

# Numerical analysis on Distributions of Temperature and Flow Velocity in an Inertance Pulse Tube Cryocooler

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**Abstract**— The inertance pulse tube cryocooler (IPTC) is well known as one of the most effective cryocooler for the cooling of infrared imaging detectors and superconducting technologies. The reciprocating flow in an IPTC was modeled with a computational thermal fluid dynamics. Two dimensional modeling consists of the flows in the cylinder of compressor, a regenerator, a pulse tube, heat exchangers, an inertance tube, and a gas reservoir. The user-defined function was applied to the compression and expansion stroke of piston in the compressor. It was confirmed that the temperature at the cold heat exchanger decayed to the lowest temperature with an oscillating convergence due to the compression and expansion of piston in the compressor. The spatial distributions of temperature and flow velocity are also compared for the piston at the top dead center and bottom dead center during compression and expansion strokes.

**Keywords**—inertance, pulse tube, cryocooler, reciprocating flow

## I. Introduction

The cryocoolers for very low temperature below 100K have been widely used for spacecraft missions, military purposes, radio frequency filter for super conduction, and medical equipment. For the high sensitivity in an infrared imaging system or MRI, the cryogenic cooling is required to reduce the background photon noise and any residual dark current or thermal noise induced in the detector itself [1]. In various types of cryocooler, the pulse tube cryocooler (PTC) has many advantages in enough reliability and durability for spacecraft application due to its low vibration and noise level[2]

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It repeats the compression and expansion of refrigerant by a gas piston without any moving parts in the cold stage. There are two types of approach for the theoretical analysis on the PTC: the thermo-acoustic analysis and the multi-dimensional numerical analysis. The former can suggest the physical process and predict the overall system performance, consequently optimize the design of PTC system.

However, it is not sufficient to understand the complicated fluid field and thermal energy transfer in the components of system due to complex secondary flows which can affect the capability of the PTC. Lee et al. [3] compared several different modeling approaches qualitatively and concluded that a complete solution to the 2-D differential equations is the most appropriate. Potratz [4] showed that a 2-D axisymmetric CFD(Computational Fluid Dynamics) model was useful during the process of identifying and correcting flow mal-distribution problems. Cha [5] showed that the implementation of a 2-D axisymmetric model using a commercial CFD code (FLUENT) could predict the second order effects correctly. Taylor et al. [6] focused on the development of a 2-D axisymmetric CFD model of the pulse tube and flow transitioning components within a PTC with the goal of developing a design tool that can be applied expeditiously to the component within the overall system.

## II. Modeling and Numerical conditions

### A. Configuration of IPTC system

The IPTC system for the numerical analysis consists of following: a compressor produces the sinusoidal pressure wave with a displacer, a transfer line which connects the compressor and heat exchanger, a warm end heat exchanger (WHX1) which removes the heat produced during compression of refrigerant in the compressor. The regenerator which has to transfer the maximum enthalpy flow to the pulse tube locates between the WHX1 and a cold end heat exchanger (CHX) which absorbs the cooling load. A pulse tube is connected to CHX for the transfer of the enthalpy flow to another warm end heat exchanger (WHX2) which cools down the heat at the end of pulse tube. An inertance to control the phase of pressure wave and mass flow locates between CHX and the reservoir as a buffer volume. The schematic configuration of IPTC is illustrated in Fig.1. A linear compressor was adopted in this IPTC model as the compressor

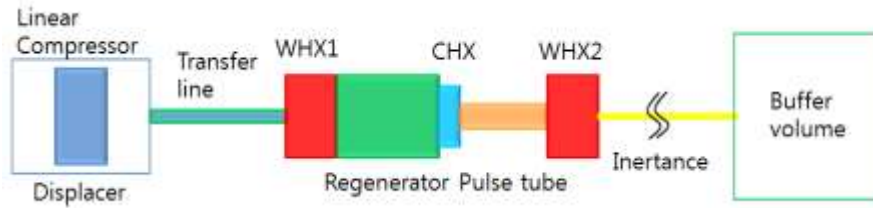


Fig. 1 Schematic configuration of the inertance pulse tube cryocooler

for refrigerant and was simplified only with the displacer (piston) and cylinder.

### B. Governing equations

Two-dimensional compressible fluid model is applied in this calculation. The governing equations are as follows.

The continuity equation is given by

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{u}) = 0 \quad (1)$$

where  $\rho$  is density,  $t$  is time, and  $\vec{u}$  is velocity vector.

The axial and radial momentum conservation of equation :

$$\frac{\partial (\rho \vec{u})}{\partial t} + \nabla \cdot (\rho \vec{u} \vec{u}) + \nabla P - \nabla \cdot (\tau) = 0 \quad (2)$$

where  $P$  is static pressure and  $\tau$  is stress tensors.

Assuming that the working fluid is Newtonian, the constitutive relation for shear stress-strain rate is :

$$\tau = \mu \left\{ \langle \nabla \vec{u} + \nabla \vec{u}^T \rangle - \frac{2}{3} \nabla \cdot \vec{u} I \right\} \quad (3)$$

where  $\mu$  is viscosity and  $I$  is unit tensor

The energy conservation:

$$\nabla \cdot (k \nabla T + \tau \cdot \vec{u}) - \frac{\partial (\rho E)}{\partial t} - \nabla \cdot (\vec{u} (\rho E + P)) = 0 \quad (4)$$

Table 1 Dimensions of each component

Components	Radius(mm)	Length(mm)
Linear compressor	9.54	7.5
Transfer line	1.55	101.0
WHX 1	4.0	20.0
Regenerator	4.0	38.1
CHX	3.0	5.7
Pulse tube	2.5	60.0
WHX 2	4.0	10.0
Inertance tube	0.425	684.1
Buffer volume	13.0	130.0

where  $k$  is thermal conductivity and  $T$  is temperature.

$$E = h - \frac{P}{\rho} + \frac{u^2}{2} \quad (5)$$

where  $h$  is enthalpy.

The regenerator and heat exchangers can be more appropriately modeled using porous media methods. The porous media was modeled using the volume-averaged conservation equations for mass and momentum. The gravity and external forces are neglected.

The volume averaged continuity equation :

$$\frac{\partial (\alpha \rho)}{\partial t} + \nabla \cdot (\alpha \rho \vec{u}) = 0 \quad (6)$$

where  $\alpha$  is porosity of the porous medium.

The momentum equation of porous medium:

$$\frac{\partial (\alpha \rho \vec{u})}{\partial t} + \nabla \cdot (\alpha \rho \vec{u} \vec{u}) + \alpha \nabla P - \nabla \cdot (\alpha \tau \cdot \vec{u}) \quad (7)$$

$$- \mu \beta^{-1} \cdot \vec{u} + \frac{C}{2} \cdot |\vec{u}| \vec{u} = 0$$

where  $\beta$  and  $C$  are the permeability parameter and inertial resistance factor for the porous medium, which can be obtained

through the relevant correlations or experimental data.

$$\nabla \cdot ((\alpha k_f + \lambda k_s) \nabla T + (\tau \cdot \alpha \vec{u}) - \frac{\partial}{\partial t} \quad (8)$$

$$(\alpha \rho_f E_f + \lambda \rho_s E_s) - \nabla \cdot (\alpha \vec{u} (\rho_f E_f + P)) = 0$$

where  $\lambda = (1 - \alpha)$ , the subscript  $f$  means the fluid, and the subscript  $s$  means the solid

### C. Numerical scheme

The axisymmetric 2-D analysis is adopted with the commercial CFD package FLUENT, which can solve the complicated multi-dimensional problems with the transient flow and transport phenomena, two-phase flow, and volumetrically-generating sources. It numerically solves the continuum fluid and energy equations without arbitrary assumptions [Cha]. For the mesh generation of system shape, ICEM CFD program was applied. The configuration of model with mesh generation is illustrated schematically in Fig. 2.

The typical total number of meshes is approximately 8,000 which is thought to be insufficient to obtain reliable

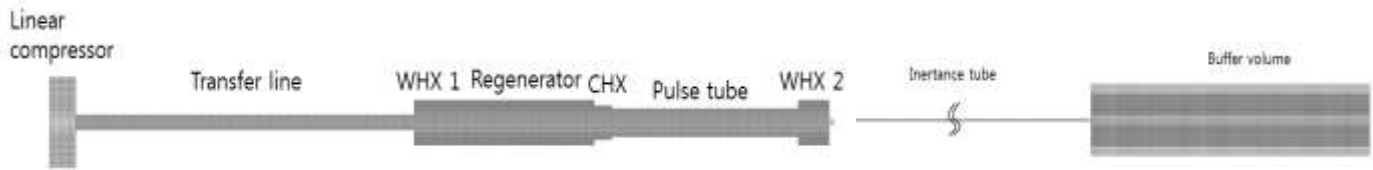


Fig. 2 2-dimensional axi-symmetric mesh generation for IPTC model

results. However, it can reduce the computational time and show the reasonable tendency in flow and temperature distribution in the each components of IPTC system with a relatively sufficient accuracy. PISO scheme is applied for the pressure-velocity coupling and PRESTO! for spatial discretization of pressure. First order upwind scheme is used for others. The time step for the calculation is set 0.0005sec and the convergence criterion for the continuity and momentum equation is 1.0E-5 and for the energy equation is 1.0E-7.

### III. Results and Discussion

The compression and expansion of piston in the cylinder of compressor was modeled with an user defined function for the moving of piston,

$$piston\ displacement = X_{amplitude} * \sin(\omega * t) \quad (9)$$

where  $X_{amplitude}$  means the amplitude of piston displacement and  $\omega$  is the angular velocity. In this work,  $X_{amplitude}$  is 0.0065m and  $\omega$  is 251.33rad/s since the frequency of piston is 40Hz. The operating conditions of IPTC are shown in the TABLE 2.

Fig. 3 shows the history of averaged temperature at the cold heat exchanger. It drops rapidly by 6K per each cycle during initial 1~2sec after starting and after 10seconds it drops

relatively slowly at each cycle and decayed to 90K. After 40 seconds it didn't show any meaningful temperature drop with the moving of piston and converged to the lowest temperature it can produce. As the piston compresses and expands, the temperature at the cold heat exchanger repeats the rise and drop in temperature oscillatory. More detail changes in averaged temperature history for 1second is shown in Fig. 4.

The changes in temperature distribution in the IPTC system is shown in Fig. 5. It illustrates the temperature contours in IPTC system for the compression and expansion of piston in the compressor after 30seconds. The lowest temperature is produced in the cold heat exchanger before the pulse tube and the compressor side shows the warmest temperature. The temperature at the warm heat exchangers before the regenerator and after the pulse tube show the

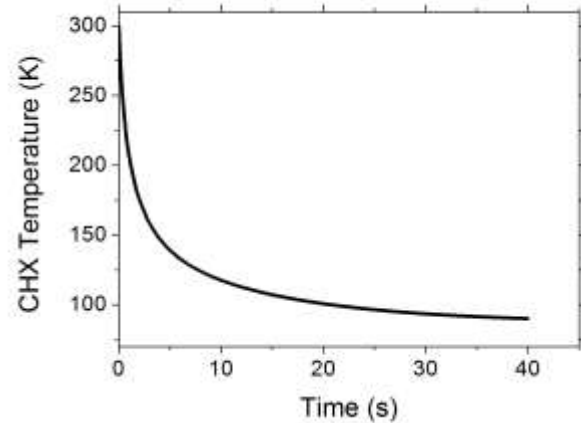


Fig. 3 Averaged temperature history at the cold

Table 2. Operating conditions and properties of materials

WHX1 wall temperature (K)	293
WHX2 wall temperature (K)	293
Regenerator material	Stainless steel
$\beta(m^2)$	2.2529E-11
C ( $m^{-1}$ )	120,000
Regenerator porosity	0.692
WHX1, CHX, WHX2 material	Copper
$\beta(m^2)$	1.345E-9
C ( $m^{-1}$ )	8147
WHX1, CHX, WHX2 porosity	0.68
Initial temperature (K)	300
Initial pressure (MPa)	2.78

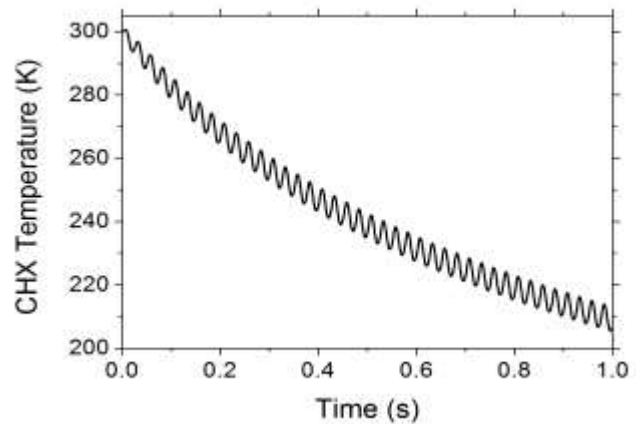
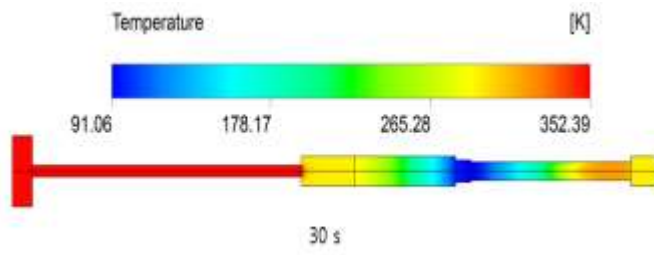
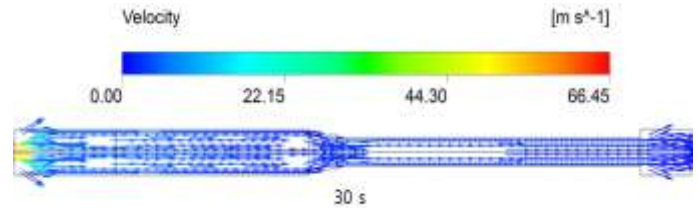


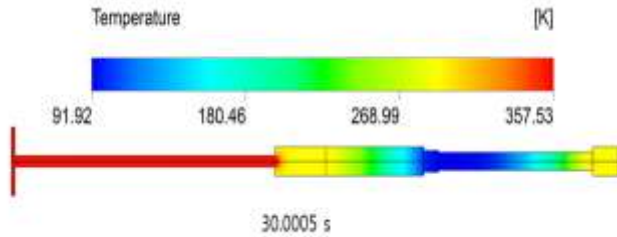
Fig. 4 Detailed averaged temperature history at the cold heat exchanger



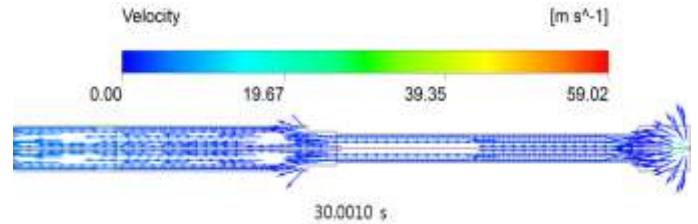
(a) Piston at the middle of stroke during compression



(a) Velocity distribution during compression of piston

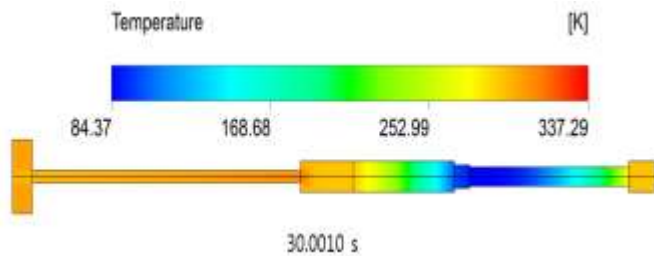


(b) Piston at the top dead center



(b) Velocity distribution during expansion of piston

Fig. 6 Flow velocity distribution in regenerator and pulse tube



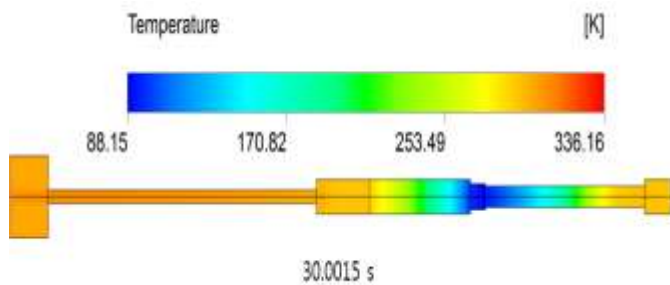
(c) Piston at the middle of stroke during expansion

## IV. Conclusions

2-D axisymmetric modeling was applied to investigate the transient temperature and flow velocity distribution in the IPTC system. The temperature dropped rapidly at the initial stage of operation, especially during 10seconds and the dropping rate decreased after 10seconds. The temperature at the cold heat exchanger decayed exponentially and oscillatory with the compression and expansion of reciprocating piston. The temperature at the pulse tube entrance showed lowest temperature and at the outlet of pulse tube it showed relatively warmer temperature distribution. It also showed that the flow velocity at the entrance of regenerator indicated the fastest velocity distribution and no flow to the side wall during compression stroke. Through this analysis it can also improve the cooling performance of IPTC.

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(d) Piston at the bottom dead center

Fig. 5 Temperature distributions during the compression and expansion stroke of piston after 30seconds

warmer temperature distribution.

The flow velocity distributions in the regenerator and pulse tube when the piston is at the middle of compression and expansion stroke are illustrated in Fig. 6. For the compression stroke the velocity in the entrance of regenerator indicates the fastest velocity distribution and it doesn't show any flow to the side wall. From this result, the divergent shape of regenerator entrance might be beneficial to the performance of cryocooler. During the expansion stroke, it also shows relatively faster velocity distribution in the region of the warm heat exchanger after the pulse tube.

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