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Risk Pooling with Transshipment Under Fill-Rate Based Inventory Decisions

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***Abstract*—This paper considers a supply chain consisting of one manufacturer and multiple retailers. The retailers can use transshipment as a recourse action to satisfy their unmet demands, and they base their inventory decisions on a fill rate constraint. We study the impact of transshipment on supply chain members' performance. We show that the impacts of transshipment are different under fill rate based inventory decisions from under profit-maximization based inventory decisions. In particular, when the retailers have only one purchasing opportunity, transshipment always hurts the manufacturer because retailers order less with transshipment. Further, the transshipment hurts the manufacturer more whenever the value of the transshipment for the retailers is high. On the other hand, for an infinite horizon case when retailers have multiple ordering opportunities, the retailers' expected ordering quantities remain unchanged with transshipment; however, the manufacturer still benefits from transshipment because its demand becomes less variable with transshipment.**

***Keywords*—transshipment, risk pooling, supply chain**

I. Introduction

Inventory sharing through transshipment is an effective risk pooling strategy to manage demand uncertainty by using virtual centralization of stocks to satisfy multiple demand streams. However, this strategy comes at a cost. In particular, an information system has to be implemented to achieve inventory transparency among participating firms. In order to justify the investment in information systems and additional administrative activities to facilitate transshipment, one has to address the question of whether transshipment benefits the participating firms and, if it indeed benefits them, when transshipment benefits the participating firms most. In some industries, an upstream manufacturer has to invest in the information systems for its retailers. Consequently, understanding the possible impact of risk pooling among downstream firms on the performance of the upstream manufacturer is vital.

Analytical studies of risk pooling traditionally compare two distinct models: a no-risk pooling model, where each stream of supplies satisfies a corresponding stream of demands, and a risk-pooling model, where a common stream of supplies satisfies multiple streams of demands. The impact of risk pooling is then drawn by comparing the optimal inventory solutions for the two different models, and the optimal inventory decisions for both models are derived by assuming a decision maker optimizes his or her expected profit or cost. Practitioners, however, often make inventory decisions by trading off inventory cost against customer service levels. The natural question then is: What is the impact of risk pooling when inventory decisions are based on customer service level constraints instead of profit maximization?

This paper focuses on one particular form of risk pooling, transshipment, under the assumption that inventory decisions are made under a constraint on the fill rate (i.e., the fraction of demand met immediately from on-hand stocks), and studies the impact of transshipment on a supply chain that consists of one upstream firm (manufacturer) and multiple downstream firms (retailers) among which inventories can be transshipped. Our analysis demonstrates that the effects of transshipment on the supply chain members' performance are quite different under fill rate based inventory decisions from under the profit-maximization based inventory decisions. We show that, as expected, transshipment benefits the retailers more when the retailers' demands are less correlated or more retailers participate in inventory transshipment. However, under the fill rate based inventory decisions, the impacts of transshipment on the manufacturer's performance are not affected by the product margin as in the profit maximization framework. In particular, if the retailers have only one purchasing opportunity, they always order less with transshipment. Consequently, transshipment among retailers always hurts the manufacturer. Moreover, transshipment hurts the manufacturer more when the retailers' demands are less correlated or more customers participate in inventory transshipment. For the infinite horizon with lost sales case, the retailers' expected orders remain the same with transshipment, and the retailers' orders become less variable with transshipment. Consequently, transshipment benefits the manufacturer.

When the transshipment cost is zero, this paper is a fill rate version of Eppen (1979). Papers that study the performance of inventory systems with transshipment in a central decision maker framework typically assume that the inventory decisions are made to either maximize the profit or to minimize the cost; see, for example, [2],[4],[6],[9] , [10],[13], and [15], among others. The reader is referred to Van Mieghem and Rudi (2002) for an overview of related literature. When inventory decisions are based on profit/cost optimization, the optimality condition for the inventory levels prescribes a Type-I service level (in-stock probability). This work, to our knowledge, is the first one that studies a fill rate model of risk pooling with transshipment. The remainder of the paper is organized as follows. Section II presents the

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II. Model and Notation

This paper considers a supply chain consisting of one manufacturer and n retailers. The retailers purchase a product from the manufacturer and sell the product to their customers. The retailers' inventories are replenished periodically, and the replenishment activities are controlled by a central planner (the distributor). At the beginning of each period, the distributor places an order with the manufacturer, then the orders are delivered immediately to all the retailers after which demand materializes. Retailers' demands in each period are random. Let D_i be the demand of retailer i in a generic period and denote $D=(D_1,\ldots,D_n)$ as the demand vector. D is a continuous random vector and $E[D_i]=\mu_i$ for $i=1,2,\ldots,n$. No assumption is made on the correlation structure between D_i's in the same period; however, the demands in different periods are assumed to be independent and identical. When transshipment is used, if the demand at one retailer cannot be satisfied with local inventories, it may be satisfied using inventories from another retailer through transshipment at a per-unit transshipment cost τ. The transshipment time is assumed to be negligible.

This paper considers two cases in terms of time horizons. In the first case, we assume that the distributor has a single purchasing opportunity for the selling season. In the second case, we study the situation where retailers can make multiple purchases in an infinite horizon setting. In the single period case, the distributor decides on the order quantity for each retailer such that the fill rate is at least β and the expected total inventory cost (the sum of inventory holding cost and transshipment cost) is minimized. In the infinite horizon case, the retailers' inventory replenishment follows a base stock policy, and the distributor determines the base stock levels such that the fill rate is at least β and the expected total inventory cost per period is minimized.

The fill rate of the system is defined as the fraction of demand of all retailers met immediately from on-hand stocks or through transshipment. Consequently, the fill rate we consider in this paper is the overall fill rate of the n retailers. This is a proper definition for our problem because a central planner controls the inventory decisions for all retailers. One may think of the n retailers as different retailing stores in one metropolitan area operated by a firm (such as Circuit City). In such cases, the firm cares more about the overall fill rate than the fill rate of each individual store.

In both the single period and the infinite horizon cases, the inventory holding cost is assessed based on the inventory level at the end of each period, the holding cost is assumed to be linear in the amount of leftover inventories, and the per-unit inventory holding cost is h. We further assume that $h>\tau$, that is, it is economically profitable to transship a product ex post. For a random variable D, denote $F_D(.)$ as its distribution with $F_D^{-1}(\cdot)$ as the quantile function. This paper uses boldface letters throughout to denote n-dimensional vectors of the corresponding variables; for example, $\boldsymbol{D}=(D_1,\ldots,D_n)$, $\boldsymbol{Q}=(Q_1,\ldots,Q_n)$, and so on.

III. Single Period Analysis

In this section, we study the case where the retailers order only once from the manufacturer.

A. Retailers' Optimal Order Quantities

Without transshipment, each retailer satisfies its own demand. For a given order quantity Q_i and a demand realization D_i at retailer i, the amount of retailer i's demand that can be satisfied immediately from on-hand stocks is $\min(D_i,Q_i)$ and the leftover inventory is $(Q_i-D_i)^+$, where $x^+=\max(x,0)$. (We use the same notation for a random variable and its realization throughout the paper, but their meanings are clear in the context and should cause no confusion.) Note that $\min(D_i,Q)=D_i-(D_i-Q)^+$, and $E[D_i]=\mu_i$. So, for the order vector Q, the retailers' fill rate is $1-\sum_{i=1}^{n}E[D_i-Q_i]^+/\sum_{i=1}^{n}\mu_i$. Because no transshipment is involved, the expected total inventory cost of all retailers is $C^N(Q)=h\sum_{i=1}^{n}E[Q_i-D_i]^+$. The retailers' problem can be formulated as $\min_{Q\geq 0}\{C^N(Q)|1-\sum_{i=1}^{n}E[D_i-Q_i]^+\sum_{i=1}^{n}\mu_i\geq\beta\}$.

Clearly, $C^N(Q)$ increases in Q. So, at optimality, the retailers' order quantities must satisfy $\sum_{i=1}^{n}E[D_i-Q_i]^+=(1-\beta)\sum_{i=1}^{n}\mu_i$. Therefore, we can reformulate the retailers' problem as

$$\min_{Q\geq 0} h\sum_{i=1}^{n}E[Q_i-D_i]^+, \quad (1)$$

$$\text{s.t. } \sum_{i=1}^{n}E[D_i-Q_i]^+=(1-\beta)\sum_{i=1}^{n}\mu_i. \quad (2)$$

A straightforward Lagrangian analysis shows that the optimal order quantity Q_i^N of retailer i without transshipment must satisfy, for some $\alpha^N\in[0,1]$,

$$Q_i^N=F_{D_i}^{-1}(\alpha^N), \quad (3)$$

$$\sum_{i=1}^{n}E[D_i-F_{D_i}^{-1}(\alpha^N)]^{+1}=(1-\beta)\sum_{i=1}^{n}\mu_i. \quad (4)$$

We now determine the retailers' order quantities with transshipment. For an order vector Q and demand realizations D, the total sales after transshipment is $\sum_{i=1}^{n}D_i-(\sum_{i=1}^{n}D_i-\sum_{i=1}^{n}Q_i)^+$. It follows that the expected total sales is $\sum_{i=1}^{n}\mu_i-E[\sum_{i=1}^{n}D_i-\sum_{i=1}^{n}Q_i]^+$. Therefore, the fill rate for the order vector Q is

$$1-E[(\sum_{i=1}^{n}D_i-\sum_{i=1}^{n}Q_i)^+]/\sum_{i=1}^{n}\mu_i \quad (5)$$

Because the total sales without transshipment is $\sum_{i=1}^{n}\min(D_i,Q_i)$, the total transshipment quantity is $\sum_{i=1}^{n}(D_i-Q_i)^+-(\sum_{i=1}^{n}D_i-\sum_{i=1}^{n}Q_i)^+$. Clearly, the total leftover inventory after transshipment is $(\sum_{i=1}^{n}D_i-\sum_{i=1}^{n}Q_i)^+$. Therefore, the expected inventory cost under transshipment is

$$C^T(Q) = (h-\tau)E[\sum_{i=1}^n Q_i - \sum_{i=1}^n D_i]^+ + \tau\sum_{i=1}^n E[Q_i - D_i]^+. \quad (6)$$

It is easy to see that $C^T(Q)$ increases in Q. So, the retailers' optimal order quantities Q^T with transshipment must satisfy $E[\sum_{i=1}^n D_i - \sum_{i=1}^n Q_i]^+ = (1-\beta)\sum_{i=1}^n \mu_i$. Consequently, the retailers' problem with transshipment can be formulated as

$$\min_{Q\geq 0} (h-\tau)E[\sum_{i=1}^n Q_i - \sum_{i=1}^n D_i]^+ + \tau\sum_{i=1}^n E[Q_i - D_i]^+$$

$$\text{s.t. } E[\sum_{i=1}^n D_i - \sum_{i=1}^n Q_i]^+ = (1-\beta)\sum_{i=1}^n \mu_i. \quad (7)$$

It follows that the optimal solution to the above problem must satisfy, for some $\alpha^T \in [0,1]$,

$$Q_i^N = F_{D_i}^{-1}(\alpha^T), \quad (8)$$

$$\sum_{i=1}^n E[D_i - F_{D_i}^{-1}(\alpha^T)]^{+1} = (1-\beta)\sum_{i=1}^n \mu_i. \quad (9)$$

From (8) and (9), we see that the per-unit transshipment cost τ does not affect retailers' optimal ordering quantities. This can be explained as follows: In our model, the retailer's total expected cost increases with the retailers' order quantities. As a result, the central planner will use as much transshipment as possible ex post such that the order quantities can be minimized while the fill rate constraint can still be satisfied. Consequently, the magnitude of the per-unit transshipment cost does not affect the retailers' order quantities. Note that this result is based on the assumption that the marginal transshipment cost is smaller than the per-unit inventory holding cost.

Equations (3)-(4) and (8)-(9) are key results for the analysis in the remainder of the paper. They reduce the retailer's problems into one-dimensional optimization problems. Now the retailers' optimal order quantities can be found by searching for α^N and α^T, and the impact of transshipment can be studied by comparing α^N and α^T.

B. *Impacts of Transshipment*

We first examine the effects of transshipment on the manufacturer's performance. Because the manufacturer's performance is determined by the retailers' total order quantity, the impact of transshipment can be studied by examining the relationship between the retailers' order quantities under transshipment and no transshipment. Proposition 1 shows that retailers always order less with transshipment; therefore, transshipment always hurts the manufacturer. Proposition 1 also characterizes the effects of demand variability and correlation on the retailers' order quantities. To compare the variability and correlation of different demand distributions, we use the concepts of convex order and supermodular order. Specifically, a demand vector $\boldsymbol{D}^1$ is said to be less variable (or less correlated) than a demand vector $\boldsymbol{D}^2$ if $\boldsymbol{D}^1$ is less than $\boldsymbol{D}^2$ in convex order (or in supermodular order, respectively); see, for example, [7], for more information on stochastic comparison.

Proposition 1 *(i) Transshipment always hurts the manufacturer. (ii) With transshipment, the retailers' optimal order quantities increase in demand variability and demand correlation. (iii) With transshipment, the order quantity per retailer decreases in the number of retailers if all retailers are identical. (iv) Transshipment hurts the manufacturer even more when retailers' demands are less correlated.*

From Proposition 1 we see that the impacts of transshipment on the manufacturer's performance for the single period problem clearly depend on how the retailer's ordering decisions are made. It is well known that, when the retailers' ordering decisions are based on profit maximization, the margin of the product is a defining factor on whether the manufacturer benefits from transshipment and the effects of problem parameters (such as demand variability, demand correlation, or the number of participating retailers) on the manufacturer's performance. When the retailers' ordering decisions are fill rate based, the role the product margin plays on the impact of transshipment disappears. Next, we study the value of transshipment to retailers and examine the effects of problem parameters on the value of transshipment by comparing the retailers' inventory cost with and without transshipment.

Let C^N be the retailers' expected inventory cost without transshipment at optimality. Then, using (2), we obtain

$$C^N = h\sum_{i=1}^n Q_i^N - h\beta\sum_{i=1}^n \mu_i. \quad (10)$$

Similarly, let C^T be the retailers' expected inventory cost with transshipment at optimality. Then,

$$C^T = (h-\tau)E[\sum_{i=1}^n Q_i^T - \sum_{i=1}^n D_i]^+ + \tau E[\sum_{i=}^n (Q_i^T - D_i)^+]. \quad (11)$$

It follows from (10) and (11) that

$$C^N - C^T = h\sum_{i=1}^n (Q_i^N - Q_i^T) - \tau(\sum_{i=1}^n E[D_i - Q_i^T]^+ - \sum_{i=1}^n E[D_i - Q_i^N]^+). \quad (12)$$

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$C^N - C^T$ is the reduction in the inventory cost of the retailers as a result of transshipment. By studying the behavior of $C^N - C^T$, we can characterize the impact of transshipment on the retailer's performance.

Proposition 2 *(i) Transshipment always benefits the retailers. (ii) Retailers benefit more from transshipment if retailers' demands are less correlated or more retailers participate the transshipment. (iii) With transshipment, the retailers' cost increases with respect to demand variability.*

Comparing Propositions 1and 2, we see that transshipment affects supply chain members in different ways: it benefits the retailers but hurts the manufacturer. Further, when demands are less correlated, transshipment benefits the retailer more, but hurts the manufacturer more.

IV. Multi-Period Analysis

This section considers the case where the retailers have multiple ordering opportunities. As in [1], we consider the infinite horizon case with lost sales and assume the retailers' replenishment lead time is zero. We assume that the retailers' inventory replenishment follows a base-stock policy, and the base-stock levels are set such that the retailers' expected inventory cost is minimized and the fill rate is at least β.

Without transshipment, for a given base-stock level S_i, the expected sales of retailer i in one period is $E[\min(D_i, S_i]$ and the expected inventory level per period is $S_i - E[\min(D_i, S_i]$. Thus, the retailers' problem can be formulated as

min

$$\min \sum_{i=1}^{n} h\{S_i - E[\min(D_i, S)]\}$$

$$\text{s.t. } \sum_{i=1}^{n} E[\min(D_i, S_i)] = \beta \sum_{i=1}^{n} \mu_i.$$

Comparing the retailers' problem without transshipment in the infinite horizon case with that in the single period case we see that retailers' optimal base-stock levels S_i^N are determined by (3) and (4), which implies that $S_i^N = Q_i^N$ for all i. Further, at optimality, the retailers' expected inventory cost per period without transshipment is $\sum_{i=1}^{n} h(S_i^N - \beta\mu_i)$.

Similarly, with transshipment, the retailers' base-stock levels S_i^T are determined by (8) and (9), that is, $S_i^T = Q_i^T$. For the base-stock level S_i^T, the expected total sales is $\beta \sum_{i=1}^{n} \mu_i$, the expected total leftover inventory is $E[\sum_{i=1}^{n} S_i^T - \sum_{i=1}^{n} D_i]^+$, and the expected quantity of transshipped inventory per period is $\sum_{i=1}^{n} [S_i^T - D_i]^+ - E[\sum_{i=1}^{n} S_i^T - \sum_{i=1}^{n} D_i]^+$. So, the retailers' expected inventory cost per period with transshipment is $\tau \sum_{i=1}^{n} [S_i^T - D_i]^+ + (h - \tau) E[\sum_{i=1}^{n} S_i^T - \sum_{i=1}^{n} D_i]^+$. Therefore, our analysis for the impact of transshipment on the retailer's performance continues to hold for the infinite horizon case with Q replaced by S. Consequently, the effects of transshipment on the retailer's performance are identical to the effects in the single period case.

To see the impact of transshipment on the manufacturer's performance, we compare the order streams from the retailers with or without transshipment. Because a base-stock policy is followed and unsatisfied demand is lost, the retailers' total order from the manufacturer in each period is their total sales in the previous period in steady state. Therefore, retailers order $O^N = \sum_{i=1}^{n} \min(D_i, S_i^N)$ in each period without transshipment and order $O^T = \min(\sum_{i=1}^{n} D_i, \sum_{i=1}^{n} S_i^T)$ with transshipment. O^N and O^T are the manufacturer's demands without and with transshipment. In order to understand the impact of transshipment on the manufacturer's performance, we compare O^N and O^T.

Proposition 3 *(i) $O^T \leq_{cx} O^N$, that is, the manufacturer's demands become less variable with transshipment. (ii) With transshipment, the manufacturer's demands become less variable when the retailers' demands become less correlated. (iii) With transshipment, the manufacturer's demands become less variable when the retailers' demands become less variable.*

Thus, transshipment reduces the demand variability of the manufacturer. Because variability, in general, has a deteriorating effect on the performance of stochastic inventory systems, the manufacturer is better off with transshipment. By Propositions -, both the retailers and the manufacturer benefit from the transshipment for the infinite horizon case; the retailers benefit from the transshipment because their inventory cost is reduced while the manufacturer benefits from the transshipment because its demand variability decreases. Further, both the manufacturer and the retailers benefit more from transshipment when the retailers' demand correlation is low.

From Propositions 1 and 3, we see that transshipment affects the manufacturer in totally different ways in the two cases we have examined. In the single period setting, retailers order less because transshipment allows them to provide a higher fill rate with the same amount of inventory. As a result, transshipment hurts the performance of the manufacturer in the single period case. In the infinite horizon case, retailers's "effective" demands become less variable under transshipment, which translates into lower demand variability for the manufacturer. However because of the fill rate constraints, the retailers' expected order quantities remain the same.

In practice, in order to encourage retailers to participate in the transshipment, some manufacturers either pay for the transshipment of their products among retailers or agree to buy back the leftover inventories at the end of a product's life cycle (effectively reducing the inventory holding cost for the retailers) or do both; see, for example, [14]. From (1), we see that the reduction in the inventory cost of the retailers as a result of transshipment increases in inventory holding cost (h) and decreases in the per-unit transshipment cost (τ). Therefore, paying for the transshipment cost encourages retailers to participate the transshipment is quite savvy, because it effectively reduces the demand variability of the manufacturer (Proposition). However, buying back the leftover inventory may actually discourage the retailers' participation in inventory transshipment, thereby, hurting the manufacturer.

V. Conclusion

We study the impact of transshipment among retailers on the performance of supply chain members when the inventory decisions are fill rate based using stochastic comparison approach. Our analysis demonstrates that while the impact of transshipment on the retailer is similar to the case under the profit maximization framework, its impact on the performance of the manufacturer is different from under the profit maximization framework. In particular, the product margin does not affect the impact of transshipment when the inventory decision is fill rate based. In our analysis, the retailers' ordering decisions are made by a central planner, it would be interesting to study the case where retailers' ordering decisions are made individually in future research.

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