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# Dynamic Response of a Simply Supported Plate Due to Excitation at Different Points

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***Abstract*—The rectangular plate vibration has been of immense interests to the researchers for a long time. In this literature, an investigation is performed to measure the dynamic response at the different points on a rectangular plate due to excitation at different points. The possibility of occurring different modes of vibration at the different points on the plate according to the position of excitation is demonstrated and explained from the modal shape of the plate obtained from the modal analysis. A conventional approach is performed to obtain the modes and the modes are compared with the FEM results. An analytical equation is used to obtain the dynamic deflection of harmonic excitation for free vibration to observe the occurring of different modes and also used to validate the results obtained from the finite element method. Then a constant damping condition is simulated to understand properly the variation of amplitude at different points of plate for different excitation conditions.**

***Keywords*—simply supported plate, dynamic response, finite element method (FEM), harmonic excitation**

## I. Introduction

Rectangular plates are the main parts in various structures of bridges, ships, buildings, hydraulic equipment etc.; The vibration has been observed in these structures due to moving cars, earthquake, huge moving air, etc. Many vibrating equipment, such as motors, compressor, etc. are also installed on the horizontal or vertical plates.

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The dynamic response of the structure defined by special frequency spectrum consisting of natural frequencies and mode shapes can be found by knowing the geometrical shapes, mass distribution, stiffness and boundary condition of the plates [1].The vibration analysis of rectangular plates with various uniform and non-uniform boundary conditions has received considerable attention. Most results are well documented in a book [2] and a variety of computational methods have been employed successfully for such analysis.

The computational methods to analysis the vibration have been found in different literatures are: Rayleigh Ritz method [3], methods of finite strips [4], spline finite strip [5] the series expansion for orthotropic plate [6], generalized differential quadrants [7]. The method of superposition was proposed [8] to examine free vibration analysis of cantilever plates in 1976, and rectangular plates with a combination of clamped and simply supported edge conditions are also demonstrated in literature [9].

Recently some papers are demonstrated the dynamic response of plates subjected to harmonic excitation. A literature analyzed/observed the dynamic response of a plate structure without an elastic foundation under moving loads [10]. This work showed how the amplitude of vibration can change with changing of points of load at the load points. Lin and Gbadeyan [11] discussed the dynamic behavior of beam and rectangular plate under moving loads. Takabataka [12] investigated the dynamic response of rectangular plate with stepped thickness subjected to moving loads. An algorithm based on finite element approach has been developed to study the transient response of plates with arbitrary boundary conditions and subjected to moving loads [13]. The dynamic behavior of an orthotropic plate simply supported on a pair of parallel edges and under a system of moving loads is analyzed based on Lagrange equation and modal superposition [14]. A procedure incorporating the finite strip method, together with a spring system has been developed and applied to treat the dynamic response to moving accelerated point loads resting on elastic foundation is investigated and the effects of initial moving velocity, acceleration and initial load position on the response are discussed[15]. An approximate method is presented for the determination of the natural frequencies and mode shapes of rectangular clamped orthotropic plates subjected to dynamic moving loads [16]. The investigation on the dynamic characteristics at different points of a plate for the excitation at different points on the plate has not been investigated in different literature extensively.

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The study of dynamic response of plates subjected to central or offset harmonic excitation is interesting and important to understand the dynamic response of vibration at any point of a plate for any point excitation on the plate. In this research work, a finite element method has been taken for simulation to investigate the dynamic behavior of a simply supported rectangular plate for central and offset harmonic excitation. The dynamic responses have clearly been clarified by the mode shapes. The theoretical approach (equation 1 and equation 2) has been also taken into consideration for this research work. The results from simulation is verified by the conventional theoretical calculation.

# II. Simulation

Simulation has been done by finite element method for a rectangular plate shown in Fig.2. The plate dimensions and properties are: 0.6×0.6 m (L×W); thickness= .003 m; density= 7850 kg/m$^3$; Poisson's ratio=0.3. The points, denoted by letters from 1 to 9 in Fig. 2, are shown the different positions on the plate, and these points have specific node numbers after finite element meshing. The co-ordinates of the points are as: 1 (0,0), 2 (0.15m,0.15m), 3 (0.15m,0.3m), 4 (0.15m,0.45m), 5 (0.3m,0.45m), 6 (0.45m,0.45m), 7 (0.45m,0.3m), 8 (0.45m,0.15m) and 9 (0.3m,0.15m). These points have been used to fix the position of excitations and to observe the dynamic response of different points (1 to 9) for each excitation points.

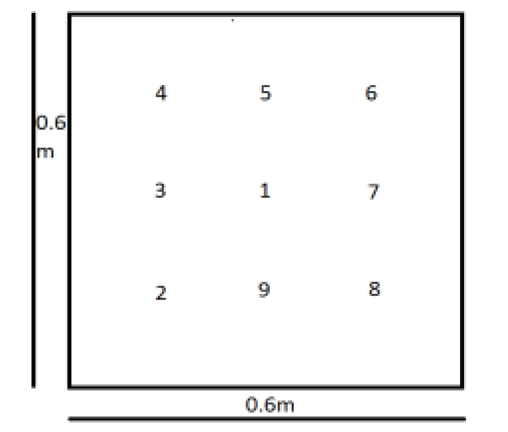


Figure 1. The square plate (a/b=1) with different points marked

## A. *Results of Modal Analysis*

### 1) Mode Shapes

The modal analysis of a simply supported square plate (a/b=1) gives us several natural frequencies at different modes. The natural frequency at different modes is given in the table 2 found from simulation. These natural frequencies and modes have been validated with the data given by the analytical method used from table 1.The mode shapes at different natural frequencies are shown in Fig.2.

TABLE 1. The Natural Frequency Comparisons for Simply Supported Square Plate at Different Modes

| Modes | Simulation (FEM) Natural Frequency(Hz) | Theoretical Natural Frequency (Hz) Using Table 2 (equation 1) |
|---|---|---|
| 1 | 40.868 | 40.8366 |
| 2 | 102.28 | 102.0915 |
| 3 | 102.28 | 102.0915 |
| 4 | 163.69 | 163.3464 |
| 5 | 204.92 | 204.1831 |
| 6 | 204.92 | 204.1831 |
| 7 | 266.36 | 265.4173 |
| 8 | 266.36 | 265.4173 |
| 9 | 349.18 | 348.78 |
| 10 | 349.18 | 348.78 |
| 11 | 369.09 | 367.5088 |
| 12 | 410.69 | 409.03 |

Fig.3 shows us the mode shapes obtained at different natural frequencies. The signs + and – are used as a way to understand the deflection of the plate. At the 1$^{st}$ mode the whole plate moves up and down from the middle which is symbolized by the + sign. The 2$^{nd}$ and 3$^{rd}$ modes are at the same natural frequencies but the modal shapes are different. A nodal line is observed which divides the plate into 2 distinct part and let them vibrate individually at the alternate deflection. So it is symbolized + and – to understand the alternate movement. From animation obtained by the simulation, in these nodal lines almost no deflection is visible. In the 4$^{th}$ mode we observed two nodal lines in the middle of the plate and the plate is divided into 4 distinct parts and these parts vibrate individually. 5$^{th}$ and 6$^{th}$ modes are at the same natural frequencies and in these modes the nodal lines divide the plate into 9 distinct square parts and all of them do not vibrate. In the 5$^{th}$ mode 4 out of 5 parts vibrate and other 5 parts do not, whereas in the 6$^{th}$ mode, those 5 non-vibrating parts (5$^{th}$ mode) vibrate and the other 4 vibrating parts (5$^{th}$ mode) do not vibrate. When portions do not vibrate we symbolized them without any sign (+/-) and we have denoted them as the nodal space, and these spaces do not show vibration. Similarly 7$^{th}$ and 8$^{th}$ mode are at the same natural frequency but introduce 3 nodal lines.9$^{th}$ and 10$^{th}$ modes are also at the same natural frequency and a distinct character of vibration in these 2 modes are shown as Fig.2. It is observed from the animation that the deflection do not start from the middle of the each space rather it is on a side. The last mode is the 11$^{th}$ mode which has 4 nodal lines and no nodal spaces.

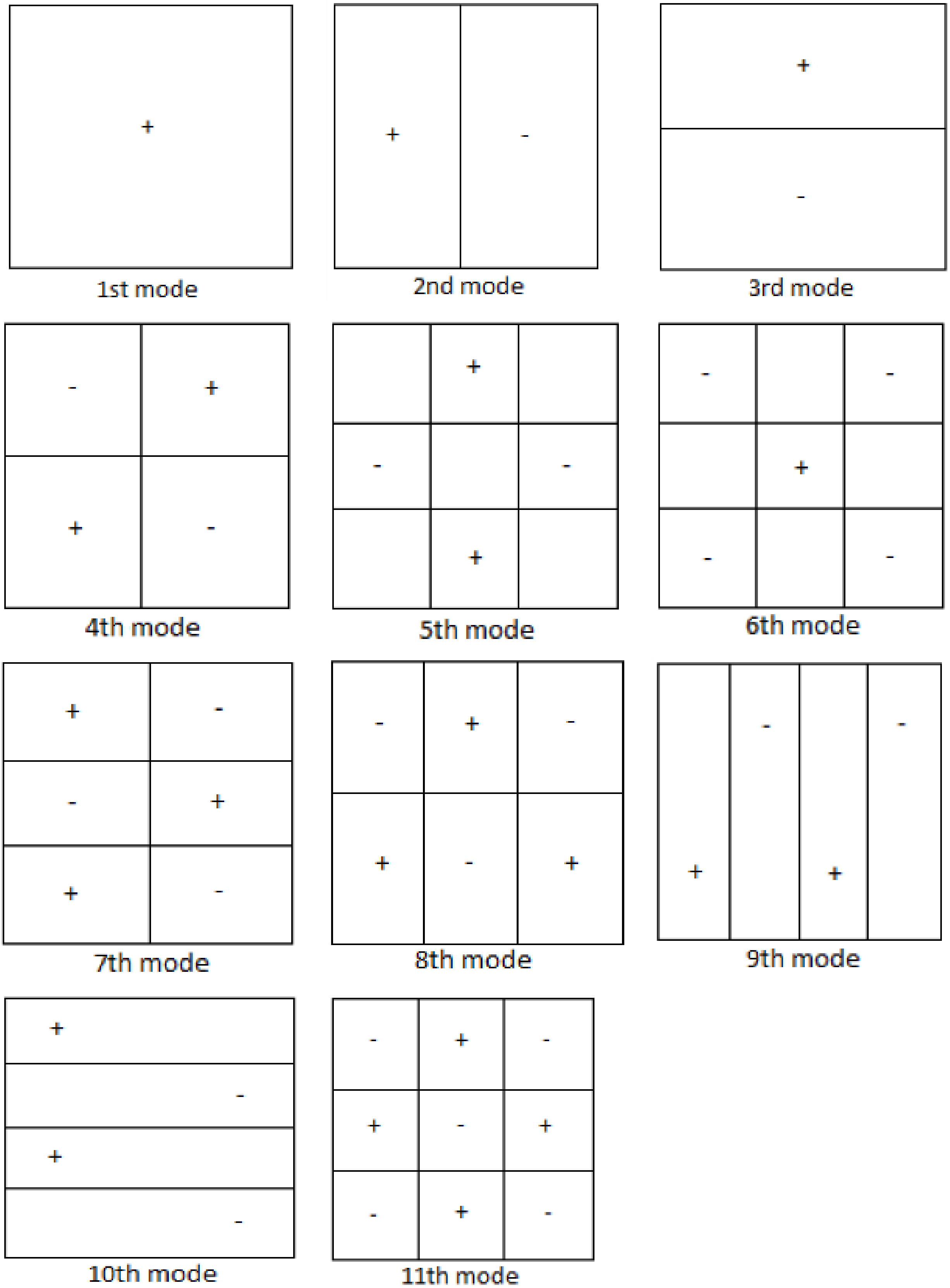


Figure 2. The mode shapes in the first 11 natural frequencies.

## B. *Results of Harmonic Analysis*

### 1) Harmonic analysis without damping :

Point 1, 2 and 3 of Fig.1 are considered for harmonic analysis of simply supported plate. Having each points' excitation, $f \sin(\omega t)$ and $f$ = 50 N, the dynamic response at these three points have been observed by simulation and theory. The other points can be discussed from the symmetry of the plates and will be discussed in this literature later.

#### a) Dynamic Response of Point 1

Displacement vs Frequency curve at point 1 is shown in Fig.4 for applying excitation at point 1, point 2, and point 3. For the excitation at point 1 and point 2, vibration is distinctly visible in 40.858 Hz, 204.92 Hz, and 369.09 Hz whose corresponding modes are respectively $1^{st}$, $5/6^{th}$ and $11^{th}$. For the excitation at point 3, vibration is visible in 40.868 Hz and 369.09 Hz whose corresponding modes are 1 and 11. Except these modes, other modes are not occurring. The occurring of vibration at different points can be explained from Fig.2. If the excitation points and response points both or any of them are not located at the nodal lines or nodal spaces for any specific mode, the vibration can occur at those response points at that mode. Now, for the excitation points 1 and 2, and the response point 1, the vibration can be possible in $1^{st}$, $6^{th}$ and $11^{th}$ modes (Fig.2) which is observed in the simulation and analytical solution (Fig.3). For the excitation point 3 and response point 1, the vibration can occur only $1^{st}$ and $11^{th}$ modes (Fig.2) which is observed in Fig.3.

#### b) Dynamic Response of point 2

Displacement vs Frequency curve of point 2 is shown in Fig.4 for applying excitation at point 1, point 2, and point 3. For the excitation at point 1, vibration is visible in 40.868 Hz, 204.92 Hz and 369.09 Hz whose corresponding modes are respectively $1^{st}$, 5th /$6^{th}$ and $11^{th}$. And for the excitation at point 2 vibration is visible in 40.868 Hz, 102.28 Hz, 163.69 Hz, 204.92 Hz, 266.36 Hz and 369.09 Hz whose corresponding modes are $1^{st}$, 2nd /$3^{rd}$, 4th, 5th /$6^{th}$, 7th /$8^{th}$ and $11^{th}$. And for the excitation at point 3 vibration is visible in 40.868 Hz, 102.28 Hz, 266.36 Hz and 369.09 Hz whose corresponding modes are $1^{st}$, 2nd/$3^{rd}$, 7th /$8^{th}$ and $11^{th}$.Except these modes, other modes are not occurring. From the modal shapes (Fig.2) it can be explained that at the response point 2, resonance can occur at: $1^{st}$ , $6^{th}$ , $11^{th}$ modes for excitation point 1; $1^{st}$ ,$2^{nd}$ ,$3^{rd}$ ,$4^{th}$ ,$6^{th}$ , $7^{th}$ and $8^{th}$ modes for excitation point 2; $2^{nd}$ ,$7^{th}$ and $11^{th}$ modes for excitation point 3.

#### c) Dynamic Response of point 3

Displacement vs Frequency curve of point 3 is shown in Fig.5 for applying excitation at point 1, point 2 and point 3. For the excitation at point 1 vibration is visible in 40.868 Hz and 369.09Hz whose corresponding modes are $1^{st}$ and $11^{th}$. and for the excitation at point 2 vibration is visible in 40.868 Hz, 102.28 Hz and 266.36 Hz and 369.09 Hz whose corresponding modes are $1^{st}$, $2^{nd}/3^{rd}$,and $7^{th}/8^{th}$ and $11^{th}$. And for the excitation at point 3 vibration is visible in 40.868 Hz, 102.28 Hz, 266.36 Hz and 369.09 Hz whose corresponding modes are $1^{st}$, 2n/$3^{rd}$, 7th/$8^{th}$ and $11^{th}$. Similarly, if the excitation point and the response points both are not located at the nodal line or nodal space, the vibration can occur. At the

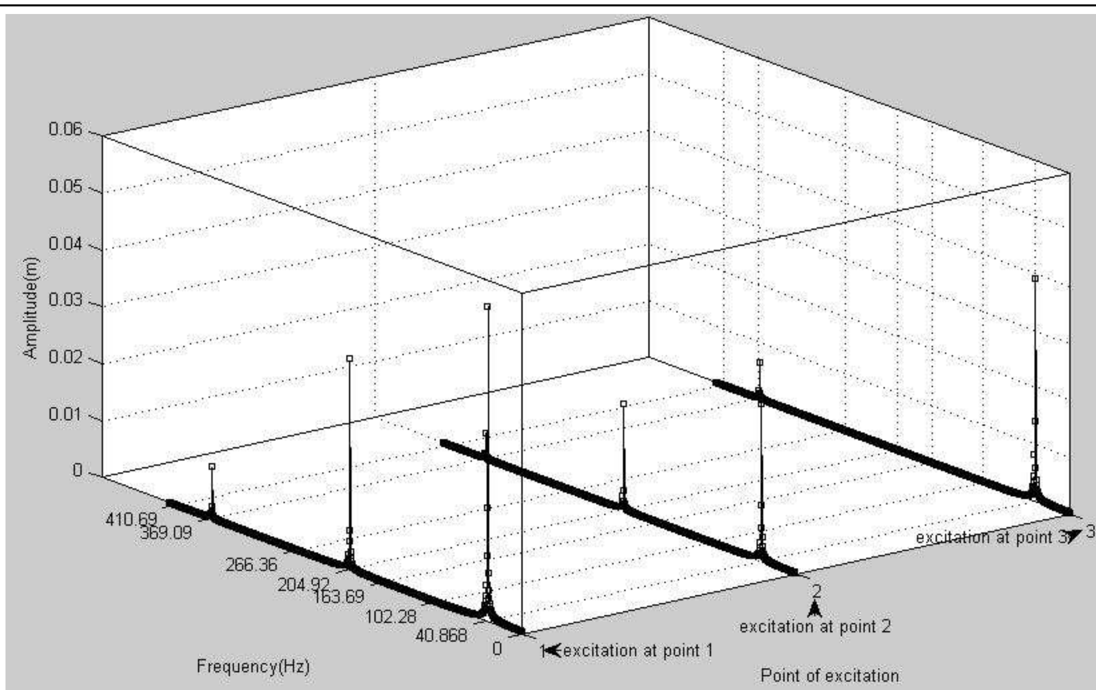


Figure 3. The dynamic response (Amplitude vs. Frequency curve) of point1 for excitation at different points. (Simulation —, analytical □ (equation 2))

response point 3, resonance can occur at: 1st and 11th modes for excitation point 1; 1st, 2nd, 3rd, 7th and 11th modes for excitation point 2; 1st, 2nd, 7th and 11th modes for excitation point 3, which is understood from Fig.2.

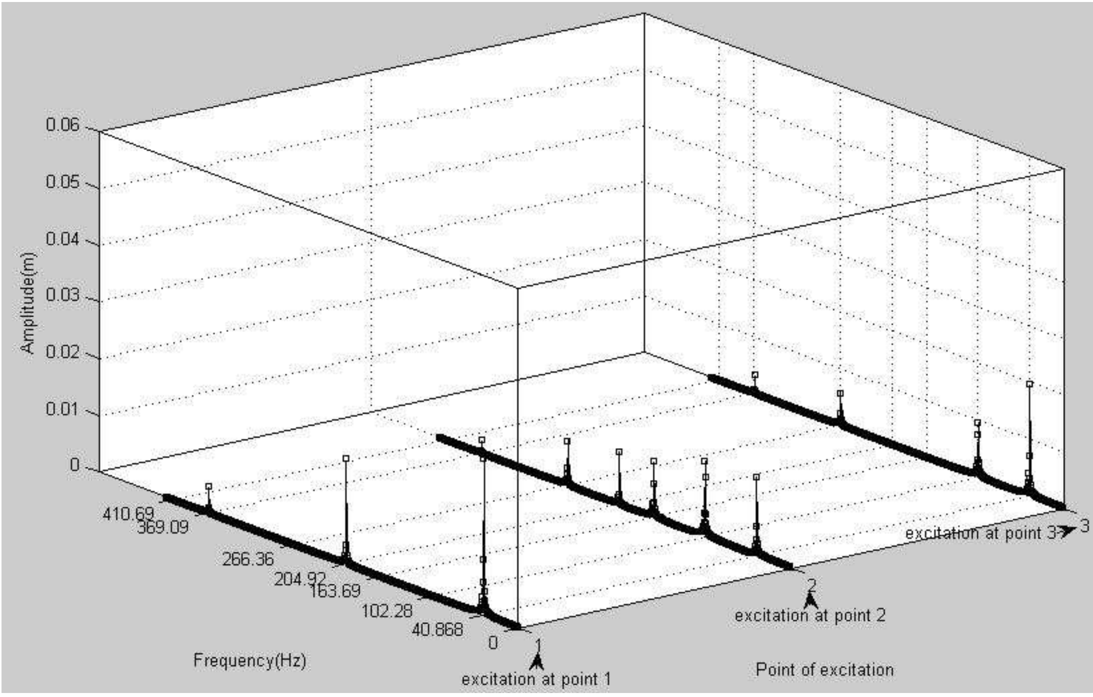


Figure 4. The dynamic response (Amplitude vs. Frequency curve) of point 2 for excitation at different points. (Simulation —, analytical □ (equation 2))

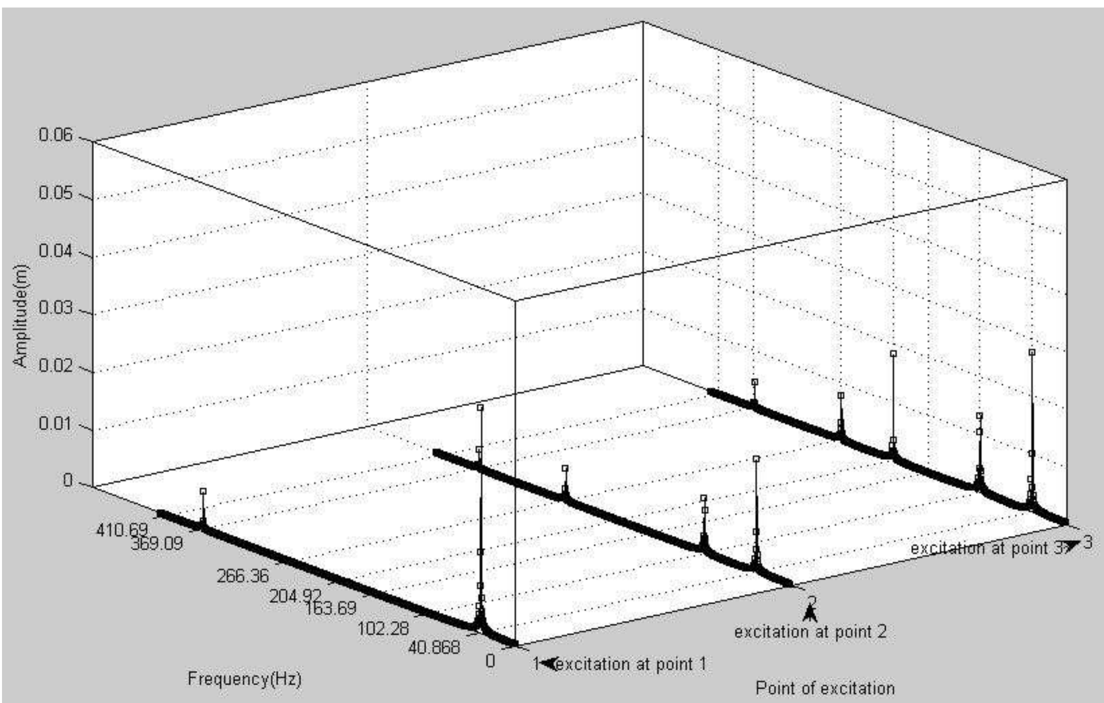


Figure 5. The dynamic response (Amplitude vs. Frequency curve) of point 3 for excitation at different points. (Simulation —, analytical □ (equation 2))

### 2) Harmonic analysis with damping ratio = $0.0040$ :

The dynamic response located at different points on a square plate due to different point's harmonic excitation with a low value of damping ratio .004 is investigated to understand the amplitude variation at different modes.

## a. Dynamic Response of Point 1, Point 2 and Point 3

Fig.6, 7 and 8 shows the frequency vs amplitude curve for damping ratio 0.004. The amplitude of 1st mode is maximum for all response points when excitation is applied at any points (1, 2 and 3) (Fig.6, Fig.7 and Fig.8).

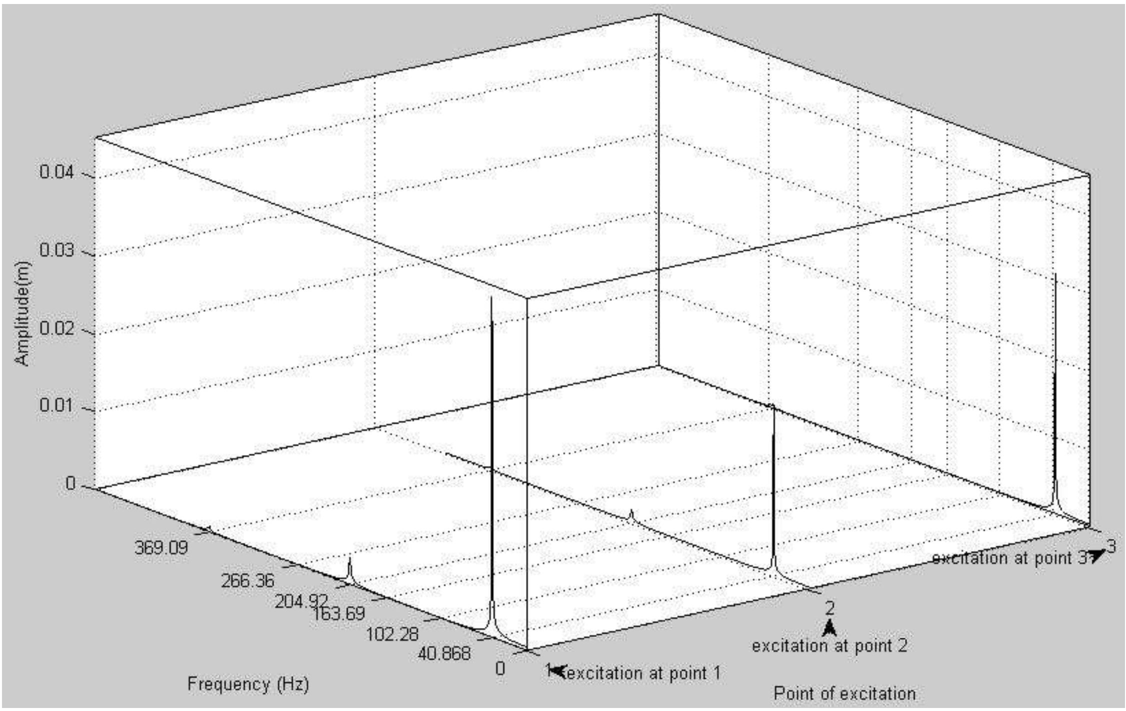


Figure 6. The dynamic response (Amplitude vs. Frequency curve) of point1 for excitation at different points using a damping ratio($\xi = .0040$).

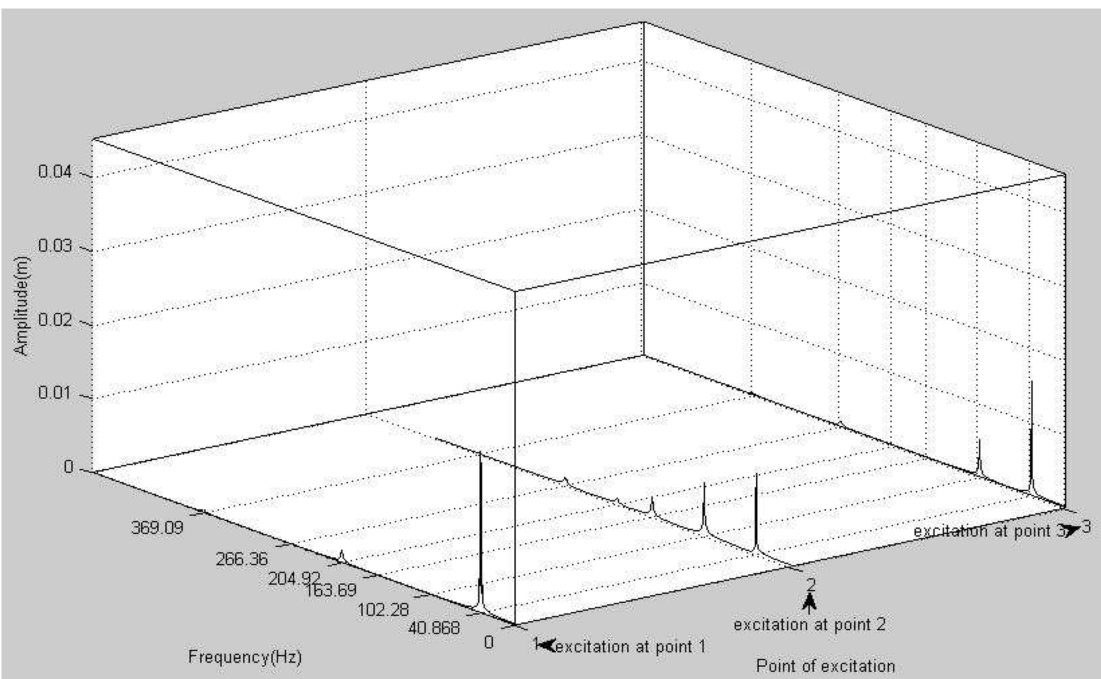


Figure 7. The dynamic response (Amplitude vs. Frequency curve) of point 2 for excitation at different points using a damping ratio ($\xi = .0040$)

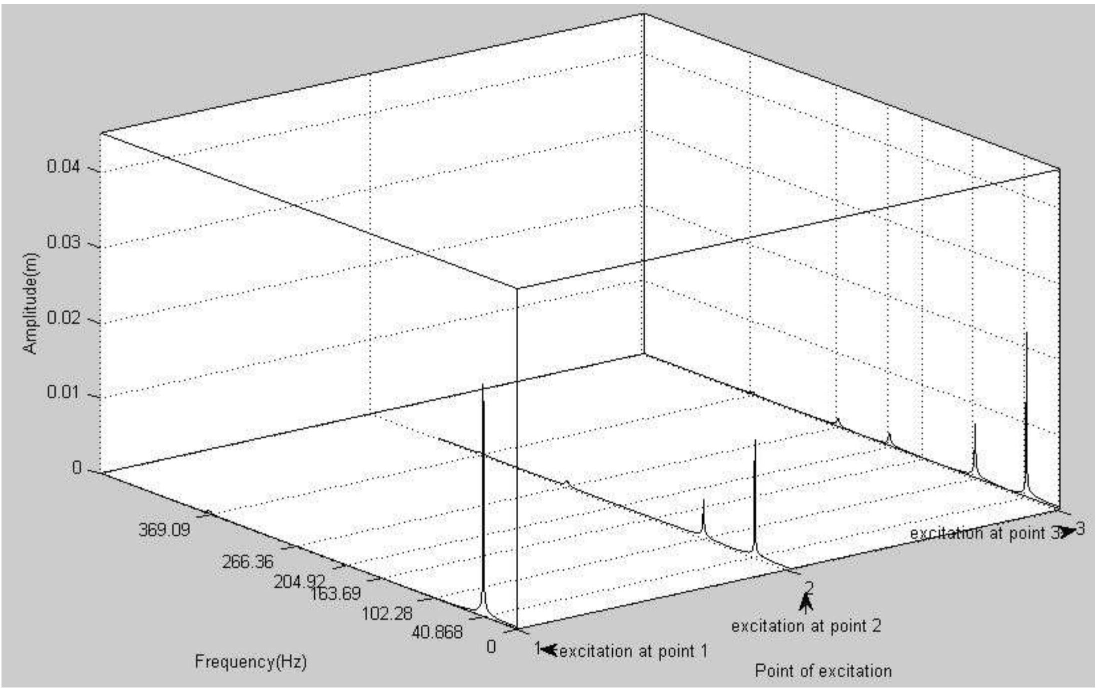


Figure 8. The dynamic response (Amplitude vs. Frequency curve) of point 3 for excitation at different points using a damping ratio$(\xi = .0040)$.

In the 1st mode, the whole plate moves up and down (Fig.2), so irrelevant of excitation point the amplitude at point 1 is more than any other points. Fig.6 also shows that the amplitude at point 1, when excitation is applied at point 3 is more than the amplitude at point 1 when excitation is applied at point 2. Because point 2 is far away from point 1 than point 3. It is observed that at the 6th mode, amplitude at point 1 for excitation at point 1 is higher than the amplitude for excitation at point 2. In other modes the amplitude shown is very small for the dynamic response of point 1 with this damping ratio. At the 2nd mode the amplitude of point 2 (Fig.7) for the excitation at point 2 is more than excitation at point 3. It may be explained from Fig.2 that dynamic response of point 2 for the excitation at point 2 is affected by the 2nd mode and 3rd mode. But dynamic response of point 2 for the excitation at point 3 and dynamic response of point 3 for the excitation at point 2 are only affected by 2nd mode.

## III. Validation:

When the plate is simply supported from all the sides, the natural frequency of the plate is [2] [17]

$$\omega_{mn} = \pi^2 \left[\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2\right] \sqrt{\frac{D}{\rho h}} \qquad (1)$$

For example, for a square plate where a/b=1, for$\omega_{mn} a^2 \sqrt{\frac{\rho h}{D}}$ we get the values

Table 2. The dimensionless frequency parameter for simply supported square plate obtained.

| | n | | |
|---|---|---|---|
| m | 1 | 2 | 3 |
| 1 | 19.72 | 49.30 | 98.60 |
| 2 | 49.30 | 78.88 | 128.17 |
| 3 | 98.60 | 128.17 | 177.47 |

From this table of dimensionless natural frequency we can find natural frequency of different modes of any plate using the specifications of the plate as shown in Table 1.

As for example, to find the natural frequency of the plate we need m=1 and n=1

Then $\omega_{11} a^2 \sqrt{\frac{\rho h}{D}} = 19.72;\ f_{11} = 40.8366\ Hz$

And

Dynamic Response of Rectangular Plate Using Steady-State Harmonic Green's Function [17] (analytical)

$$u_3(x,y,t) = \frac{4F}{\rho hab}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{\sin\frac{m\pi x_1}{a}\sin\frac{n\pi y_1}{b}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}}{\omega_{mn}^2\sqrt{\left[1-\left(\frac{\omega}{\omega_{mn}}\right)^2\right]^2+4\zeta_{mn}^2\left(\frac{\omega}{\omega_{mn}}\right)^2}} \qquad (2)$$

# IV. Conclusion:

Dynamic response of free vibration at different points of plate has been investigated for different excitation points. Then damped harmonic investigation by simulation has also been introduced.

*A.* Investigation for free vibration has shown that resonance occurs at some modes depending upon the excitation points and response points. These occurrences have been explained by the nodal line and space of mode shapes. It is observed that to occur the resonance vibration, the response point and the excitation point both must be on the outside of nodal line and nodal space.

*B.* The analytical equation validated the simulation result for free vibration for different excitation points.

*C.* The damped harmonic analysis has been demonstrated the amplitude at different modes for different boundary value conditions. The results showed that the amplitude is usually more at the first mode irrespective of the excitation points. In addition, some modes of vibration are suppressed.

*D.* From these investigations, it is possible to select the position to install any machinery on a plate to avoid the resonance of the plate for the specific operating frequency. For a range of operating frequency, it is also possible to select the positions to install machinery for minimizing the occurrences of different modes of vibration.

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