

Properties of Gyroscope Motion About One Axis

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Abstract - The known gyroscope theory based on principle of the rate change in the angular momentum of a spinning rotor. Practice demonstrates this mathematical model cannot describe other gyroscope effects and shows that there are other unsolved problems, which is spawned steady myth of the gyroscope mystery. These phenomena attract new researchers to solve gyroscope problems. The nature of gyroscope effects is more complex than represented in publications and known mathematical models do not match the actual motions and forces of the gyroscope effects. Latest studies of forces that involved in motions of a gyroscope demonstrate that there are four dynamical components, which act simultaneously and interrelated. The gyroscope with the external torque applied is experienced the resistance torque generated by the centrifugal and Coriolis forces and the precession torque generated by the inertial forces and the rate change of the angular momentum of the spinning rotor. These four dynamical components is represented the fundamental principles of gyroscope theory. Interrelation of this torques demonstrates the new unknown effects of the gyroscope. In case of the blocking the action of the precession torques resistance torques are deactivated. The mathematical model of forces acting in the gyroscope practically tested and the results validated the theoretical approach.

Keywords – gyroscope theory

I. Introduction

Gyroscope was invented by different civilizations of people thousand years ago and still attracts attention by the astonishing behavior and amazing by unusual gyroscope properties. Simple gyroscopes are represented the top toys and were not use as instruments yet scientists realize to study and better understand of spin and then to develop of a vast number of practical applications in aviation, space, ship and other industries. Numerous publications in area a gyroscope studies represent it as a device for measuring or maintaining orientation, based on the principles of angular momentum of the spinning rotor [1-4]. The gyroscope properties are relayed in many engineering calculations of rotating parts [5-8].¹

The fundamental principle of the gyroscope theory derived by famous mathematician L. Euler in 1765 in his work on the dynamics of rigid bodies. Later, Sir Isaac Newton and

many other famous and outstanding scientists developed and added new interpretations for the gyroscope phenomena, which display in the rotor's persistence of maintaining its plane of rotation [9-10].

However, known gyroscope theory and its modifications do not match practice of the gyroscope forces and motions, i.e., no true gyroscope theory more than 200 years [11-13]. This is unusual phenomenon in classical mechanics and probably the authority of the famous scientists dominated over other researchers, which did not try refuting settled formulations of gyroscope theory.

The nature of the gyroscope physics is more complex than presented in encyclopaedias, textbooks and numerous publications. Analyses of the motions of gyroscope devices demonstrate that the load torque applied to the gyroscope generates the four fundamental components that are pseudo centrifugal, Coriolis and inertial forces of the spinning rotor and the rate change in the angular momentum. These fundamental components formulate the gyroscope effects. The centrifugal and Coriolis forces generate the resistance torque in change the rotor's location. The inertial force of the spinning rotor generates the net of precession torques.

The rate change in the angular momentum of the spinning rotor involves in resistance and precession torques. The simultaneous action of this group of forces and their interrelations has not been described in the physics of gyroscope effects. Based on new fundamental approaches the gyroscope effects are represented by new mathematical model [14-18]. This model is well-matched with practical results that conducted on the Super Precision Gyroscope model "Brightfusion Ltd". This paper represents unknown interrelations of the gyroscope dynamical components which are new properties of gyroscope and action of the described above forces. New gyroscope properties practically tested and validated new mathematical models of acting forces and motions.

II. Methodology

New studies of the gyroscope motions and forces demonstrates the defined toques based on the action of the centrifugal, inertial, Coriolis forces and the rate change in the angular momentum are acting simultaneously on the spinning rotor. The equations of the resistance and precession torques and percentage of acting in the gyroscope are represented in Table I.

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The action of the internal torques is validated by practical tests of the gyroscope. The tests on the angular velocities of gyroscope motions were conducted on the base of the Supper Precision Gyroscope “Brightfusion LTD” with the counter-balance (Fig. 1), which the technical data that represented in Table II. The weight of the counter-balance is accepted close to the weight of the gyroscope. This design enables to get slow turn of the gyroscope and conduct the measurement of the time turn. The time spent on the gyroscope motion about the axis ox measured by the digital stopwatch model Alba 1/100 Sec PUL SW01.

The velocity of the spinning rotor measured by the Optical Multimeter Tachoprobe Model 2108/LSR Compact Instrument Ltd. with range of measurement 0 – 60,000.00 rpm. The tests of the gyroscope motions are conducted for the location of the gyroscope axis on 60^0 to the horizontal line. The technical parameters of the gyroscope components (Fig. 1, Table II) were calculated with aim to use for the equations of the gyroscope torques (Table 1). The gyroscope assembled with several components with ability to rotate about two axes on the fixed supports. The assembled test stand with the gyroscope and counter-balance is represented in Fig 2. The scheme of the test stand with inclined gyroscope on angle γ , acting forces and sizes of gyroscope components is represented in Fig. 3.

The gyroscope linked by the one side with the axle and other side the axle connected with the counter-balance G . The axle s is fixed on the centre beam b with ability of free rotation about the axis ox on the spherical fixed supports A and B that located on the vertical arms. Centre beam is installed on the frame that assembled with two arms and one horizontal bar. The bar has ability of rotation about the fixed pivot C (vertical axis oy) on the platform.

The action of the gyroscope, axle and the counter weights represent the load torque T that generates the gyroscope resistance torque and precession torque about the point o , which is the centre of coordinate system $\Sigma oxyz$ for the test stand of the gyroscope. The angular velocities of the gyroscope about the axes ox should be determined in case of the blocking the gyroscope rotation about axis oy .

The scheme of the test stand with inclined gyroscope on angle γ , acting forces and sizes of gyroscope components is represented in Fig. 3. The action of the torque T_x about the axis ox which is positive, generated by the weight of the test stand and gyroscope components is represented by the following equation:

$$T_x = (Wgl - sgl_s/2 - Gga)\cos\gamma \tag{1}$$

where W is the gyroscope weight; s is the axle weight, G is the weight of the counter-balance; g is the gravity acceleration, l is the distances of location the centre gravity of the gyroscope, l_s is the length of the axle; a , is the distances of location the centre gravity of the counter-balance and γ is the angle of the axle inclination.

Analysis of components of Eq. (1.) shows that the acting torque T_x is variable and depends on the angle γ . The weight of the gyroscope is acting positively by the counter clock wise direction. The weight of the counter-balance and the axle are acting negatively by the clock wise direction. Other torques acting in the gyroscope stand are represented by the several components which are negative. One of them is friction torque generated by the friction forces of the weight components in the supports A and B that is represented by the following equation:

$$T_{fW} = (W + s + b + G)(d/2)gf \tag{2}$$



Fig. 1. Supper Precision Gyroscope “Brightfusion LTD” with the counter-balance

TABLE I. EQUATIONS OF TORQUES ACTING IN GYROSCOPE

Type of the torque	Equation	Percentage of action, %
The torque generated by the centrifugal forces	$T_{ci} = T_{in} = 2\left(\frac{\pi}{3}\right)^2 J\omega\omega_i$	34.95
The torque generated by the inertial forces		34.95
The torque generated by the Coriolis forces	$T_{cr} = (8/9)J\omega\omega_i$	14.16
The torque generated by the rate change of the angular momentum	$T_{am} = J\omega\omega_i$	15.94
Total		100

where J is the mass moment of inertia of the spinning rotor, ω is the angular velocity of the spinning rotor, ω_x is the angular velocity of the gyroscope motion or precession about axis ox .

TABLE II. Technical data of Super Precision Gyroscope,

		Parameters and components	Numerical data
Weight, W	Spinning components		0.1159 kg
	Aluminium frame		0.0294 kg
	Gyroscope,		0.1453 kg
	Total gyroscope		0.146 kg
Mass moment of inertia, J kgm ²	About axis oz	Spinning components	0.5543873×10^{-4}
		Aluminium frame	$0.70871974 \times 10^{-4}$
	About axis ox	Spinning components	1.3981904×10^{-4}
		Total	2.1069097×10^{-4}

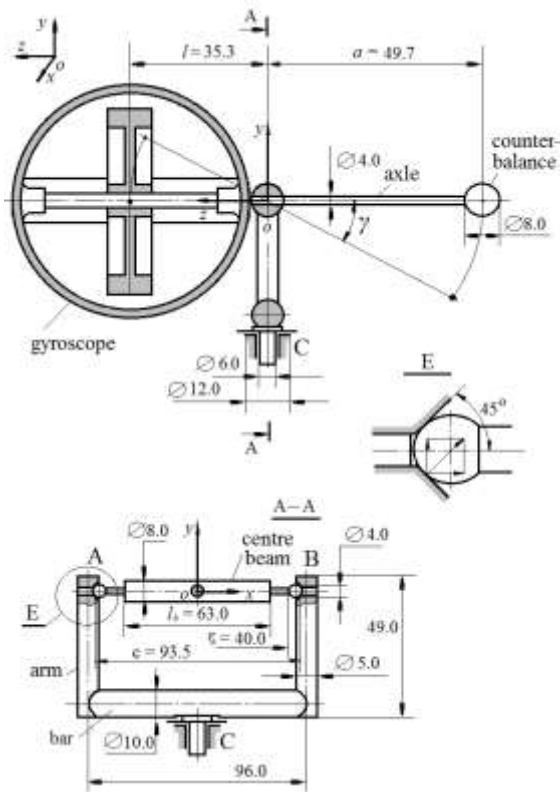


Fig. 2. Geometrical parameters of the gyroscope components

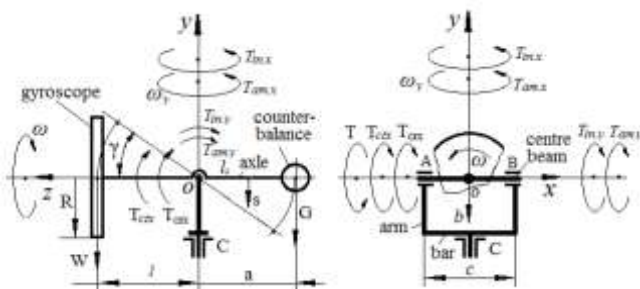


Figure 3. The forces and torques acting on the gyroscope with the counter-balance.

where b is the weight of the centre beam; f is the sliding friction factor in the supports A and B of diameter d ; other components are as specified above.

The acting torque T_x generates resistance torques of the centrifugal T_{ct} and Coriolis T_{cr} forces about axis ox and precession torques of the inertial forces T_{in} and the rate change of the angular momentum T_{am} of the spinning rotor about axis oy . The resistance torques do not act on the support and do not generate the friction force and torque at the support A and B . This property of the gyroscope is pointed at [1]. The precession torques are acting and causing the precession angular velocity ω_x of the gyroscope about the axes ox .

The precession torques about the axis oy are causing the additional loads on the supports A and B that lead to the increasing friction forces in the supports that are acting

negatively. Based on this information, the equation of the friction torques in the supports A and B is represented by the following equation:

$$T_{fx} = F_{px} \times f \times \frac{d}{2} = 2 \times \frac{T_{px}}{c} \times f \times \frac{d}{2} = \frac{T_{px}d}{c} \times f \quad (3)$$

where $F_{px} = T_{px}/c = (T_{in,x} + T_{am,x})/c$ is the reaction of the one pivot on action of precession torque, T_{px} precession torque about the axis oy , c is the distance between contact points of the centre beam journals with supports A and B (Fig. 3); other parameters are as specified above. The gyroscope precession torque about the axis oy is represented by the following equation:

$$T_{px} = T_{in,x} + T_{am,x} = 2 \left(\frac{\pi}{3} \right)^2 J \omega \omega_x + J \omega \omega_x \quad (4)$$

The gyroscope resistance torque about the axis ox is represented by the following equation:

$$T_{rx} = T_{ct,x} + T_{cr,x} = 2 \left(\frac{\pi}{3} \right)^2 J \omega \omega_x + \frac{8}{9} J \omega \omega_x \quad (5)$$

where all parameters are as specified above.

The technical parameters of the test stand and gyroscope are represented in Tables I, II, and III and in Fig. 2.

The friction torque in sliding chamfered bearings of the support A and B about axis ox is represented from the components of the friction forces (Eqs. (2) and (3)) which combining yield the following modified equation:

$$T_{fx} = T_{fw} + T_{fpx} = (W + s + b + G)gf \frac{d}{2 \cos 45^\circ} + \left\{ 2 \left(\frac{\pi}{3} \right)^2 + 1 \right\} \times 2 \times \frac{J \omega \omega_x}{c} \times \frac{fd}{2 \cos 45^\circ} \quad (6)$$

where 45° is angle of the chamfered sliding bearing, other parameters are as specified above.

TABLE III. DATA OF GYROSCOPE “BRIGHTFUSION LTD”.

Parameters	Weight, kg	Mass moment of inertia about axes ox , $J \text{ kgm}^2$
Gyroscope, W	0.1453	$J_W = (2/3)m_s r_s^2 + m_s l^2 + (mR^2/4) + ml^2 = (2/3)*0.0294*0.03^2 + 0.0294*0.0353^2 + (0.1159*0.03093^2/4) + 0.1159*0.0353^2 = 2.2641624*10^{-4} \text{ kg.m}^2$
Counter-balance, G	0.097	$J_G = m_G r^2/2 + m_G a^2 = 0.097*0.004^2/2 + 0.097*0.05^2 = 2.43276*10^{-4}$
Axle, s	0.005	$J_s = m_s l^2/3 = 0.005*0.05^2/3 = 0.00416666*10^{-4}$
Centre beam with journals and screw, b	0.028	$J_b = m_b r^2/2 = 0.028*0.004^2/2 = 0.00224*10^{-4}$
Total	0.2753	

The test of the gyroscope with the counter-balance on motion about axis ox when the rotation of the frame about axis oy is blocked demonstrates one phenomenon. This phenomenon is represented in the gyroscope rotation about the axis ox that is carrying out with high angular velocity, which larger when the gyroscope rotation is free about two axes. Analysis of acting torques on the gyroscope enables to explain this phenomenon by the following reason.

In case of the blocking the gyroscope rotation about axis oy the following torques is involved in process:

- the torque generated by the inertial forces and the rate change of the angular momentum of the spinning rotor with the precession about axis ox , receives the reactive counter torque from the supports.
- the blocking the gyroscope rotation about axis oy leads to deactivation of the resistance torque generated by the centrifugal, inertial and Coriolis forces and rate change of the angular momentum about axis ox .
- deactivation of resistance torques about axis ox leads to increasing the angular velocity of the gyroscope turn about the axis ox .

Full blocking of gyroscope motions about one axis leads to full deactivation of the precession and resistance torques.

The first tests were conducted for the gyroscope with the spinning rotor and blocked one axis of rotation. The second test conducted for the gyroscope motion which rotor does not spin. This type of tests is focused on validation of the statement that resistance torques that generated by the pseudo forces is deactivated in case of the blocking of rotation about axis ox . The tests are conducted with record of the time of the gyroscope turn about axis ox for both cases.

The gyroscope with the spinning rotor turns about axis ox in case of blocking its precession about axis oy . The motion about axis ox with corrections based on the action of the gyroscope components weight and the friction forces is represented by the following equation:

$$(J_{xw} + J_{xG} + J_{xs} + J_{xb}) \frac{d\omega_x}{dt} = \left(Wl - s \frac{l_x}{2} - Ga \right) g \cos \gamma - \quad (7)$$

$$(W + s + b + G)gf \frac{d}{2 \cos 45^\circ} - \left\{ 2 \left(\frac{\pi}{3} \right)^2 + 1 \right\} \frac{fd}{2c \cos 45^\circ} J\omega\omega_x$$

where all parameters are as specified above.

Equation (7) is used for the test of the gyroscope motion that conducted with counter-balance of the mass G (Table III). The location of the gyroscope axis is accepted $\gamma = 60^\circ$ and coefficient of friction in sliding supports is $f = 0.1$. Substituting defined parameters that represented in Table II and III and Fig. 3 into Eq. (7), transforming and simplification yield the following result:

$$4.703329 \times 10^{-4} \frac{d\omega_x}{dt} = 0.001902551 \cos \gamma - \quad (8)$$

$$7.638713336 \times 10^{-4} - 1.071048 \times 10^{-6} \omega\omega_x$$

Equation (8) enables to define the initial angular velocity of gyroscope precession about axis ox . Separating variables and transforming for the differential Eq. (8) gives the following equation:

$$\frac{d\omega_x}{\frac{1776.345224 \cos \gamma - 713.199906}{\omega} - \omega_x} = 0.002277 \omega dt \quad (9)$$

Taking integrals from Eq. (9) at definite limits yields the following

$$\int_0^{\omega_x} \frac{d\omega_x}{\frac{1776.345224 \cos \gamma - 713.199906}{\omega} - \omega_x} = 0.002277 \omega \int_0^t dt \quad (10)$$

Left integral of Eq. (10) is tabulated and presents integral

$\int \frac{dx}{a-x} = -\ln x + C$. Right integral is simple and integrals

have the following solution:

$$-\ln \left(\frac{1776.345224 \cos \gamma - 713.199906}{\omega} - \omega_x \right) \Big|_0^{\omega_x} = 0.002277 \omega t \Big|_0^t$$

giving

$$1 - \frac{\omega_x}{\frac{1776.345224 \cos \gamma - 713.199906}{\omega}} = e^{-0.002277 \omega t} \quad (11)$$

Solving Eq. (11) gives the equation of the angular velocity for the gyroscope about the axis ox as the result of the action of the gyroscope weight

$$\omega_x = \left(\frac{1776.345224 \cos \gamma - 713.199906}{\omega} \right) (1 - e^{-0.002277 \omega t}) \quad (12)$$

Expression $e^{-0.002277 \omega t}$ of Eq. (12) contents the angular velocity of the spinning which value is around $n = 10000 - 30000$ rpm. Hence, the formula $e^{-0.002277 \omega t}$ has a small magnitude and can be neglected. Then, the equation of the angular velocity for the gyroscope about the axis ox as the result of the action of the gyroscope weight is represented by the following equation:

$$\omega_x = \frac{1776.345224 \cos \gamma - 713.199906}{\omega} \quad (13)$$

where all parameters are as specified above.

The angular velocity of the gyroscope about the axis ox is variable and depends on the angle γ of location the gyroscope axis. The time of gyroscope turn on angle γ about axis ox is derived from the following equation:

$$t = \gamma / \omega_x \quad (14)$$

where t is time of gyroscope turn about axis ox , γ is the angle of the turn.

The change of the time of the gyroscope motion with the change of the its angle location represented by the following differential equation:

$$dt = \frac{d\gamma}{\omega_x} \quad (15)$$

Substituting Eq. (14) into Eq. (15) and transforming enable to represent the integral form of Eq. (15) is as follows:

$$\int_0^t dt = \frac{\omega}{1776.3452244} \int_0^\gamma \frac{d\gamma}{\cos \gamma - 0.401498} \quad (16)$$

The right integral can be converted to integrals of rational functions using the versatile trigonometric substitution $x = \tan(\gamma/2)$. Then, the

derivative $dx = \frac{d\gamma}{2\cos^2(\gamma/2)}$. Using the trigonometric

identities $\cos \gamma = \frac{1 - \tan^2(\gamma/2)}{1 + \tan^2(\gamma/2)}$, $\cos^2 \frac{\gamma}{2} = \frac{1 + \cos \gamma}{2}$,

substituting into Eq. (19)

and transforming yield the following integral equation:

$$\int_0^t dt = -8.033598 \times 10^{-4} \omega \int_0^\gamma \frac{dt}{t^2 - 0.653486^2} \quad (17)$$

Integrals (17) is tabulated and represented by the following equation: $\int \frac{dt}{x^2 - a^2} = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right|$. Substituting expression of x into Eq. (17) and transforming yield the following equation:

$$t \Big|_0^t = -6.146725 \times 10^{-4} \omega \times \ln \left| \frac{\tan(\gamma/2) - 0.653486}{\tan(\gamma/2) + 0.653486} \right| \Big|_0^\gamma \quad (18)$$

Substituting the given speed of the gyroscope rotor $\omega = 10,000$ rpm and inclination of the gyroscope axis on angle $\gamma = 60^\circ$ into Eq. (18), transforming and calculating yield the time of the gyroscope turn to horizontal location of the gyroscope axis is as follows:

$$t = -6.146725 \times 10^{-4} \times \frac{10000 \times 2\pi}{60} \times \left\{ \ln \left| \frac{\tan(60^\circ/2) - 0.653486}{\tan(60^\circ/2) + 0.653486} \right| - \ln \left| \frac{\tan(0^\circ/2) - 0.653486}{\tan(0^\circ/2) + 0.653486} \right| \right\} = 1.79s$$

For the gyroscope with the stopped rotor, the equation of the gyroscope motion about axis ox does not includes the component of the friction torque generated by the inertial torque and the rate change of the angular momentum

acting about axis oy . The equation of the gyroscope motion is represented by the following expression:

$$(J_{xw} + J_{xG} + J_{xs} + J_{xb}) \frac{d\omega_x}{dt} = \left(Wl - s \frac{l_s}{2} - Ga \right) g \cos \gamma - (W + s + b + G) g f \frac{d}{2 \cos 45^\circ} \quad (19)$$

The acceleration of the gyroscope with the stopped rotor turn about axis ox is calculated by Eq. (19) that after transforming yields the following result:

$$4.703329 \times 10^{-4} \varepsilon_x = 0.001902551 \cos 60^\circ - 7.638713336 \times 10^{-4}$$

, then $\varepsilon_x = 0.39845 \text{ rad/s}^2$

The time of the gyroscope turn about axis ox is defined by equation:

$$\gamma = \varepsilon t^2 / 2 \quad (20)$$

where γ is the angle of the gyroscope turn, ε is the angular acceleration.

Substituting defined results into Eq. (22) and transforming gives the time of the gyroscope turn about axis ox is as follows:

$$t = \sqrt{\frac{2\gamma}{\varepsilon_x}} = \sqrt{\frac{2 \times 60^\circ \times \pi / 180^\circ}{0.39845}} = 1.32s$$

Practical tests of the time motion of gyroscope with the counter-balance from 60° of axis inclination to horizontal recorded and represented in Table IV.

Results of the theoretical calculations and practical tests of the gyroscope turn time that represented in Table IV are well matched in spite on differences. The reason of differences in results can be represented by simplification in calculations and by magnitude of the friction coefficient of sliding supports that defined empirically.

The time of the turn of the gyroscope with spinning rotor is larger than with the stopped rotor. This is result of the action of the friction force in the sliding supports, generated by the action of the precession torque. Also, this result is validation of the analytical statement that blocking of the rotation of the gyroscope about axis oy leads to deactivation of the resistance torques generated by the pseudo forces acting about axis ox .

TABLE IV. TEST RESULTS OF GYROSCOPE

Time of the gyroscope turn on 60° about axis ox , (seconds)	Tests, s	Theoretical, s
With the spinning rotor	1.43	1.79
The spinning rotor is stopped	0.97	1.32

References

v. Results and Discussion

The load torque applied to the gyroscope leads to an angular velocity of precessions and generates the torques based on action the centrifugal, inertial, Coriolis forces and the rate changes in the angular momentum of the spinning rotor. Based on the action of these torques is formulated the mathematical models for the resistant and precession torques about axis ox at accepted systems of coordinates. The new analytical approach to gyroscopic problems demonstrates that centrifugal, inertial and Coriolis forces of the mass elements for the spinning rotor are really active physical components as its the rate change in the angular momentum. These forces generate the torques that act simultaneously and result in the resistance and precession torques, respectively. Experimental tests and results of the gyroscope torques that conducted for the gyroscope of one axis turn demonstrates that blocking of motion deactivate of pseudo torques.

vi. Conclusion

The gyroscope theory in classical mechanics is one of the most complex and intricate in terms of analytical solutions. The known mathematical models in the gyroscope theory are mainly based on the actions of the load torque and the rate change in the angular momentum of the spinning rotor. The known gyroscope theory does not give correct answer on numerous practical problems. The known gyroscope theory and mathematical models for gyroscope effects do not consider the action of the pseudo centrifugal, inertial and Coriolis forces of the rotating mass of spinning rotor, which play a critical role.

New mathematical models for the gyroscope effects lead to new properties and will thus be useful for modeling the behavior of the gyroscopic devices. The experimental tests and mathematical models of the gyroscope effects for the gyroscope with blocked rotation about one deactivate the pseudo resistance torques. The new analytical approach for the gyroscopic effects describes its new properties

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