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Design of spacecraft control systems in the class of two-parameteric structurally stable mappings using Lyapunov function

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Abstract — In this paper, we present a method increasing the potential of robust stability of control systems of spacecraft with an approach to the construction of two-parameter system in the class of structurally stable maps. Research of robust stability is produced by using the method of Lyapunov functions, based on geometric interpretation of the second Lyapunov method and definition of system stability in the state space.

For research of the stability of steady states of spacecraft control system we using the ideas of the second method A.M. Lyapunov and define components of the gradient vector of the Lyapunov function. During the research, we will represent Equations of state in deviations relative to the stationary states. The general problem **of Lyapunov functions for investigation of the systems with improved capacity of robust stability** is defined and a condition of stability is given.

We propose a method for construct the control system of spacecraft, built in the two-parameter class of structurally stable, which will be sustained indefinitely in a wide range of uncertain parameters of the control object. This work presents novelty theoretical fundamental results assisting in analyzing of the behavior of control systems, meaning of robust stability.

***Keywords*—control system, robust stability, increased potential of robust stability, structurally stable mappings, geometric interpretation.**

I. Introduction

The problem of stability under conditions of uncertainty is a center piece in the design of automatic control systems. They are widely used in almost all areas of manufacturing and technology: mechanical engineering, energy, electronics, chemical, biological, metallurgical and textile industries, transportation, robotics, aircraft, space systems, military equipment and high-precision technologies, etc. At the same time, uncertainty can be caused by imprecise knowledge of the true values of the parameters of the controlled plant or their unpredictable changes in the process of operation of the system. Therefore, the problem of robust stability [1,2,3] is one of the most pressing issues in control theory and is of great practical interest. In general, research of robust stability includes identification of specific constraints for changing parameters of the control system, which preserve stability. These constraints are determined by the region of stability for uncertain parameters of control systems [1,2,3].

Real control systems are nonlinear, and one of the basic properties of nonlinear dynamic systems is generation of deterministic chaos [4,5,6]. Chaotic systems represent the class of models with uncertainty. Conditions of robust stability of these systems allow the presence of instability regions of the stationary states of the control system. The general problem of conditions of suppression or elimination of chaotic oscillations from the process development scenarios by application of control efforts still remains unsolved [7,8,9].

It is generally recognized that real control system are designed and fabricated under the conditions of significant parametric uncertainty; therefore, increasing robust stability characteristics of the system [10,11,12] is one of the key factors that prevents it from causing deterministic chaos, which forms strange attractors [4] . In dynamic systems with linear approximation it manifests itself as the loss of stability of the zero steady state.

Therefore, when significant uncertainty is an issue, it becomes crucial to develop models and methods of control system design with sufficiently wide range of robust stability, called control systems with high potential of robust stability [10,11,12]. The concept of control system with increased potential of robust stability is based on the results of catastrophe theory [13,14], which studies basic structurally-stable mappings.

This article is focused on design and research of control systems with high potential of robust stability applied to dynamic objects with uncertain parameters. It discusses an approach to control system synthesis in the class of two-parameter structurally stable mappings [15,16,17], that allows maximum increase of the limit of robust stability characteristics of the system.

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Recent research demonstrates that robustness of linear or non-linear control systems can be successfully studied by using the method of Lyapunov functions [18,19], based on geometric interpretation of the second Lyapunov method and definition of system stability in the state space. It is assumed that the origin of our system of coordinates corresponds to natural motion of the system, and state equations are written with respect to perturbations, i.e. deviations in perturbed state from the unperturbed [18,19,20,21]. Therefore, state equations express the rate of change of the vector of perturbation (deviations). In case of a stable system, the vector that shows the rate of change of disturbance is directed toward the origin. At the same time, the vector of gradient of the desired Lyapunov function is directed towards the greatest increase of the function, i.e. for a stable system will always be pointed in the opposite direction. This allows us to represent the original dynamic system in the form of a gradient system, and the Lyapunov function in the form of the potential energy surface [13] from the catastrophe theory. Research of robust stability of control system with uncertain parameters is based on the theorem of asymptotic stability [18,19].

II. Main mathematical model

Let us consider a linear model of a spacecraft [22] and construct a control system with high potential for robust stability in the class of two-parameter structurally stable mapping [12,13], i.e., a system described by the following equations [10,11]:

$$\begin{cases} \frac{dx_1}{dt} = x_2, \\ \frac{dx_2}{dt} = -ax_1^4 - ak_1^1x_1^2 + ak_1^2x_1 - ax_2^4 - ak_2^1x_2^2 + ak_2^2x_2, \\ \frac{dx_3}{dt} = x_4, \\ \frac{dx_4}{dt} = -bx_3^4 - bk_3^1x_3^2 + bk_3^2x_3 - bx_4^4 - bk_4^1x_4^2 + bk_4^2x_4, \\ \frac{dx_5}{dt} = x_6, \\ \frac{dx_6}{dt} = -cx_5^4 - ck_5^1x_5^2 + ck_5^2x_5 - cx_6^4 - ck_6^1x_6^2 + ck_6^2x_6. \end{cases} \quad (1)$$

The control law for each channel of the spacecraft is proposed in the form of two-parameter structurally stable mappings [11,12] in the form:

$$u_i = -x_i^4 k_i^1 x_i^2 + k_i^2 x_i, \; i = 1,...,6. \quad (2)$$

We can demonstrate that the system given by (1) and (2) can be used to determine the area of robust stability of the spacecraft control system with respect to parameter variations and provide extremely wide range of stability for uncertain parameters, i.e., the control system prevents the spacecraft from entering the mode of deterministic chaos [15,16,17], even under wide-range variations of uncertain parameters.

Find the steady states of the system from the following equations:

$$\begin{cases} x_{2S} = 0, \\ ax_{1S}^4 + ak_1^1x_{1S}^2 - ak_1^2x_{1S} + ax_{2S}^4 + ak_2^1x_{2S}^2 - ak_2^2x_{2S} = 0, \\ x_{4S} = 0, \\ bx_{3S}^4 + bk_3^1x_{3S}^2 - bk_3^2x_{3S} + bx_{4S}^4 + bk_4^1x_{4S}^2 - bk_4^2x_{4S} = 0, \\ x_6 = 0, \\ cx_{5S}^4 + ck_5^1x_{5S}^2 - ck_5^2x_{5S} + cx_{6S}^4 + ck_6^1x_{6S}^2 - ck_6^2x_{6S} = 0. \end{cases} \quad (3)$$

One stationary state of the system (3) is

$$x_{1S} = 0, \; x_{2S} = 0, \; x_{3S} = 0, \; x_{4S} = 0, \; x_{5S}, \; x_{6S} = 0. \quad (4)$$

Other stationary states of the system (1) can be determined by solving the equation

$$x_{iS}^3 + k_i^1 x_{iS} - k_i = 0, \; i = 1,...,6. \quad (5)$$

As we know from catastrophe theory [13] the solution of equation (5) correspond to critical points of the cusp catastrophe function defined by the formula

$$f(x_{iS}, k_i^1, k_i^2) = -x_{iS}^4 - k_i^1 x_{iS}^2 + k_i^2 x_{iS} = 0, \quad i = 1,...,6. \quad (6)$$

Critical points, double-degenerate critical and triple-degenerate critical points of the cusp catastrophe (6) are found by equating the respective first, second and third derivatives of degeneracy (6) to zero.

Condition (6) is satisfied at critical points

$$4x_{iS}^3 + 2k_i^1 x_{iS} - k_i^2 = 0, i = 1,...,6 \quad (7)$$

and

$$12x_{iS}^2 + 2k_i^1 = 0, i = 1,...,6 \quad (8)$$

and also at double-degenerate critical points. Condition (7), (8) and

$$12x_{iS} = 0, \; i = 1,...,6 \quad (9)$$

are satisfied at triple-degenerate critical points.

Position of the point in the parameter space that describes the function with triple-degenerate critical point is defined as

$$(9) \Rightarrow x_{iS} = 0 \overset{(8)}{\Rightarrow} k_i^1 = 0 \overset{(7)}{\Rightarrow} -k_i^2 = 0, \; i = 1,...,6. \quad (10)$$

The relevant function $(x_{iS}, 0, 0) = x_{iS}^4$ has a triple-degenerate critical point at the origin.

Points in the parameter space, which parameterize functions with double-degenerate critical point, are determined from the equation (8), (7).

$$(8) \Rightarrow k_i^1 = -6x_{iS}^2 \overset{(7)}{\Rightarrow} k_i^2 = 8x_{iS}^3, i = 1,...,6. \quad (11)$$

If the position of the double-degenerate critical point is denoted by x_{iS}, equation (11) gives the values k_i^1 and k_i^2, which describe the functions with double-degenerate critical point x_{iS}.

Equation (11) define parametric representation of the relationship between k_i^1 and k_i^2 , which describes the function with a double-degenerate critical point x_{iS} .

More direct expression that links k_i^1 and k_i^2 can be obtained by excluding x_{iS} from (11):

$$\left(\frac{k_i^1}{6}\right)^{\frac{1}{2}} = x_{iS} = \left(\frac{k_i^2}{8}\right)^{\frac{1}{3}}, \; i = 1,...,6 \quad \text{or}$$

$$\left(\frac{k_i^1}{3}\right)^{\frac{1}{2}} = x_{iS} = \left(\frac{k_i^2}{2}\right)^{\frac{1}{3}}, \; i = 1,...,6,$$

$$\left(\frac{k_i^1}{3}\right)^{3} = \left(\frac{k_i^2}{2}\right)^{2}, \; i = 1,...,6.$$

Hence parametric relationship between k_i^1 and k_i^2 will be determined by the equation

$$\left(\frac{k_i^1}{3}\right)^{3} - \left(\frac{k_i^2}{2}\right)^{2} = 0, \; i = 1,...,6. \qquad (12)$$

As known by elementary algebra, a cubic equation (5) can have up to three real solutions of the form

$$x_{iS}^2 = A_i + B_i, x_{iS}^{3,4} = \frac{A_i + B_i}{2} \pm j\frac{A_i - B_i}{2}\sqrt{3},$$

where $A_i = \sqrt[3]{\frac{k_i^2}{2} + Q_i}, B_i = \sqrt[3]{\frac{k_i^2}{2} - Q_i},$

$$Q_i = \left(\frac{k_i^1}{3}\right)^{3} - \left(\frac{k_i^2}{2}\right)^{2}, \; i = 1,...,6.$$

Hence, taking into account (12), equation (5) has a solution:

$$x_{iS}^2 = 2\sqrt[3]{\frac{k_i^2}{2}} \text{ and } x_{iS}^{3,4} = \sqrt[3]{\frac{k_i^2}{2}}, \; i = 1,...,6. \qquad (13)$$

From (12) can find $k_i^1 = 3\left(\frac{k_i^2}{2}\right)^{\frac{2}{3}}$.

III. The robust stability conditions for spacecraft control system based on geometric approach of the Lyapunov function

3.1 Stability of stationary states (4)

To investigate robust stability of the steady states (4) and (13) of the system given by (1), we can use the approach reported in [15,16,17] that relies on geometric interpretation of the method of Lyapunov functions. Using the geometric interpretation of the gradient vector of the Lyapunov function and the velocity vector of the system under study, we find:

$$\frac{\partial V_1(x)}{\partial x_1} = 0, \frac{\partial V_1(x)}{\partial x_2} = -x_2, ..., \frac{\partial V_1(x)}{\partial x_6} = 0,$$
$$\frac{\partial V_2(x)}{\partial x_1} = ax_1^4 + ak_1^1 x_1^2 - ak_1^2 x_1,$$
$$\frac{\partial V_2(x)}{\partial x_2} = ax_2^4 + ak_2^1 x_2^2 - ak_2^2 x_2,$$
$$\frac{\partial V_2(x)}{\partial x_3} = 0, ..., \frac{\partial V_2(x)}{\partial x_6} = 0,$$
$$\frac{\partial V_3(x)}{\partial x_1} = 0, \frac{\partial V_3(x)}{\partial x_2} = 0, \frac{\partial V_3(x)}{\partial x_3} = 0,$$
$$\frac{\partial V_3(x)}{\partial x_4} = -x_4, \frac{\partial V_3(x)}{\partial x_5} = 0, \frac{\partial V_3(x)}{\partial x_6} = 0,$$
$$\frac{\partial V_4(x)}{\partial x_1} = 0, ..., \frac{\partial V_4(x)}{\partial x_3} = bx_3^4 + bk_3^1 x_3^2 - bk_3^2 x_3,$$
$$\frac{\partial V_4(x)}{\partial x_4} = bx_4^4 + bk_4^1 x_4^2 - bk_4^2 x_4, \frac{\partial V_4(x)}{\partial x_5} = 0, \frac{\partial V_4(x)}{\partial x_6} = 0,$$
$$\frac{\partial V_5(x)}{\partial x_1} = 0, ..., \frac{\partial V_5(x)}{\partial x_5} = 0, \frac{\partial V_5(x)}{\partial x_6} = -x_6,$$
$$\frac{\partial V_6(x)}{\partial x_1} = 0, ..., \frac{\partial V_6(x)}{\partial x_4} = 0, \frac{\partial V_6(x)}{\partial x_5} = cx_5^4 + ck_5^1 x_5^2 - ck_5^2 x_5,$$
$$\frac{\partial V_6(x)}{\partial x_6} = cx_6^4 + ck_6^1 x_6^2 - ck_6^2 x_6.$$

Full time derivative of the Lyapunov vector-function can be found by taking the state equation (1) into consideration as follows:

$$\frac{dV(x)}{dt} = \frac{\partial V(x)}{\partial x}\frac{dx}{dt} = -x_2^2 - \left(ax_1^4 + ak_1^1 x_1^2 - ak_1^2 x_1\right)^2 -$$
$$-\left(ax_2^4 + ak_2^1 x_2^2 - ak_2^2 x_2\right)^2 - \left(bx_3^4 + bk_3^1 x_3^2 - bk_3^2 x_3\right)^2 - \qquad (14)$$
$$-\left(bx_4^4 + bk_4^1 x_4^2 - bk_4^2 x_4\right)^2 - x_4^2 -$$
$$-\left(cx_5^4 + ck_5^1 x_5^2 - ck_5^2 x_5\right)^2 - \left(cx_6^4 + ck_6^1 x_6^2 - ck_6^2 x_6\right)^2 - x_6^2.$$

Expansion of the velocity vector into the coordinate form results in the following components

$$\left(\frac{dx_1}{dt}\right)_{x_1}=0,\left(\frac{dx_1}{dt}\right)_{x_2}=x_2,\ldots,\left(\frac{dx_1}{dt}\right)_{x_6}=0,$$
$$\left(\frac{dx_2}{dt}\right)_{x_1}=-ax_1^4-ak_1^1x_1^2+ak_1^2x_1,$$
$$\left(\frac{dx_2}{dt}\right)_{x_2}=-ax_2^4-ak_2^1x_2^2+ak_2^2x_2,$$
$$\left(\frac{dx_2}{dt}\right)_{x_3}=0,..,\left(\frac{dx_2}{dt}\right)_{x_6}=0,$$
$$\left(\frac{dx_3}{dt}\right)_{x_1}=0,\ldots,\left(\frac{dx_3}{dt}\right)_{x_4}=x_4,$$
$$\left(\frac{dx_3}{dt}\right)_{x_5}=0,\left(\frac{dx_3}{dt}\right)_{x_6}=0,$$
$$\left(\frac{dx_4}{dt}\right)_{x_1}=0,\left(\frac{dx_4}{dt}\right)_{x_2}=0,$$
$$\left(\frac{dx_4}{dt}\right)_{x_3}=-bx_3^4-bk_3^1x_3^2+bk_3^2x_3,$$
$$\left(\frac{dx_4}{dt}\right)_{x_4}=-bx_4^4-bk_4^1x_4^2+bk_4^2x_4,$$
$$\left(\frac{dx_4}{dt}\right)_{x_5}=0,\left(\frac{dx_4}{dt}\right)_{x_6}=0,$$
$$\left(\frac{dx_5}{dt}\right)_{x_1}=0,\ldots,\left(\frac{dx_5}{dt}\right)_{x_5}=0,\left(\frac{dx_5}{dt}\right)_{x_6}=x_6,$$
$$\left(\frac{dx_6}{dt}\right)_{x_1}=0,\ldots,\left(\frac{dx_6}{dt}\right)_{x_4}=0,$$
$$\left(\frac{dx_6}{dt}\right)_{x_5}=-cx_5^4-ck_5^1x_5^2+ck_5^2x_5,$$
$$\left(\frac{dx_6}{dt}\right)_{x_6}=-cx_6^4-ck_6^1x_6^2+ck_6^2x_6.$$

Full time derivative (14) of the vector function is guaranteed to be a negative-definite function.

Using the gradient of the vector function, we can formulate the components of the Lyapunov vector function

$$V_1(x)=-\frac{1}{2}x_2^2,$$
$$V_2(x)=\frac{1}{5}ax_1^5+\frac{1}{3}ak_1^1x_1^3-\frac{1}{2}ak_1^2x_1^2+\frac{1}{5}ax_2^5+\frac{1}{3}ak_2^1x_2^3-\frac{1}{2}ak_2^2x_2^2,$$
$$V_3(x)=-\frac{1}{2}x_4^2,$$
$$V_4(x)=\frac{1}{5}bx_3^5+\frac{1}{3}bk_3^1x_3^3-\frac{1}{2}bk_3^2x_3^2+\frac{1}{5}bx_4^4+\frac{1}{3}bk_4^1x_4^3-\frac{1}{2}bk_4^2x_4^2,$$
$$V_5(x)=-\frac{1}{2}x_6^2,$$
$$V_6(x)=\frac{1}{5}cx_5^5+\frac{1}{3}ck_5^1x_5^3-\frac{1}{2}ck_5^2x_5^2+\frac{1}{5}cx_6^5+\frac{1}{3}ck_6^1x_6^3-\frac{1}{2}ck_6^2x_6^2.$$

The Lyapunov vector function can be presented in the scalar form as follows

$$\begin{aligned}V(x)=V_1(x)+,\ldots,+V_6(x)=\frac{1}{5}ax_1^5+\frac{1}{3}ak_1^1x_1^3-\frac{1}{2}ak_1^2x_1^2+\\+\frac{1}{5}ax_2^5+\frac{1}{3}ak_2^1x_2^3-\frac{1}{2}(ak_2^2+1)x_2^2+\frac{1}{5}bx_3^5+\frac{1}{3}bk_3^1x_3^3-\\-\frac{1}{2}bk_3^2x_3^2+\frac{1}{5}bx_4^5+\frac{1}{3}bk_4^1x_4^3-\frac{1}{2}(bk_4^2+1)x_4^2+\frac{1}{5}cx_5^5+\\+\frac{1}{3}ck_5^1x_5^3-\frac{1}{2}ck_5^2x_5^2+\frac{1}{5}cx_6^5+\frac{1}{3}ck_6^1x_6^3-\frac{1}{2}(ck_6^2+1)x_6^2.\end{aligned}\qquad(15)$$

According to the Morse theorem [13,14], function (15) can be replaced by a quadratic form. By skipping relatively simple, but time-consuming expansion of the function given by (15) in the vicinity of the steady state (4) and finding the elements of the Hessian matrix, the quadratic form is written as:

$$\begin{aligned}V(x)\approx-\frac{1}{2}ak_1^2x_1^2-\frac{1}{2}(ak_2^2+1)x_2^2-\frac{1}{2}bk_3^1x_3^2-\\-\frac{1}{2}(bk_3^2+1)x_4^2-\frac{1}{2}ck_5^2x_5^2-\frac{1}{2}(ck_6^2+1)x_6^2.\end{aligned}$$

Condition for the existence of a positive-definite Lyapunov vector function is defined by the inequalities:

$$k_1^2<0,k_2^2<-\frac{1}{2a},k_3^2<0,k_4^2<-\frac{1}{2b},k_5^2<0,k_6^2<-\frac{1}{2c}.\qquad(16)$$

3.1 Stability of stationary states (13)

As the next step, we investigate robust stability of another steady state given by (13). To achieve this, the state equation (1) is written to describe deviation with respect to the steady state (13). By skipping the time-consuming procedure of expansion and finding derivatives at stationary points, the state equation can be written for deviations as:

$$\begin{cases} \frac{dx_1}{dt} = x_2, \\ \frac{dx_2}{dt} = -ax_1^4 - 4a\sqrt[3]{\frac{k_1^2}{2}}x_1^3 - 9a\sqrt[3]{\left(\frac{k_1^2}{2}\right)^2}x_1^2 - 4ak_1^2x_1 - \\ -ax_2^4 - 4a\sqrt[3]{\frac{k_2^2}{2}}x_2^3 - 9a\sqrt[3]{\left(\frac{k_2^2}{2}\right)^2}x_2^2 - 4ak_2^2x_2, \\ \frac{dx_3}{dt} = x_4, \\ \frac{dx_4}{dt} = -bx_3^4 - 4b\sqrt[3]{\frac{k_3^2}{2}}x_3^3 - 9b\sqrt[3]{\left(\frac{k_3^2}{2}\right)^2}x_3^2 - 4bk_3^2x_3 - bx_4^4 - \\ -4b\sqrt[3]{\frac{k_4^2}{2}}x_4^3 - 9b\sqrt[3]{\left(\frac{k_4^2}{2}\right)^2}x_4^2 - 4bk_4^2x_4, \\ \frac{dx_5}{dt} = x_6, \\ \frac{dx_6}{dt} = -cx_5^4 - 4c\sqrt[3]{\frac{k_5^2}{2}}x_5^3 - 9c\sqrt[3]{\left(\frac{k_5^2}{2}\right)^2}x_5^2 - 4ck_5^2x_5 - cx_6^4 - \\ -4c\sqrt[3]{\left(\frac{k_6^2}{2}\right)}x_6^3 - 9c\sqrt[3]{\left(\frac{k_6^2}{2}\right)^2}x_6^2 - 4ck_6^2x_6. \end{cases}$$

Let us introduce the vector function $V(x)$ with components $V_1(x), V_2(x), V_3(x), V_4(x), V_5(x), V_6(x), x = \|x_1,\dots,x_6\|^T$.

Next, we find components of the gradient vector

$$\frac{\partial V_1(x)}{\partial x_1} = 0, \frac{\partial V_1(x)}{\partial x_2} = -x_2, \dots, \frac{\partial V_1(x)}{\partial x_6} = 0,$$

$$\frac{\partial V_2(x)}{\partial x_1} = ax_1^4 + 4a\sqrt[3]{\frac{k_1^2}{2}}x_1^3 + 9a\sqrt[3]{\left(\frac{k_1^2}{2}\right)^2}x_1^2 + 4ak_1^2x_1,$$

$$\frac{\partial V_2(x)}{\partial x_2} = ax_2^4 + 4a\sqrt[3]{\frac{k_2^2}{2}}x_2^3 + 9a\sqrt[3]{\left(\frac{k_2^2}{2}\right)^2}x_2^2 + 4ak_2^2x_2,$$

$$\frac{\partial V_2(x)}{\partial x_3} = 0, \dots, \frac{\partial V_2(x)}{\partial x_6} = 0,$$

$$\frac{\partial V_3(x)}{\partial x_1} = 0, \dots, \frac{\partial V_3(x)}{\partial x_4} = -x_4,$$

$$\frac{\partial V_3(x)}{\partial x_5} = 0, \frac{\partial V_3(x)}{\partial x_6} = 0,$$

$$\frac{\partial V_4(x)}{\partial x_1} = 0, \frac{\partial V_4(x)}{\partial x_2} = 0,$$

$$\frac{\partial V_4(x)}{\partial x_3} = bx_3^4 + 4b\sqrt[3]{\frac{k_3^2}{2}}x_3^3 + 9b\sqrt[3]{\left(\frac{k_3^2}{2}\right)^2}x_3^2 + 4bk_3^2x_3,$$

$$\frac{\partial V_4(x)}{\partial x_4} = bx_4^4 + 4b\sqrt[3]{\frac{k_4^2}{2}}x_4^3 + 9b\sqrt[3]{\left(\frac{k_4^2}{2}\right)^2}x_4^2 + 4bk_4^2x_4,$$

$$\frac{\partial V_4(x)}{\partial x_5} = 0, \frac{\partial V_4(x)}{\partial x_6} = 0$$

$$\frac{\partial V_5(x)}{\partial x_1} = 0, \dots, \frac{\partial V_5(x)}{\partial x_5} = 0, \frac{\partial V_5(x)}{\partial x_6} = -x_6$$

$$\frac{\partial V_6(x)}{\partial x_1} = 0, \dots, \frac{\partial V_6(x)}{\partial x_4} = 0,$$

$$\frac{\partial V_6(x)}{\partial x_5} = cx_5^4 + 4c\sqrt[3]{\frac{k_5^2}{2}}x_5^3 + 9c\sqrt[3]{\left(\frac{k_5^2}{2}\right)^2}x_5^2 + $$

$$+ 4ck_5^2x_5 + cx_6^4 + 4c\sqrt[3]{\frac{k_6^2}{2}}x_6^3 + 9c\sqrt[3]{\left(\frac{k_6^2}{2}\right)^2}x_6^2 + 4ck_6^2x_6.$$

Specify the projection of the speed vector on the coordinate axes

$$\left(\frac{dx_1}{dt}\right)_{x_1} = 0, \left(\frac{dx_1}{dt}\right)_{x_2} = x_2, \dots, \left(\frac{dx_1}{dt}\right)_{x_6} = 0,$$

$$\left(\frac{dx_2}{dt}\right)_{x_1} = -ax_1^4 - 4a\sqrt[3]{\frac{k_1^2}{2}}x_1^3 - 9a\sqrt[3]{\left(\frac{k_1^2}{2}\right)^2}x_1^2 - 4ak_1^2x_1,$$

$$\left(\frac{dx_2}{dt}\right)_{x_2} = -ax_2^4 - 4a\sqrt[3]{\frac{k_2^2}{2}}x_2^3 - 9a\sqrt[3]{\left(\frac{k_2^2}{2}\right)^2}x_2^2 - 4ak_2^2x_2,$$

$$\left(\frac{dx_2}{dt}\right)_{x_3} = 0, \dots, \left(\frac{dx_2}{dt}\right)_{x_6} = 0,$$

$$\left(\frac{dx_3}{dt}\right)_{x_1} = 0, \dots, \left(\frac{dx_3}{dt}\right)_{x_4} = x_4, \left(\frac{dx_3}{dt}\right)_{x_5} = 0, \left(\frac{dx_3}{dt}\right)_{x_6} = 0,$$

$$\left(\frac{dx_4}{dt}\right)_{x_1} = 0, \left(\frac{dx_4}{dt}\right)_{x_2} = 0,$$

$$\left(\frac{dx_4}{dt}\right)_{x_3} = -bx_3^4 - 4b\sqrt{\frac{k_3^2}{2}}x_3^3 - 9b\sqrt[3]{\left(\frac{k_3^2}{2}\right)^2}x_3^2 - 4bk_3^2x_3,$$

$$\left(\frac{dx_4}{dt}\right)_{x_4} = -bx_4^4 - 4b\sqrt{\frac{k_4^2}{2}}x_4^3 - 9b\sqrt[3]{\left(\frac{k_4^2}{2}\right)^2}x_4^2 - 4bk_4^2x_4,$$

$$\left(\frac{dx_5}{dt}\right)_{x_1} = 0, \dots, \left(\frac{dx_5}{dt}\right)_{x_5} = 0, \left(\frac{dx_5}{dt}\right)_{x_6} = x_6,$$

$$\left(\frac{dx_6}{dt}\right)_{x_1} = 0, \dots, \left(\frac{dx_6}{dt}\right)_{x_4} = 0,$$

$$\left(\frac{dx_6}{dt}\right)_{x_5} = -cx_5^4 - 4\sqrt[3]{\frac{k_5^2}{2}}x_5 - 9c\sqrt[3]{\left(\frac{k_5^2}{2}\right)^2}x_5^2 - 4ck_5^2x_5,$$

$$\left(\frac{dx_6}{dt}\right)_{x_6} = -cx_6^4 - 4c\sqrt[3]{\left(\frac{k_6^2}{2}\right)}x_6^3 - 9c\sqrt[3]{\left(\frac{k_3^2}{2}\right)^2}x_6^2 - 4ck_6^2x_6.$$

Using this representation of the gradient vector of the Lyapunov function and the velocity vector (17) and its projections on the coordinate axes full time derivative of the Lyapunov function can be written as

$$\frac{\partial V_i(x)}{\partial x_j}\left(\frac{dx_i}{dt}\right)_{x_j} = \frac{dV(x)}{dt} = \frac{\partial V(x)}{\partial x}\frac{dx}{dt} =$$

$$= \sum_{i=1}^{6}\sum_{j=1}^{6}\frac{\partial V_i(x)}{\partial x_j}\left(\frac{dx_i}{dt}\right)_{x_j} =$$

$$= -\left(ax_1^4 + 4a\sqrt[3]{\frac{k_1^2}{2}}x_1^3 + 9a\sqrt[3]{\left(\frac{k_1^2}{2}\right)^2}x_1^2 + 4ak_1^2x_1\right)^2 -$$

$$-\left(ax_2^4 + 4a\sqrt[3]{\left(\frac{k_2^2}{2}\right)}x_2^3 + 9a\sqrt[3]{\left(\frac{k_2^2}{2}\right)^2}x_2^2 + 4ak_2^2x_2\right)^2 - x_2^2 -$$

$$-\left(bx_3^4 + 4b\sqrt[3]{\frac{k_3^2}{2}}x_3^3 + 9b\sqrt[3]{\left(\frac{k_3^2}{2}\right)^2}x_3^2 + 4bk_3^2x_3\right)^2 -$$

$$-\left(bx_4^4 + 4b\sqrt[3]{\frac{k_4^2}{2}}x_4^3 + 9b\sqrt[3]{\left(\frac{k_4^2}{2}\right)^2}x_4^2 + 4bk_4^2x_4\right)^2 - x_4^2 -$$

$$-\left(cx_5^4 + 4c\sqrt[3]{\frac{k_5^2}{2}}x_5^3 + 9c\sqrt[3]{\left(\frac{k_5^2}{2}\right)^2}x_5^2 + 4ck_5^2x_5\right)^2 - \quad (18)$$

$$-\left(cx_6^4 + 4c\sqrt[3]{\frac{k_6^2}{2}}x_6^3 + 9c\sqrt[3]{\left(\frac{k_6^2}{2}\right)^2}x_6^2 + 4ck_6^2x_6\right)^2 - x_6^2.$$

With this approach, full time derivative of the Lyapunov vector function (18) is guaranteed to be negative-definite; therefore, sufficient condition for the Lyapunov asymptotic stability is always met.

Using the components of the gradient vector, we can build components of the Lyapunov vector function:

$$V_1(x) = -\frac{1}{2}x_2^2,\ V_2(x) = \frac{1}{5}ax_1^5 + a\sqrt[3]{\frac{k_1^2}{2}}x_1^4 +$$

$$+3a\sqrt[3]{\left(\frac{k_1^2}{2}\right)^2}x_1^3 + 2ak_1^2x_1^2 + \frac{1}{5}ax_2^5 + a\sqrt[3]{\frac{k_2^2}{2}}x_2^4 +$$

$$+3a\sqrt[3]{\left(\frac{k_2^2}{2}\right)^2}x_2^3 + 2ak_2^2x_2^2,\quad V_3(x) = -\frac{1}{2}x_4^2,$$

$$V_4(x) = \frac{1}{5}bx_3^5 + b\sqrt[3]{\frac{k_3^2}{2}}x_3^4 + 3b\sqrt[3]{\left(\frac{k_3^2}{2}\right)^2}x_3^2 +$$

$$+2bk_3^2x_3^2 + \frac{1}{5}bx_4^5 + b\sqrt[3]{\frac{k_4^2}{2}}x_4^4 + 3b\sqrt[3]{\left(\frac{k_4^2}{2}\right)^2}x_4^3 + 2bk_4^2x_4^2,$$

$$V_5(x) = -\frac{1}{2}x_6^2, V_6(x) = \frac{1}{5}cx_6^5 + c\sqrt[3]{\frac{k_6^2}{2}}x_6^4 + 3c\sqrt[3]{\left(\frac{k_6^2}{2}\right)^2}x_6^3 +$$

$$+2ck_6^2x_6^2 + \frac{1}{5}cx_5^5 + c\sqrt[3]{\frac{k_6^2}{2}}x_5^4 + 3c\sqrt[3]{\left(\frac{k_5^2}{2}\right)^2}x_5^3 + 2ck_5^2x_5^2.$$

The Lyapunov function in the scalar form can be represented as

$$V(x_1,\dots,x_6) = \frac{1}{5}ax_1^5 + a\sqrt[3]{\frac{k_1^2}{2}}x_1^4 + 3a\sqrt[3]{\left(\frac{k_1^2}{2}\right)^2}x_1^4 + 2ak_1^2x_1^2 +$$

$$+\frac{1}{5}ax_2^5 + a\sqrt[3]{\frac{k_2^2}{2}}x_2^4 + 3a\sqrt[3]{\left(\frac{k_2^2}{2}\right)^2}x_2^3 + \frac{1}{2}\left(4ak_2^2 - 1\right)x_2^2 +$$

$$+\frac{1}{5}bx_3^5 + b\sqrt[3]{\frac{k_3^2}{2}}x_3^4 + 3b\sqrt[3]{\left(\frac{k_3^2}{2}\right)^2}x_3^2 + 2bk_3^2x_3^2 + \frac{1}{5}bx_4^5 +$$

$$+b\sqrt[3]{\frac{k_4^2}{2}}x_4^4 + +3b\sqrt[3]{\left(\frac{k_4^2}{2}\right)^2}x_4^3 + \frac{1}{2}\left(4bk_4^2 - 1\right)x_4^2 + \frac{1}{5}cx_5^5 +$$

$$+c\sqrt[3]{\frac{k_6^2}{2}}x_5^4 + 3c\sqrt[3]{\left(\frac{k_5^2}{2}\right)^2}x_5^3 + 2ck_5^2x_5^2 + \frac{1}{5}cx_6^5 + c\sqrt[3]{\frac{k_6^2}{2}}x_6^4 +$$

$$+3c\sqrt[3]{\left(\frac{k_6^2}{2}\right)^2}x_6^3 + \frac{1}{2}\left(4ck_6^2 - 1\right)x_6^2. \quad (19)$$

According to the Morse theorem [13,14], after performing time-consuming calculation of the Hessian matrix, the Lyapunov function (19) can be represented in the following quadratic form.

$$V(x) \approx 2ak_1^2x_1^2 + \frac{1}{2}\left(4ak_2^2 - 1\right)x_2^2 + 2bk_3^2x_3^2 +$$
$$+\frac{1}{2}\left(4bk_4^2 - 1\right)x_4^2 + 2ck_5^2x_5^2 + \frac{1}{2}\left(4ck_6^2 - 1\right)x_6^2. \quad (20)$$

The condition of robust stability of the system given by (17) can be obtained by taking into account the negative-definite time derivative (18) of the Lyapunov function (19) or (20) in the form

$$ak_1^2 > 0,\ k_2^2 > \frac{1}{4a},\ bk_3^2 > 0,\ k_4^2 > \frac{1}{4b}, ck_5^2 > 0,\ k_6^2 > \frac{1}{4c}. \quad (21)$$

Thus, the control system of the spacecraft, built in the two-parameter class of structurally stable mappings for a linear system is stable for indefinitely wide ranges of parametric uncertainty and; therefore, it assures avoidance of deterministic chaos in the system. Stationary state (5) is stable when parameters of the spacecraft vary in the range defined by (16) and the stationary state (13) and in various combinations of them (5) acquires the properties of the steady state (5), and at the same time, they are not sustainable. Stationary states of the spacecraft (13) will be stable only if the inequality (21).

IV. Conclusion

In this research was created control system with increased potential linearized robust stability of the spacecraft with uncertain parameters, with the approach to creation of control system in two-parametric class of structurally stable mappings of catastrophe theory and shown maximum increase of potential of robust stability.

For the study of robust stability of control systems with a high potential is applied a new approach to the creation of Lyapunov's vector function based on the geometric interpretation of the theorem on asymptotic stability in the state space. Terms of robust stability control system with increased potential robustness space vehicle obtained in the form of simple inequalities that define the conditions for the existence of Lyapunov's vector function. The control system can ensure the stability of any change of uncertain parameters of the spacecraft.

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