Topology Optimization using Integrated Multi-Point Approximation with application

[Abdurahman M. Hassen; Hai Huang]

Abstract— **Multi-point approximation (MA) [1] and GA together with two-level multipoint approximation concepts was used. The topology variables of the trusses are optimized, through GA in the external layer of the first level approximation, while the cross-sectional areas of bars are optimized in the internal layer, which is solved by the dual method in the second level approximation. But singularity of MA may take place. The Integrated Multi-point Approximation (IMA) developed by using two approximate functions in two different specified domains, to avoid the singularity, the developed function can be used for both topology and sizing. Its accuracy of the new function studied [2],[3], results were satisfying, for topology optimization and sizing; in this study application examples are demonstrated, to show the validity and the efficiency of the proposal, for topology optimization as well as sizing.**

Keywords— **Structural Optimization, Multi-point approximation, GA, Topology Optimization**

I.   Introduction

Generally, on discrete structures such as trusses, the topology optimization is concerned with finding an optimal configuration of structure, within a specified domain. The weight of a structure is often taken as objective function. The most difficulty in such problem is that there may exist many local optimal solutions, as well as a singularity problem. Hajela & Lee [4] developed an approach based on a two level genetic algorithm; in one level they satisfy kinematics stability constraints, followed by response constraints at the second level, to generate near optimal structural topologies. Global search algorithm by Ringertz [5] based on the branch and bound algorithm, effective for a problem with multiple local optimal solution. Sankaranarayana & Haftka [6] used the simultaneous analysis and design (SAND) approach. The SAND approach treats the equilibrium equations as equality constraints, with the nodal displacements used as design variables, in addition to the cross sectional areas of truss members, as a result of the method the design variables increase substantially. KanGAL [7] used GA based optimization, with fixed-length vector of design variables,

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representing member areas and change in nodal coordinates, this mostly leading to near optimum. Sakamoto [8] used hybrid method composed by the genetic algorithm, to optimize the layout and the cross-sectional area of truss members, but this method not suitable for large structures, because required a large number of function evaluations and structural analysis. In the recent work [9], an exponent modified function is introduced to original MA; GA and two-level multipoint approximation (MA) by Huang [1] are coupled. The method is to process the multi-point approximate function into two levels, with a layered optimization strategy. The topology variables of the trusses are optimized, through GA in the external layer, of the first-level approximation that avoids the use of repeated finite element analysis, while the cross-sectional areas of bars are optimized in the internal layer, which is solved by the dual method, in the second level approximation. The original MA concept shows high quality approximation in sizing, results of application examples are highly competitive, but singularity may take place, when design variable approaches to zero, and adaptive parameters are negative values. In this study, the IMA is proposed. IMA uses two approximate functions (MA and its modified), each of them implemented in different specified domain, to avoid the singularity of the MA, and to gain the best features of both approximate functions; as a result the high quality of IMA is increased, as unity function to be used for topology and sizing. A numerical and application truss examples are used to verify the efficiency of the proposal.

II.   IMA function used in structural topology optimization

A.   *The mathematical model*

IMA [2],[3] is a reliable and effective model for both sizing and topology optimization. The accuracy of the proposed IMA was tested, through a series of explicit and implicit functions [2], and a series of classical truss examples are solved by the model for sizing and topology [3]. The results were comparable, and concluded that IMA can integrate the accuracy of original MA in a domain $x_i \in (x_{it}, x_u]$, and its modifications in a domain $x_i \in [0, x_{it})$; IMA can also have its independent accuracy, when some components from both domains are shared in one iteration. Where x_i is the design variable; x_{it} is the expansion point, and x_u is the upper limit. At p-th stage of the first-level approximate problem is presented as follows: -

$$\begin{cases} Find & X = \{x_1, x_2, ..., x_n\}^T \\ & \alpha = \{\alpha_1, \alpha_2, ..., \alpha_n\}^T \\ Min & f(X) = \sum_{i=1}^{n} f_i(X) \\ St & \tilde{g}_j^{(p)}(X) \le 0 \qquad j = 1,...,J_1 \\ & \alpha_i x_{i(p)}^L + (1-\alpha_i)x_i^b \le x_i \qquad i = 1,...,n \\ & x_i \le \alpha_i x_{i(p)}^U + (1-\alpha_i)x_i^b \\ & \alpha_i = 0 \quad or \quad \alpha_i = 1 \end{cases}$$ $\cdots(1)$

$$x_{i(p)}^U = \min\{x_i^U, \tilde{x}_{i(p)}^U\} \qquad \cdots(2)$$

$$x_{i(p)}^L = \max\{x_i^L, \tilde{x}_{i(p)}^L\} \qquad \cdots(3)$$

Where X and α are the vectors of cross-sectional size variables, and topology variables, respectively; j_1 is the number of active constraint; n is the group number of linked bars; $x_{i(p)}^U$ and $x_{i(p)}^L$ are the upper and lower bounds of the size variables; x_i^b is a small value to substitute the cross sectional size of the removed bar; $f_i(X)$ is the weight of bars in a group of α_i; $\tilde{x}_{i(p)}^U$ and $\tilde{x}_{i(p)}^L$ are the move limits; $\tilde{g}_j^{(p)}(X)$ represents the approximated constraint function, which is stable even x_i reaches zero. The functions are summarized here as follows: -

$$\tilde{g}_j^{(p)}(X) = \sum_{t=1}^{H}\{g(X_t) + \tilde{g}_{IMA.}(X)\}h_t(X) \qquad \cdots(4)$$

Where,

$$\tilde{g}_{IMA}(X) = \tilde{g}_{MA}(X) + \tilde{g}_{MMA}(X) \qquad \cdots(5)$$

And,

$$\begin{cases} \tilde{g}_{MA}(X) = \dfrac{1}{r_{to}}\sum_{\substack{i=1 \\ i \ne k}}^{n} \dfrac{\partial g(X_t)}{\partial x_i} X_{it}^{1-r_{to}}\left(x_i^{r_{to}} - x_{it}^{r_{to}}\right) & \cdots x_i \ge x_{it} \\ \tilde{g}_{MMA}(X) = \dfrac{1}{r_{tM}}\sum_{\substack{k=1 \\ k \ne i}}^{n} \dfrac{\partial g(X_t)}{\partial x_i}\left(1 - e^{-r_{tM}(x_k - x_{it})}\right) & \cdots x_k < x_{it} \end{cases}$$ $\cdots(6)$

Where, $x_{it}(t=1,...,H; \quad i=1,...,n)$ are the known points; H is the number of points to be counted; n is the number of design variables in a point; $g(X_t)$ is the function values. And, $h_t(X)$ is the weighting function, which can be determined as: -

$$h_t(X) = \dfrac{\overline{h}_t(X)}{\sum_{l=1}^{H}\overline{h}_t(X)} \qquad t = 1,...,H, \qquad \cdots(7)$$

$$\overline{h}_t(X) = \prod_{\substack{S=1 \\ S \ne l}}^{H}(X - X_S)^T(X - X_S) \quad l = 1,...,H \qquad \cdots(8)$$

The exponent r_{to} and r_{tM} are the adaptive parameters, to control the non-linearity of the approximation, to be found from the following equations respectively: -

$$f(r_{to}) = g(X_H) - \left[g(X_t) + \dfrac{1}{r_{to}}\sum_{i=1}^{n}\dfrac{\partial g(X_t)}{\partial x_i}x_{it}^{1-r_{to}}\left(x_{iH}^{r_{to}} - x_{it}^{r_{to}}\right)\right] \qquad \cdots(9)$$

$$f(r_{tM}) = g(X_H) - \left[g(X_t) + \dfrac{1}{r_{tM}}\sum_{i=1}^{n}\dfrac{\partial g(X_t)}{\partial x_i}\left(1 - e^{-r_{tM}(x_{iH} - x_i)}\right)\right] \qquad \cdots(10)$$

Where, X_H is the present point; usually a domain for $r_{to} \& r_{tM} \in [a, 0)$ & $r_{to} \& r_{tM} \in (0, b]$ should be given, $a \& b$ are lower and upper limitations for r_{to} and r_{tM}. In this study the upper and lower limits are defined as -3.5 and 3.5 respectively.

B. *The layered strategy*

Because of mixed variables, a layered strategy was introduced. The topology variables of the trusses are optimized using GA technique, in the external layer, where the finite element is avoided; using this technique, the problem in (1) is transferred into minimum problem with penalty function R.

$$\min F_1 = f(X) + R\sum_{j=1}^{j_1}\left[\max\left(\tilde{g}_j^{(p)}(X), 0\right)\right]^2 \qquad \cdots(11)$$

Only the topology variables α are optimized in this layer. Then the problem transferred into internal layer, where the cross-sectional areas of the topology bars are optimized, through second-level approximation, and solved by the dual method.

III.   Numerical examples

Different examples from literature are chosen to demonstrate the validity, and to compare the efficiency of the IMA. The comparison is made on two aspects, cross-sectional sizing, and topology optimization.

A. *The Ten-Node, 2D Truss ground structure*

IMA is applied to the ten-node truss ground structure "Fig. 1", with ground structure of all possible interconnection a total of 34 members; parameters details refer to reference [7], the example was also solved in [3].
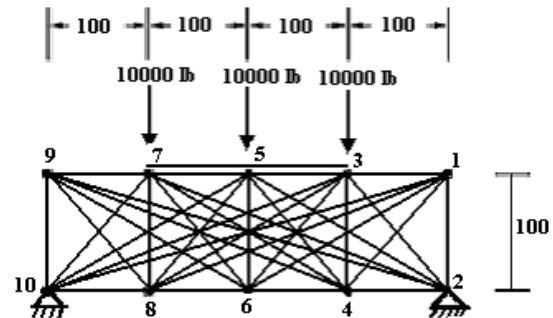

Figure 1.   Ten-Node truss ground structure

The Optimized Ten-node truss ground structure is shown in "Fig. 2", as well as the optimized solution from ref. [7]. For

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the objective function and cross-sectional area of members of the optimized truss are tabulated in table I. The optimized solution "Fig. 2", shows present study has almost same topology as ref. [7], the number of members are less and no overlapping members as comparing with ref. [7]. The cross-sectional areas are different for those overlapping members, but are almost identical for others, and the objective function is comparable.

TABLE I.   CROSS-SECTIONAL AREA OF THE OPTIMIZED TEN-NODE TRUSS GROUND STRUCTURE

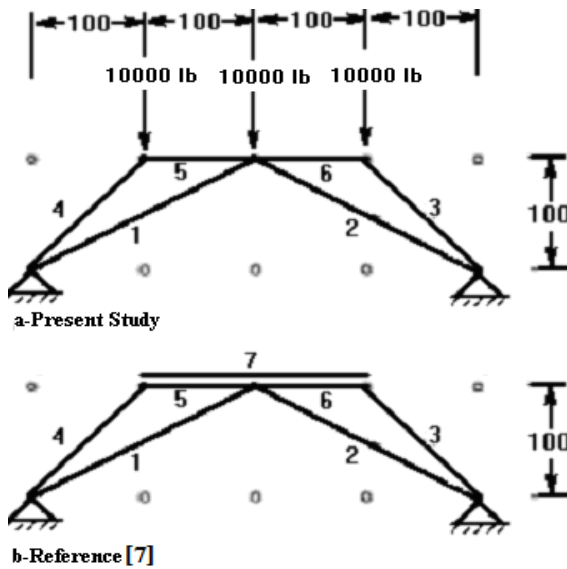| Member No.	Cross-sectional area (in2)	
	Present study	*Ref. [5]*
1	0.446612343	0.477
2	0.446612343	0.477
3	0.565546022	0.566
4	0.565546022	0.566
5	0.399648979	0.082
6	0.399648979	0.082
7	0	0.321
Weights [lb]	44.2708	44.033

Figure 2.   The Optimized Ten-Node Truss ground structure

B. The ground structure of a 9-nodes Truss

The truss ground structure of nine nodes is shown in "Fig. 3", for details see reference [8]. The Optimized Nine-node truss ground structure is shown in "Fig. 4" as well as the topology-optimized solution from ref. [8], for the objective function and cross-sectional areas of members of the optimized truss are tabulated in table II. The topology-optimized solution "Fig. 4" shows that IMA has same topology as ref. [8], and the cross-sectional areas of the

members are slightly different where the objective function increased by 8%
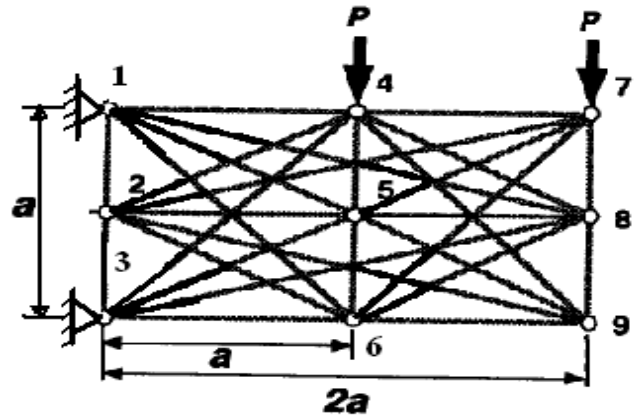

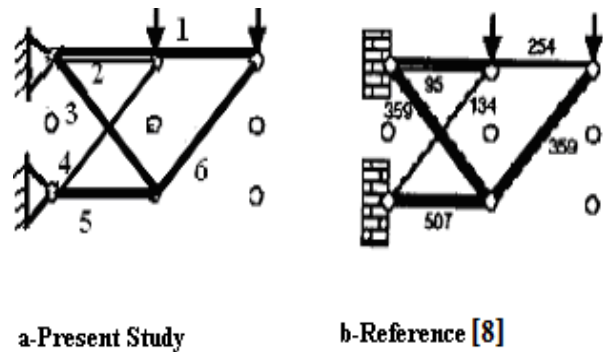Figure 3.   The Nine-Node Truss ground structure

Figure 4.   The Optimized Nine-Node Truss ground structure

TABLE II.   CROSS-SECTIONAL AREA OF THE OPTIMIZED NINE-NODE TRUSS GROUND STRUCTURE

Member No.	Cross-sectional area	
	Present Study	*Ref. [8]*
1	264.156	254
2	105.1604	95
3	379.5246	359
4	137.4901	134
5	539.5367	507
6	377.2747	359
Objective Function	0.5459	0.502

IV.   Helicopter Tail-Boom Structure

The structure of the Tail-Boom was discussed by Haug & Arora [10]. It is known as enclose tail-boom, consists of longitudinal members, cross members, and a skin cover. This type of structure is vulnerable to blasts that occur inside or near the skin. An open truss type structure is considered to avoid such damage. The objective is to design a minimum

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weight structure that meets multiple performance requirements. Two conceptual designs are considered here both depend on open tail-boom. First conceptual design is same as considered in ref. [10], which is studied here for verification purpose. Second conceptual design is suggested. Size optimization is done for bars cross-sectional areas; results are compared with NASTRAN results.

A. *Truss idealization and Results for Open Tail-Boom by Haug & Arora*

The basic configuration and end sections of the tail-boom as well as the maximum flight loads to be supported by the tail-boom structure are shown "Fig. 5". The tail-boom is mainly consisting of six sections; every section has four main longitudinal members, crossed with members. A truss idealization for the open tail-boom is shown in "Fig. 6". The structure is modeled as 28 points and 108 members, with 72 degrees of freedom. Variable linking is manipulated to reduce the modeling fabrications. For other details such as elements numbering and member definitions …etc. see ref. [10]. The design parameters to be calculated are the cross-sectional areas of the members. A lower bound constraint is also imposed on design variables. Material properties and design data for the structure is given in table III.

The results of the open truss helicopter tail-boom optimized for the cross-sectional areas are tabulated in table IV . The iteration history is tabulated in table V. The initial design variables are started with 1.0 in^2. Results are compared with Haug & Arora work for optimum design. Comparing critical constraint of the lower limit on design variables, one can realize that the same members are on the lower limit, except for member No. 40 in which the present work is on lower limit where Haug & Arora isn't. The objective function is almost identical, table IV.
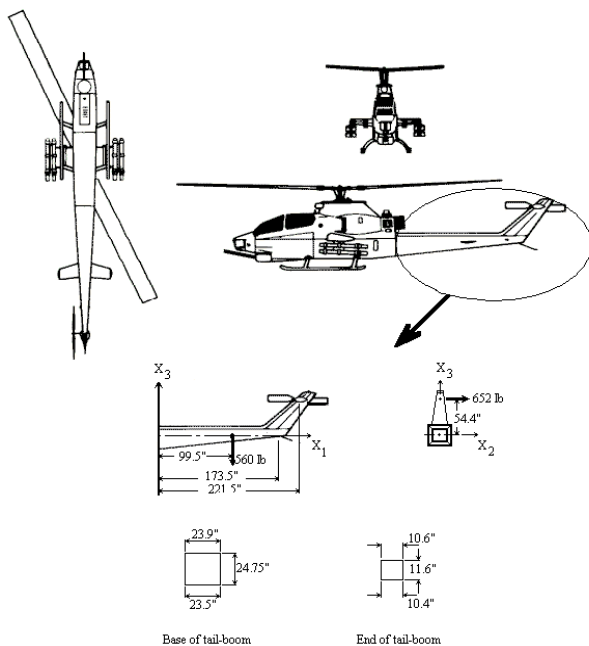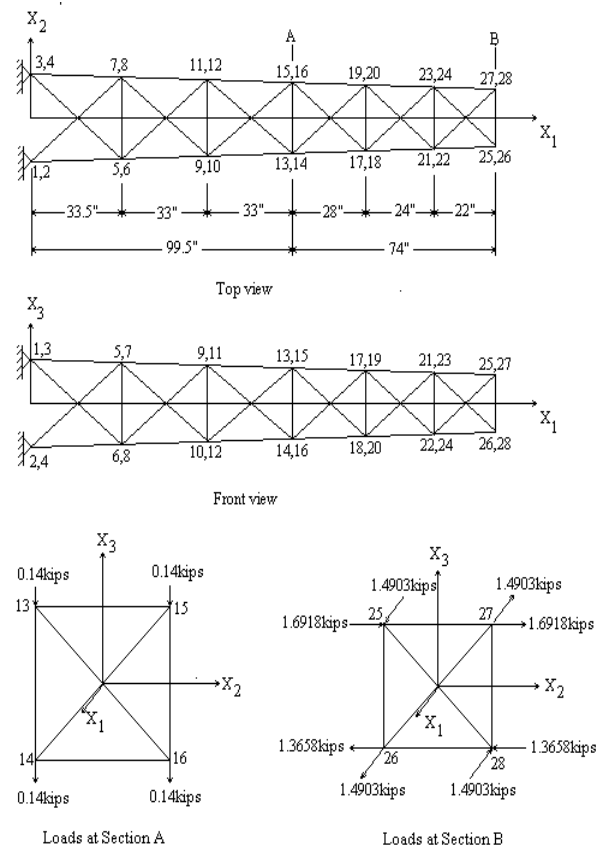
Figure 5.   Geometry of helicopter tail-boom

Figure 6.   Truss idealization for the open tail-boom

B. *Proposed Truss Idealization for open Tail-Boom*

Basically, the proposal is reducing the longitudinal members to three members for each section "Fig. 7". Fabrication process can be reduced to a large percent. Flight loads are same as given in ref. [10]. Redistribution of loads on finite element model were determined, "Fig. 8". Data and material properties shown in table III are used.
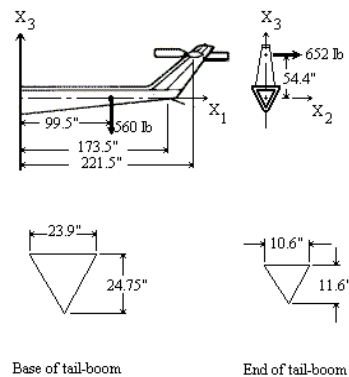
Figure 7.   Proposed Truss for the open tail-boom

C. *Modeling and Results of Proposed Truss Idealization for Open Tail-Boom*

The structure is modeled as 21 points and 72 members, with 54 degrees of freedom. The element numbering system for a typical section is shown in "Fig. 9". To maintain symmetry and to facilitate fabrication of the structure, variable linking is manipulated to reduce the modeling. 72 members of the structure are divided into a total of 42 groups and each group is assigned to a design variable. Therefore, each section of the structure (shown in "Fig. 8") has seven design variables. Group members of same cross-sectional areas are used to maintain symmetry in the structure.

The proposal of open truss helicopter tail-boom is optimized. The cross-sectional areas results are tabulated in table VI. Iteration history data are shown in table VII. For comparison purpose NASTRAN also used to optimize the idealized proposed truss for open tail-boom. Finite element model by NASTRAN is shown in "Fig. 10", final optimized stress tensor is shown in "Fig. 11". It is well obvious that the proposed method has same final objective value comparing with NASTRAN, and has less iteration number. One can note that members on lower limit boundary are more than NASTRAN. No critical stress constraint. One can conclude here that the objective function is reduced by 11.34% from truss idealization by Haug & Arora. Moreover, the number of members are reduced by 33.333%, that means fabrication process is reduced by great deal.

V. Conclusion

Apparently, the results from the ground trusses structures examples bring out that the IMA results, compared with the published one are comparable and satisfying, for both topology optimization as well as cross sectional sizing. Moreover, the application trusse structures results, also assure that the IMA results are satisfied and comparable. However, the IMA can be very useful for both, sizing and topology optimization.
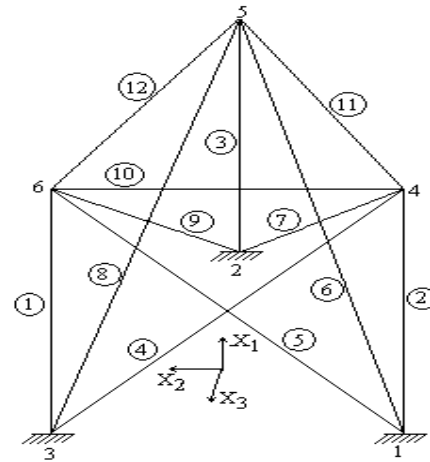

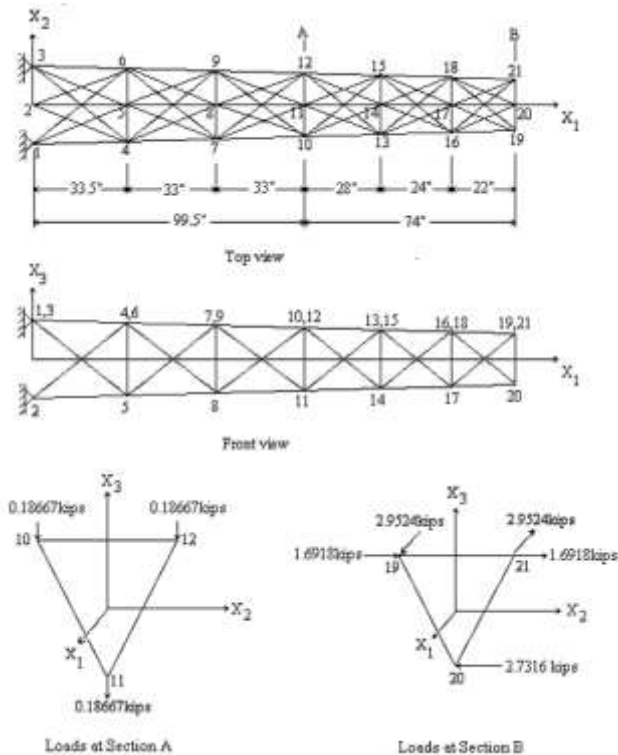Figure 9.   Member numbering for the first section

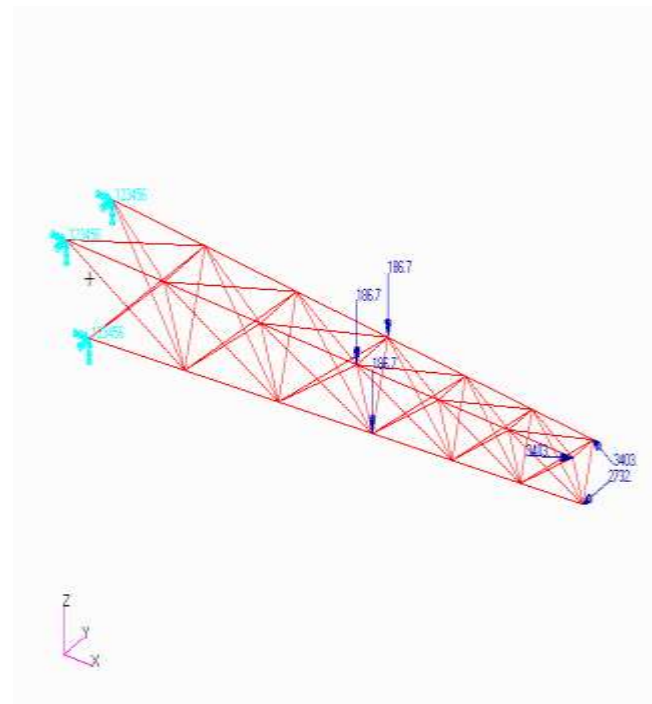Figure 8.   Proposed Truss idealization for the open tail-boom

Figure 10. Finite element model of Proposed Truss idealization for the open tail-boom

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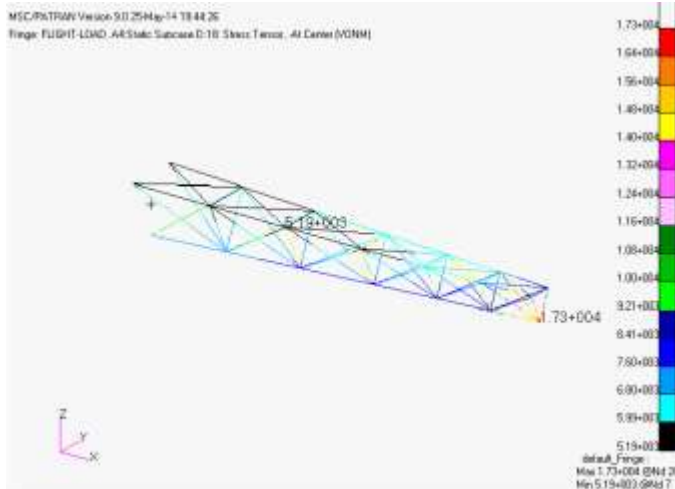Publication Date : 30 April, 2015

Figure 11. Final optimized stress tensor results of Proposed Truss idealization for the open tail-boom

TABLE III.   DESIGN DATA FOR OPEN TRUSS HELICOPTER TAIL BOOM

Material 2024-T3 aluminum alloy	
Models of Elasticity	10.5*103ksi
Stress limits	±25.0ksi
Material density	0.1 lb/in3
Displacement Limits	±0.5 in
Lower bound	0.0415in2
Upper bound	None
No. Of loading condition	One

TABLE IV.   OPTIMUM DESIGN FOR OPEN TRUSS HELICOPTER TAIL BOOM

Design variable	Cross- Sectional Area (in2)		
	Members No.	IMA	Haug & Arora
1	2,3	1.37	1.375
2	1,4	1.36	1.371
3	5,6,9,10	0.132	0.1375
4	7,8,11,12	0.154	0.1395
5	13,15	0.0415	0.0415
6	14,16	0.0877	0.0821
7	17,18	0.0415	0.0415
8	20,21	1.25	1.242
9	19,22	1.25	1.239
10	23,24,27,28	0.167	0.1741
11	25,26,29,30	0.166	0.1649
12	31,33	0.0415	0.0415
13	32,34	0.0959	0.1002
14	35,36	0.0415	0.0415
15	38,39	1.05	1.029
16	37,40	1.05	1.028
17	41,42,45,46	0.209	0.211
18	43,44,47,48	0.222	0.2295
19	49,51	0.0415	0.0415
20	50,52	0.119	0.1371
21	53,54	0.0415	0.0415
22	56,57	0.847	0.8221
23	55,58	0.847	0.8226
24	59,60,63,64	0.235	0.2365
25	61,62,65,66	0.245	0.2587
26	67,69	0.0415	0.0415
27	68,70	0.128	0.1575
28	71,72	0.0415	0.0415

Design variable	Cross- Sectional Area (in2)		
	Members No.	IMA	Haug & Arora
29	74,75	0.604	0.5806
30	73,76	0.604	0.583
31	77,78,81,82	0.265	0.2675
32	79,80,83,84	0.274	0.2883
33	85,87	0.0415	0.0415
34	86,88	0.151	0.1934
35	89,90	0.0415	0.0415
36	92,93	0.241	0.2299
37	91,94	0.246	0.209
38	95,96,99,100	0.325	0.3295
39	97,98,101,102	0.324	0.3428
40	103,105	0.0415	0.0564
41	104,106	0.0805	0.1036
42	107,108	0.197	0.1987
Objective function (lb)		105.57	105.6

TABLE V.   ITERATION HISTORY FOR OPEN TRUSS HELICOPTER TAIL BOOM

No. Of Analysis	Weights (lbs)
	IMA
1	181.8
2	117.17
3	110.39
4	106.07
5	105.71
6	105.59
7	105.57

TABLE VI.   OPTIMUM DESIGN OF THE PROPOSAL OPEN TRUSS HELICOPTER TAIL BOOM

Design variable	Cross- Sectional Area (in2)		
	Members No.	IMA	NASTRAN
1	1,2	2.16	2.1375
2	3	0.651	0.61659
3	4,5	0.299	0.3
4	6,8	0.0415	0.0415
5	7,9	0.235	0.2396
6	10	0.0415	0.0415
7	11,12	0.148	0.15341
8	13,14	1.93	1.9072
9	15	0.464	0.045978
10	16,17	0.361	0.035985
11	18,20	0.0415	0.0415
12	19,21	0.274	0.28035
13	22	0.0415	0.0415
14	23,24	0.175	0.17638
15	25,26	1.59	1.5673
16	27	0.111	0.11176
17	28,29	0.457	0.45382
18	30,32	0.0415	0.0415
19	31,33	0.367	0.37226
20	34	0.0415	0.044621
21	35,36	0.193	0.19355
22	37,38	1.24	1.2514
23	39	0.0415	0.0415
24	40,41	0.516	0.51427
25	42,44	0.0415	0.0415
26	43,45	0.364	0.3701
27	46	0.0415	0.055552
28	47,48	0.204	0.21
29	49,50	0.859	0.91595
30	51	0.0415	0.0415

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Design variable	Cross- Sectional Area (in2)		
	Members No.	IMA	NASTRAN
31	52,53	0.584	0.58139
32	54,56	0.0415	0.0415
33	55,57	0.405	0.40822
34	58	0.0415	0.12584
35	59,60	0.223	0.22875
36	61,62	0.368	0.37812
37	63	0.0415	0.0415
38	64,65	0.7	0.69669
39	66,68	0.0415	0.0415
40	67,69	0.466	0.47358
41	70	0.0415	0.15928
42	71,72	0.115	0.1435
Objective function (lb)		93.598	93.87525

TABLE VII.   THE ITERATION HISTORY OF THE PROPOSAL OPEN TRUSS HELICOPTER TAIL-BOOM

No. Of Analysis	Weights (lbs)	
	IMA	NASTRAN
1	207.24	207.24
2	232.74	225.42
3	104.25	175.74
4	101.86	152.77
5	94.593	136.61
6	93.56	124.66
7	93.684	115.84
8	93.598	110.04
9		106.533
10		102.97
11		100.17
12		98.01
13		96.41
14		95.18
15		94.66
16		94.38
17		93.957
18		93.875

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