	-1,98	2,07	-0,20	-0,26	0,33
6E	-0,82	-2,49	2,62	-0,74	-2,46	2,57	-0,08	-0,03	0,08
7E	-0,99	-2,84	3,01	-1,04	-2,83	3,02	0,05	-0,01	0,05
7B	-1,46	-2,24	2,67	--	--	--	--	--	--
6B	-1,03	-1,79	2,07	-1,33	-1,94	2,35	0,30	0,15	0,33
5B	-0,96	-1,47	1,76	-1,16	-1,76	2,11	0,21	0,28	0,35
4B	-0,87	-1,15	1,44	-0,91	-1,29	1,58	0,04	0,14	0,15
3B	-0,71	-0,93	1,17	-0,67	-1,04	1,23	-0,04	0,10	0,11
2B	-0,34	-0,43	0,55	-0,44	-0,57	0,72	0,10	0,14	0,17
1B	-0,15	-0,24	0,28	-0,30	-0,42	0,52	0,15	0,18	0,23

TABLE VIII. STATISTICS OF THE CONTRAST.

<b>CONTRAST: THEODOLITE-ANGLES DISTANCEMETER-DISTANCES</b>			
		$\square X$	$\square Y$
<b>SAMPLE</b>		31	31
<b>MEAN</b>		0,018	-0,010
<b>VARIANCE</b>		0,045	0,058
<b>STANDARD DEVIATION</b>		0,209	0,236

When it is planned a geodetic auscultation, the first approach which should be solved is to define the kind of observable to measure. Decide to measure angles or distances is not always possible, because sometimes the desired instrumentation is not available, especially at the levels of precision in which the instruments capable of conducting auscultation moves. Once selected from among Theodolites and distancemeters available, determine the error expected with both instruments is feasible, and therefore choose the most accurate, also.

Equal in this case, i.e. having a 1 mm rangefinder 1 ppm and a theodolite 0.5" angular appreciation, the election, as demonstrated in this paper, is difficult to adopt, then being able to take into account other considerations to determine the observable to measure. Among those considerations, and always respecting the accuracy as goal, can take into account observation of angles is much more demanding than the distances because the first one has a heavy dependence on the observer. However if you 'see' the target it is sufficient to make the measurement with distances as long as the line of sight must be sufficiently normal so the rangefinder can measure distance, in this case taking comments very little dependence of the observer. This results in that the amount of measurements is greater if you observe angles than if you observe distances, such as in de present research work.

## REFERENCES.

- [1] ABRAMOVICH, M.; STEGUN, I. (1972). "Handbook of mathematical functions". Dover Publicantions, Inc. New York.
- [2] BIEVRE, G.; NORGEOT, C. (2005). "Utilisation des methodes geophysiques pour l'auscultation des digues en eau. Etude de cas sur le Canal du Centre (France)". Bulletin des Laboratoires des Ponts et Chaussees, N°254, pp: 85-107.
- [3] CHUECA, M. et al. (1994). "Redes topográficas y locales". Servicio de Publicaciones de la Universidad Politécnica de Valencia.
- [4] DE LUIS RUIZ, J.M. et al. (2005). "Geodesic inspections using distance measurements. Application to La Cohilla Dam (Cantabria, Spain)". Geodetic deformation monitoring: from geophysical to engineering roles. Pp: 270-277.
- [5] DE LUIS RUIZ, J.M. (2010). "Contraste en la ejecución de auscultaciones geodésicas por métodos clásicos y con láser escáner". Universidad de Cantabria.
- [6] FAN, H. (2005). "Theoretical Geodesy". Royal Institute of Technology. Stockholm.
- [7] FAN, H. (2005). "Theory of Errors and Least Squares Adjustment". Royal Institute of Technology. Stockholm.
- [8] FERRER, R. (1992). "Mediciones en torno a pequeños Displacements que se producen en estructuras y suelos de marcado interés en la

Ingeniería Civil (Dams, muros y taludes)". Servicio de Publicaciones de la E.T.S.I. de Caminos, Canales y Puertos de Santander.

- [9] FERRER, R.; PIÑA, B. (1996). "Topografía aplicada a la Ingeniería". Instituto Geográfico Nacional de Madrid.
- [10] FERRER, R.; VALVERDE, A. (1998) "Cartografía, Geodesia y Fotogrametría". Servicio de Publicaciones de la E.T.S.I. de Caminos, Canales y Puertos de Santander.
- [11] LOPEZ-CUERVO, S. (1993). "Topografía". Mundi-prensa. Madrid.
- [12] MARTIN, L. (1987). "Topografía y replanteos". Editorial Romargraf. Barcelona.
- [13] MORITZ, H. (1984). "Sistemas de referencia en Geodesia". Curso de Geodesia Superior. (Instituto de Astronomía y Geodesia). Madrid.
- [14] MUSSIO, L. (1987). "Estrategias de cálculo del método de colocación y ejemplos calculados". IV Curso de Geodesia Superior. (Instituto Geográfico Nacional). Madrid.
- [15] OJEDA, J. L. (1984). "Métodos topográficos y oficina técnica". Edición del autor. Madrid.
- [16] SANDOVER, J.A. (1982). "Topografía". Editorial Continental. México.
- [17] SANTOS, A. (1993). "Replanteo y control de Dams de embalse". Edición del Colegio de Ingenieros Técnicos en Topografía de Madrid.
- [18] SEVILLA, M.J. (1987). "Colocación mínimos cuadrados". IV Curso de Geodesia Superior. (Instituto de Astronomía y Geofísica). Madrid.
- [19] SUMDAKOV, Y.A. (1980). "Trabajos geodésicos en la construcción de grandes obras industriales y altos edificios". Editorial Mir. Moscú.
- [20] TORGE, W. (1991). "Geodesy. 2nd Edition". Walter de Gruyter.
- [21] VALDES, F. (1993). "Topografía". CEAC. Madrid.
- [22] WOLFGANG, J. (1983). "Geodesia". Editorial TARGET. México.
- [23] ZAKATOV, P.S. (1981). "Curso de Geodesia Superior". Editorial MIR. Moscú.