International Journal of Structural Analysis & Design– IJSAD
Volume 2: Issue 1 [ISSN : 2372-4102]
Publication Date : 30 April, 2015

Dynamic Behavior of the Viscoelastic Noncylindrical Helices with a Mixed Finite Element Formulation

Nihal Eratlı, Akif Kutlu

***Abstract*—The scope of this study is to investigate linear viscoelastic vibration response of conical and hyperboloidal helices having an elliptical hollow section. A mixed finite element formulation based on the Timoshenko beam theory is implemented, where the numerical analysis are performed in the Laplace domain. Noncylindrical helix geometry is spatially discretized by the finite elements where nodes are placed on the exact geometry. The curvatures are approximated by shape functions over each element. The material constants are replaced with their complex counterparts in the Laplace domain in accordance to the correspondence principle. For the step type loading, results from the solutions performed in the Laplace domain are transformed back to the time domain numerically by employing the Modified Durbin's transformation procedure. Numerical results are presented as benchmark examples investigating the viscolelastic dynamic behavior of noncylindrical helices with elliptical hollow sections. Parametric studies are performed to analyze the influence of the helix geometry on the dynamic response of the structure.**

***Keywords*—viscoelasticity, noncylindrical helix, mixed finite element method, free vibration, Laplace space**

I. Introduction

Among various structural elements, helicoidal rods are frequently used in a wide range of engineering applications, e.g. springs and staircases. Due to their desirable functionality and aesthetical reasons helicoidal bars are the critical parts of the global structures they belong to. Therefore an optimal and reliable engineering design of helicoidal structures requires a good understanding o their dynamic characteristics. It is very common to analyze the dynamic response of the helicoidal rods under the assumption of linear elastic behavior. Instead of ignoring the internal friction in the material model, viscoelastic theory can more reliable reflect the behavior of the material under dynamic conditions. The formulation of the viscoelasticity theory is well documented by many researchers, among them, one can refer to the monographs [1] and [2]. In the case of linear viscoelastic problems, the correspondence principle ([3]) states that the equations of the viscoelastic problem defined in the Laplace space may be expressed identical to the equations of the elastic problem by replacing the elastic constants with related complex moduli regarding the chosen viscoelastic model. Numerous studies in the literature investigated the viscoelastic behavior of straight, circular and helicoidal bars having circular, square or rectangular cross-sections, e.g. [4]-[9].

Nihal Eratlı, Akif Kutlu

Department of Civil Engineering, Istanbul Technical University
Turkey

In the scope of this study, the dynamic behavior of noncylindrical helixes is investigated by employing a linear viscoelastic material model. Field equations of the helicoidal beam depend on the Timoshenko assumptions. A mixed finite element formulation is employed for the solution of the structural problem in the Laplace domain. The three parameter standard model is implemented in the formulation according to the correspondence principle to reflect the viscoelastic material behavior. The curvatures of the helicoidal geometry are approximated with the same shape functions of the finite elements, which are used to interpolate field variables over the domain. The results of the dynamic problem obtained from the Laplace domain are transformed back to the time domain numerically according to the Modified Durbin's transformation algorithm ([10]-[12]). The verification of the present formulation by means of the literature is carried out in [9]. In the presented parametric studies, the effects of the helix geometry on the dynamic response of conical and hyperboloidal helices with elliptical hollow section are investigated.

II. Formulation

A. *The Field Equations and the Mixed Formed Functional in the Laplace Space*

The geometry of an helix can be defined in the Cartesian coordinate system in terms of the helix parameters as: $x = R(\varphi)\cos\varphi$, $y = R(\varphi)\sin\varphi$ and $z = p(\varphi)\varphi$, where $p(\varphi) = R(\varphi)\tan\alpha$. Here, α is the pitch angle. For the conical helix, the centerline radius $R(\varphi)$ is expressed as

$$R(\varphi) = R_1 + \frac{(R_2 - R_1)\varphi}{2n\pi} \tag{1}$$

where n is the number of active turns, R_1 and R_2 are the bottom and top radii of the helix, respectively (See Figure 1), and for the hyperboloidal helix, the radius is

$$R(\varphi) = R_2 + (R_1 - R_2)\left(1 - \frac{\varphi}{2n\pi}\right)^2 \tag{2}$$

where R_1 and R_2 are the bottom radius and the central radius, respectively (See Figure 1). $p(\varphi)$ is a function of the horizontal angle φ and defines the step for unit angle of the helix. The field equations of the elastic cylindrical/non-cylindrical helix based on the Timoshenko beam assumptions were presented by [13] and [14] regarding Frenet coordinate system. Based on this premise, the Laplace transformed field equations can be listed as follows:

Equations of motion:

$$\left.\begin{aligned}&-\bar{\mathbf{T}}_{,s}-\bar{\mathbf{q}}+\rho A z^2\,\bar{\mathbf{u}}=\mathbf{0}\\&-\bar{\mathbf{M}}_{,s}-\mathbf{t}\times\bar{\mathbf{T}}-\bar{\mathbf{m}}+\rho\mathbf{I}\,z^2\;\bar{\boldsymbol{\Omega}}=\mathbf{0}\end{aligned}\right\}\tag{3}$$

Kinematic equations:

$$\left.\begin{aligned}&\bar{\mathbf{u}}_{,s}+\mathbf{t}\times\bar{\boldsymbol{\Omega}}-\bar{\boldsymbol{\gamma}}=\mathbf{0}\\&\bar{\boldsymbol{\Omega}}_{,s}-\bar{\boldsymbol{\kappa}}=\mathbf{0}\end{aligned}\right\}\tag{4}$$

Constitutive equations:

$$\left.\begin{aligned}&\bar{\boldsymbol{\gamma}}-\bar{\mathbf{C}}_{\gamma}\bar{\mathbf{T}}=\mathbf{0}\\&\bar{\boldsymbol{\kappa}}-\bar{\mathbf{C}}_{\kappa}\bar{\mathbf{M}}=\mathbf{0}\end{aligned}\right\}\tag{5}$$

where z is the Laplace transformation parameter. In (5) $\bar{\mathbf{u}}=\bar{u}_t\,\mathbf{t}+\bar{u}_n\,\mathbf{n}+\bar{u}_b\,\mathbf{b}$ is the displacement vector, $\bar{\boldsymbol{\Omega}}=\bar{\Omega}_t\,\mathbf{t}+\bar{\Omega}_n\,\mathbf{n}+\bar{\Omega}_b\,\mathbf{b}$ is the rotation vector, $\bar{\mathbf{T}}=\bar{T}_t\,\mathbf{t}+\bar{T}_n\,\mathbf{n}+\bar{T}_b\,\mathbf{b}$ is the force vector, $\bar{\mathbf{M}}=\bar{M}_t\,\mathbf{t}+\bar{M}_n\,\mathbf{n}+\bar{M}_b\,\mathbf{b}$ is the moment vector in the Laplace space, ρ is the density of homogeneous material, A is the area of the cross section, $\mathbf{I}$ is the moment of inertia of the cross section, $\bar{\boldsymbol{\gamma}}$ is the unit shear vector, $\bar{\boldsymbol{\kappa}}$ is the unit rotation vector, $\bar{\mathbf{C}}_{\gamma}$ and $\bar{\mathbf{C}}_{\kappa}$ are the compliance matrices in the Laplace space, related with the shear and bending deformations, respectively. $\bar{\mathbf{q}}$ and $\bar{\mathbf{m}}$ are the distributed external force and moment vectors in the Laplace space.

Incorporating the potential operator concept and the Gateaux differential ([15]), the functional of the structural problem is obtained in the Laplace space regarding (3)-(5) as,

$$\begin{aligned}\mathbf{I}(\bar{\mathbf{y}})=&-[\bar{\mathbf{u}},\bar{\mathbf{T}}_{,s}]+[\mathbf{t}\times\bar{\boldsymbol{\Omega}},\bar{\mathbf{T}}]-[\bar{\mathbf{M}}_{,s},\bar{\boldsymbol{\Omega}}]-\tfrac{1}{2}[\mathbf{C}_{\kappa}\bar{\mathbf{M}},\bar{\mathbf{M}}]\\&-\tfrac{1}{2}[\mathbf{C}_{\gamma}\bar{\mathbf{T}},\bar{\mathbf{T}}]+\tfrac{1}{2}\rho A z^2\left[\bar{\mathbf{u}},\bar{\mathbf{u}}\right]+\tfrac{1}{2}\rho\, z^2\left[\mathbf{I}\,\bar{\boldsymbol{\Omega}},\bar{\boldsymbol{\Omega}}\right]\\&-\left[\bar{\mathbf{q}},\bar{\mathbf{u}}\right]-\left[\bar{\mathbf{m}},\bar{\boldsymbol{\Omega}}\right]+[(\bar{\mathbf{T}}-\hat{\bar{\mathbf{T}}}),\bar{\mathbf{u}}]_{\sigma}+[(\bar{\mathbf{M}}-\hat{\bar{\mathbf{M}}}),\bar{\boldsymbol{\Omega}}]_{\sigma}\\&+[\hat{\bar{\mathbf{u}}},\bar{\mathbf{T}}]_{\varepsilon}+[\hat{\bar{\boldsymbol{\Omega}}},\bar{\mathbf{M}}]_{\varepsilon}\end{aligned}\tag{6}$$

Theoretical details of the functional developed for elastic and viscoelastic bars can be found in [13], [9]. The terms with hats define the known values on the boundary. The subscripts, ε and σ represent the geometric and dynamic boundary conditions, respectively.

B. *Mixed Finite Element Formulation*

A two nodded curved element is employed to discretize the beam domain. Linear shape functions namely $\phi_i=(\theta_j-\theta)/\Delta\theta$ and $\phi_j=(\theta-\theta_i)/\Delta\theta$ are adopted for i th and j th nodes of the finite element. Here $\Delta\theta=(\theta_j-\theta_i)$. The mixed finite element matrices with the sub-matrices which indicate the variation of helix geometry can be found in explicit form [13], [14]. Each node has 12 degrees of freedom namely:

$$\mathbf{X}^T=\{u_t,u_n,u_b,\Omega_t,\Omega_n,\Omega_b,T_t,T_n,T_b,M_t,M_n,M_b\}\tag{7}$$

III. Numerical Examples

Viscoelastic conical and hyperboloidal helices with fixed at both ends are analyzed (see Figure 1). These helices are subjected to impulsive step type vertical distributed load having an intensity of $q_o=1\,\text{N/m}$. The helix geometry has $n=1.5, 3.5, 7.5$ number of active turns, the height of the rod is $H=4\,\text{m}$ and the minimum radius of helix to maximum radius of helix ratios $R_{\min}/R_{\max}=0.3, 0.5, 0.7$ where $R_{\max}=2\,\text{m}$. The major and minor radii of elliptical hollow cross-section $a_1=150\,\text{mm}$, $a_2=75\,\text{mm}$ and $b_1=90\,\text{mm}$, $b_2=45\,\text{mm}$ (see Figure 2), respectively.

The viscoelastic material exhibits the standard type of distortional behavior while having elastic Poisson's ratio $\bar{\upsilon}=\upsilon=0.3$. The complex shear modulus is given by

$$\bar{G}=G\left[(\beta^G-1)\frac{z}{z+1/\tau_r^G}+1\right];\quad \beta^G=G_g/G>1\tag{8}$$

where τ_r^G G and G_g are the retardation time, the equilibrium value and the instantaneous value of relaxation function associated with shear modulus. The material parameters are $G=7\times10^5\,\text{N/m}^2$, $\tau_r^G=0.005\text{s}, 0.01\text{s}, 0.1\text{s}$, $\beta^G=3$ and the material density $\rho=1000\ \text{kg/m}^3$.

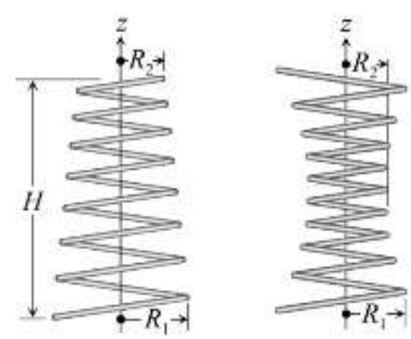

Figure 1. Conical and hyperboloidal helix geometry.

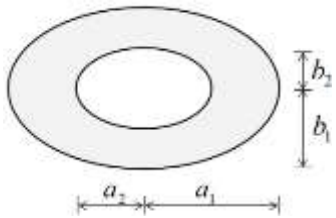

Figure 2. Elliptical hollow cross-section.

The solutions are obtained in the Laplace space and they are inverted back to the time space with the use of modified Durbin's algorithms [12]. The numerical parameters of modified Durbin's algorithm are taken as $N = 2^{11}$ and $aT = 6$. For $0 \le t \le 50\text{s}$, the time histories of the vertical displacement u_z at the middle of the helices by using 100 elements are depicted in Figure 3-Figure 5. As seen from Figure 3-Figure 4, as the number of active turns and the minimum radius of helix to the maximum radius of helix ratios increase, the magnitude and the vibration period of u_z increases. The magnitude of the vertical displacement u_z is inversely proportional with the retardation time τ_r^G (see Figure 5).

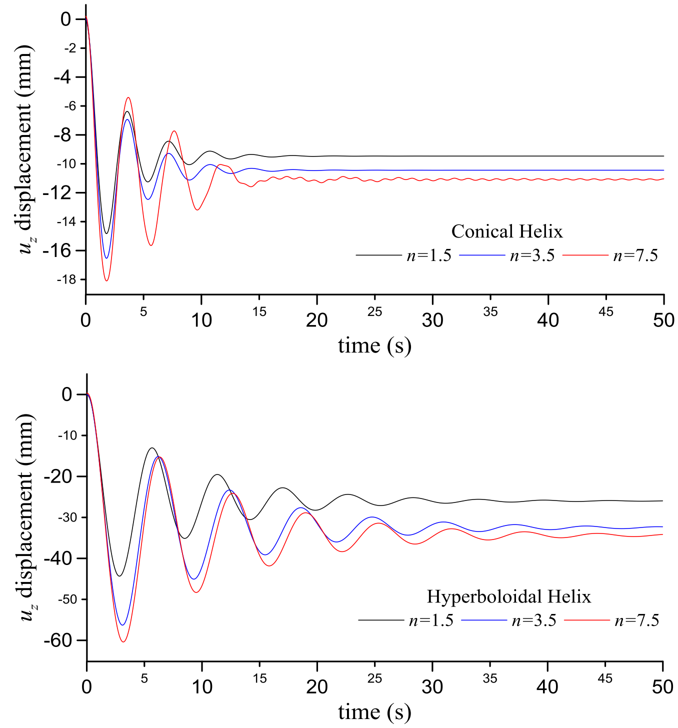

Figure 3. Time histories of viscoelastic conical and hyperboloidal helices for different number of active turns ($n = 1.5, 3.5, 7.5$).

IV. Conclusions

A mixed finite element formulation is developed for the dynamic analysis of viscoelastic conical and hyperboloidal helices based on the Timoshenko beam theory. The formulation is accomplished in Laplace space and the viscoelastic properties of a body are accounted using the correspondence principle. The viscoelastic material behavior is simulated by using the standard model. The finite element solutions are carried out in the Laplace space. The results obtained in transform space are inverted back to time space using modified Durbin's algorithm. In the presented parametric studies, the influences of the some parameters (*e.g.* the number of turns, the minimum radius of helix to the maximum radius of helix ratios, the retardation time) on viscoelastic behavior of the helicoidal structures are investigated. As far as the author's knowledge, dynamic analysis of conical and hyperboloidal helixes with elliptical hollow cross-section is original examples for the literature.

Acknowledgment

This research is supported by The Scientific and Technological Research Council of Turkey under project no 111M308. This support is gratefully acknowledged by the authors. The authors would like to thank Professor Dr. Mehmet H. Omurtag for his support.

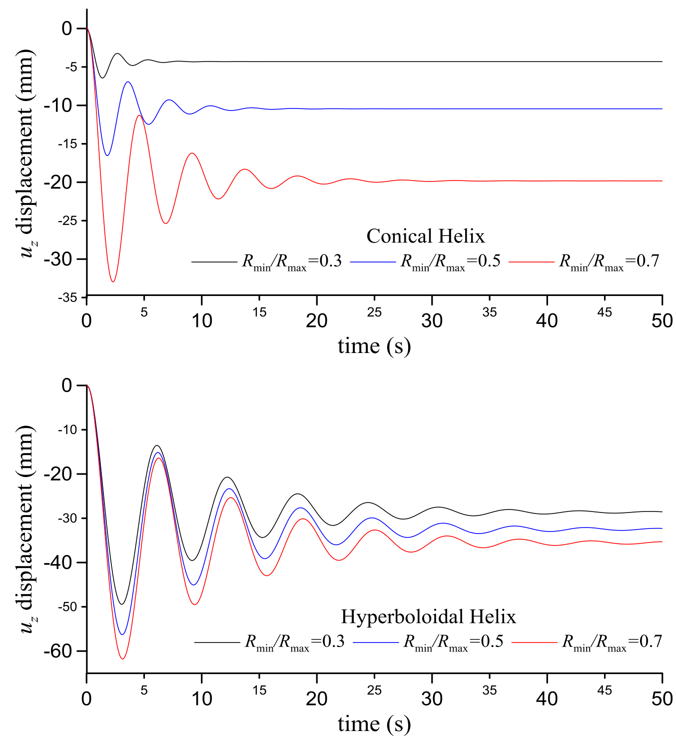

Figure 4. Time histories of viscoelastic conical and hyperboloidal helices for different minimum radius of helix to the maximum radius of helix ratios ($R_{\min} / R_{\max} = 0.3, 0.5, 0.7$).

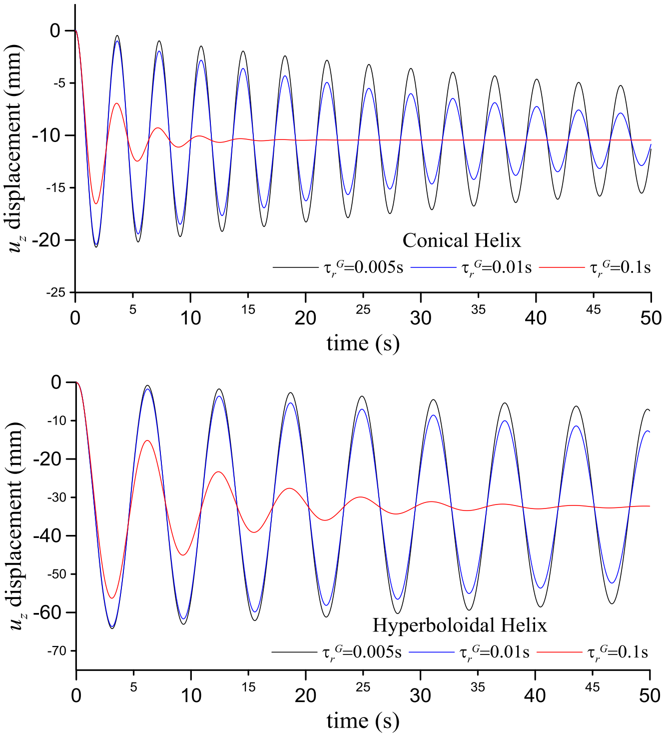

Figure 5. Time histories of viscoelastic conical and hyperboloidal helices for different retardation time ($\tau_r^G = 0.005\text{s}, 0.01\text{s}, 0.1\text{s}$).

References

[1] W. Flügge, Viscoelasticity, 2nd, rev. ed. edition, Springer, Berlin; New York, 1975.

[2] P.M. Christensen, Theory of Viscoelasticity: An Introduction, 2nd ed., Academic Press, New York, 1982.

[3] B.A. Boley, J.H. Weiner, Theory of Thermal Stresses, John Wiley and Sons, Inc, New York, 1960.

[4] Y. Aköz, F. Kadioğlu, The mixed finite element method for the quasi-static and dynamic analysis of viscoelastic timoshenko beams, Int. J. Numer. Methods Eng. 44 (1999) 1909–1932. doi:10.1002/(SICI)1097-0207(19990430)44:12<1909::AID-NME573>3.0.CO;2-P.

[5] F. Kadioglu, A.Y. Akoz, The mixed finite element for quasi-static and dynamic analysis of viscoelastic circular beams, Struct. Eng. Mech. 15 (2003) 735–752. doi:10.12989/sem.2003.15.6.735.

[6] B. Temel, F. Fırat Çalim, N. Tütüncü, Quasi-static and dynamic response of viscoelastic helical rods, J. Sound Vib. 271 (2004) 921–935. doi:10.1016/S0022-460X(03)00760-0.

[7] H. Argeso, N. Eratli, F.F. Çalım, M.H. Omurtag, Analysis of viscoelastic conical helixes via mixed finite element method, in: Baku, Azerbaijan, 2011: pp. 102–106.

[8] H. Argeso, N. Eratli, F.F. Çalım, M.H. Omurtag, Dynamic analysis of viscoelastic cylindrical helixes subjected to impulsive-sinusoidal load by using the finite element method, in: METU, Ankara, Turkey, 2012.

[9] N. Eratlı, H. Argeso, F.F. Çalım, B. Temel, M.H. Omurtag, Dynamic analysis of linear viscoelastic cylindrical and conical helicoidal rods using the mixed FEM, J. Sound Vib. 333 (2014) 3671–3690. doi:10.1016/j.jsv.2014.03.017.

[10] H. Dubner, J. Abate, Numerical inversion of Laplace transforms by relating them to the finite Fourier cosine transform, J. ACM. 15 (1968) 115–123. doi:10.1145/321439.321446.

[11] F. Durbin, Numerical inversion of Laplace transforms: an efficient improvement to Dubner and Abate's method, Comput. J. 17 (1974) 371–376. doi:10.1093/comjnl/17.4.371.

[12] G.V. Narayanan, Numerical Operational Methods in Structural Dynamics, Ph.D., University of Minnesota, 1980. http://0-search.proquest.com.divit.library.itu.edu.tr/pqdtglobal/docview/303016499/E521E750154E49ECPQ/1?accountid=11638 (accessed July 18, 2014).

[13] M.H. Omurtag, A.Y. Aköz, The mixed finite element solution of helical beams with variable cross-section under arbitrary loading, Comput. Struct. 43 (1992) 325–331. doi:10.1016/0045-7949(92)90149-T.

[14] K. Girgin, Free vibration analysis of non-cylindrical helices with, variable cross-section by using mixed FEM, J. Sound Vib. 297 (2006) 931–945. doi:10.1016/j.jsv.2006.05.001.

[15] J.T. Oden, J.N. Reddy, Variational methods in theoretical mechanics, Springer-Verlag, 1976.

About Authors:

Nihal Eratlı
Assoc.Prof.Dr.

Akif Kutlu
Dr.