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Truthful Combinatorial Double Auction For Combinatorial Exchanges

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Abstract—***First-come-first-serve (FCFS) scheme is used for selling the goods in market that is a multi-million dollar industry for any popular event. But in a competitive environment is this FCFS efficient? In earlier literature it has been shown that the auction based alternative solutions using the framework of mechanism design, a sub field of game theory [18] can provide better results against FCFS in terms of profit making and efficiency in allocation. However the solution proposed in the earlier literature can address the selling environment where an agent can give demand for a single good that is they have single/Multi unit demand. However there might be a situation where a seller wishes to sell in the combination of goods and buyer wishes to buy in some combinations. It is highly difficult to solve a combinatorial double auction where sellers and buyers bids simultaneously. In this paper we proposed two algorithms of which one is truthful produces a solution which is very near to optimal by assuming some prefix base value for each good and another algorithm provides a solution by dividing the seller sub group and buyer sub group.***

Keywords—**Double Auction, Combinatorial Exchanges, Mechanism Design.**

I. Introduction

When the available goods of any event is sold by the event organizers, huge market is created in terms of selling the available goods against a huge demand in the market. The major events comprising of sports, railways, entertainments, airlines, and many others have a huge demand of the tickets that are sold by the event organizers. By allocating the available tickets to the competing customers, the event organizers gain some profit. To maximize the sellers profit and to reach the buyers demand Auction [14] based selling Mechanisms have been introduced. There are many types of auction mechanisms were present in the market today: English auction, Dutch auction, sealed first price auction, Vickery auction, double auction and Multiunit auctions. English auction is the most common form of auction in history. Participants bid openly against one another, with each subsequent bid higher than the previous bid. The auction ends when no participant is willing to bid further, at which point the highest bidder pays their bid. Alternatively, if the seller has set a minimum sale price in advance (the 'reserve' price) and the final bid does not reach that price, the item remains unsold. Dutch auction also known as an open descending price auction. In the traditional Dutch auction the auctioneer begins with a high asking price which is lowered until some participant is willing to accept the auctioneer's price. The winning participant pays the last announced price. Sealed first-price auction is also known as a first-price sealed-bid auction (FPSB). In this type of auction all bidders simultaneously submit sealed bids so that no bidder knows the bid of any other participant. The highest bidder pays the price they submitted. This type of auction is distinct from the English auction, in that bidders can only submit one bid each. In our earlier art we proposed a truthful multiunit ticket booking scheme [15] in static environment where an agent can give demand for multiple tickets in static environment. Furthermore, as bidders cannot see the bids of other participants, they cannot adjust their own bids accordingly. Vickery auction is also known as a sealed-bid second-price auction. Double auctions are gaining attention in recent times where sellers as well as buyers will bid for common items. Multiunit auctions sell more than one identical item at the same time, rather than having separate auctions for each. This type can be further classified as either a uniform price auction or a discriminatory price auction.

Apart from the above mentioned type combinatorial auctions are gaining popularity because it leads to more efficient allocations than traditional auction mechanisms in multi-item auctions where the agents' valuations of the items are not additive [12-14]. Combinatorial auctions allow the simultaneous sale of more than one item. Bidders can place bids on a combination of goods according to personal preferences rather than just individual items. Combinatorial auctions are beneficial if complementarities exist between the items to be auctioned. Allowing bids for bundles of items is the foundation of combinatorial auctions. Bidders can select multiple items at one time and offer those items a price. Well-known combinatorial auction examples are the auctioning of Federal Communications Commission's radio spectrum licenses, the sales of airport time slots, and allocation of delivery routes, and some times during clearance sale in factory etc..,. So far many mechanisms have been introduced to motivate bidders to bid their true valuations for avoiding market manipulation. VCG mechanism is one of them which allows bidders to bid their true value in order to increase their utility but in case of combinatorial exchanges it failed to achieve truthfulness [11]. In this paper we provided two algorithms of which one provides solution by separating the seller and buyer groups and comparing with one another. Although this algorithm provides solutions and generates revenue, it may be manipulated by agents in the market. The proof of this will be provided in the later part of this paper. To overcome this we proposed another algorithm which reduces the combinatorial double auction into normal double auction by dividing the combinations proportionally with their equilibrium value. The following figure shows a typical combinatorial buyer and seller scenario.

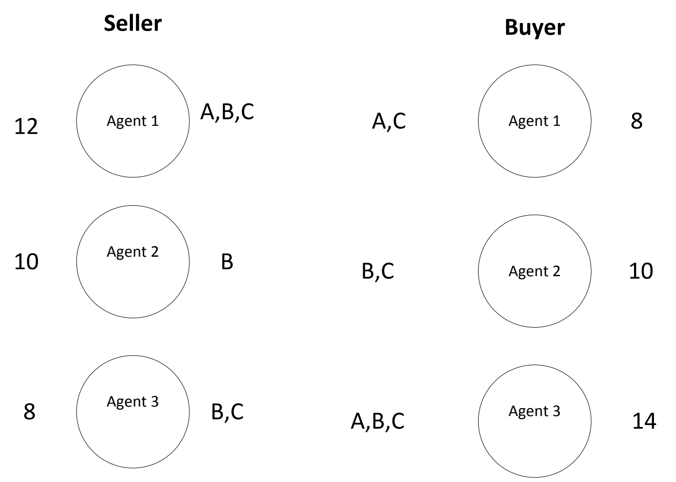

Figure 1

II. Characterization of Problem

Each Customer who wishes to buy goods is represented as buyer and each customer who wishes to sell goods is represented as Seller. Now a buyer can bid for either individual items or in the combination of items. Similarly the seller can ask for Individual or in combination of items. Let there are m sellers and n buyers $I = \{1, 2, ..m\}$ denote the set of all sellers and $J = \{1, 2, ..n\}$ denote set of all buyers. The agents may reveal their true valuation or may misreport. The type of a seller agent is described by $\widehat{\theta_i^s} = (\hat{v}_i^s)$. Here $(\hat{v}_i^s)$ is the marginal valuation vector and formally the valuation vector could be defined as $\hat{v}_i^s = < \hat{v}_{i,1}^s, \hat{v}_{i,2}^s, .. \hat{v}_{i,j}^s >$. The type of a buyer agent is described by $\widehat{\theta_i^w} = (\hat{v}_i^w)$. Here $(\hat{v}_i^w)$ is the marginal valuation vector and formally the valuation vector could be defined as$\hat{v}_i^w = < \hat{v}_{i,1}^w, \hat{v}_{i,2}^w, .. \hat{v}_{i,j}^w >$. Where$\hat{v}_{i,j}^w$ represent the bid value of agent 'i' for j^{th} item. Where $\hat{v}_{i,j}^s$ represents the ask value of agent 'i' for j^{th} item. The payment for the buyer agent 'i' is denoted by p_i^w. Similarly the payment of seller is denoted by p_i^s. The utility of buyer or seller agent 'i' is given in the Eq 1. A_k is availability of item k. D_k is demand of item k.

$$u_i^a = \hat{v}_i^a - p_i^a \quad \forall\, a \in \{w, s\} \qquad \text{Eq 1}$$

III. Existing Schemes

The main existing scheme that is prevailing in the ticket market for selling multiple tickets against the multiunit demand by the agents is FCFS. In FCFS scheme the agents line up in a queue in front of a counter and get the tickets if the tickets are not exhausted when their turn has come. In e-environment the demand is given with some electronic medium (such as Internet) and the tickets are allocated instantly until exhausted. An agent may purchase multiple tickets. The algorithm is given in FCFSM (FCFS for multiple demands).However this FCFSM does not provide equal opportunity to every agent and hence it cannot be said that efficiency in allocation is being provided. Moreover the mechanism provided here is not truthful. VCG proposed a mechanism where the agents can't gain by misreporting their valuation.

However the mechanisms stated above are only one sided auctions where only buyers/sellers can bid at once. McAfee proposed a mechanism for double auctions where asks were sorted in non-decreasing order and bids were sorted in non-increasing order. It then checks the least possible case where the transaction is possible and then leaving that transaction aside (since least profitable transaction) and then the bids above that and asks less than that value are declared as winners. It is better illustrated in the following Table 1.

Let $\hat{v}_i^w = < 2,9,8,7,3,11 >$ and $\hat{v}_i^s = < 3,5,4,8,6,11 >$

Before Sort				After Sort			
Buyer		Seller		Buyer		Seller	
w_1	2	s_1	3	w_6	11	s_1	3
w_2	9	s_2	5	w_2	9	s_4	4
w_3	8	s_3	4	w_3	8	s_2	5
w_4	7	s_4	8	w_4	7	s_5	6
w_5	3	s_5	6	w_5	3	s_3	8
w_6	11	s_6	11	w_1	2	s_6	11

Table 1

In the left part of the table it shows the buyers and sellers and their valuations and in the tight part it shows that buyers are sorted in non-increasing order and sellers are sorted in non-decreasing order and the shaded region tells us the least possible transaction that could be possible and the agents above that region are declared as winners. So in this case $w_6, w_2, w_3, s_1, s_4 s_2$ are declared as winners and buyers payment is 7$ each and sellers payment is 6$ each and auctioneers profit is (7-1) x 3 = 3$.

IV. Proposed Algorithms

In this section 2 auction based algorithms are proposed out of which 1 provides the result but not truthful and 2nd produces the truthful result but in case of threshold auction it provides result by either taking the partial dummy valuation from agents or by fixing the limits by auctioneer. The contribution of this paper is that all the concepts of MD are adopted in algorithmic framework in the proposed schemes for selling tickets in the ticket market where each agent can give demand in the combination of goods and also the comparisons of the algorithms are presented with stylized experiments. Many of the algorithms proposed earlier failed to explain the situation where goods are available in combinations and the price is truthful. In this paper two algorithms have been tried in one place and comparisons are made accordingly.

The two algorithms proposed here are:

1) Iterative Combinatorial Double Auction mechanism for Combinatorial Exchanges (ICDAM).
2) Truthful Combinatorial Double Auction Mechanism for Combinatorial Exchanges (TCDAM).

For each of the algorithm one separate auction is run and demand for each auction is collected separately. ICDAM doesn't provide truthful results so taking in to consideration the real time scenario where individual items will also be available along with combinations we designed a truthful mechanism and proof for truthfulness is given in later part of this paper which is another major contribution of this paper.

A. In ICDAM, sellers' tries to maximize their revenue and buyers' try to minimize their procurement cost. This algorithm considers the entire bids as one bundle and all asks as another bundle. It starts with a maximum procurement cost and with minimum trading cost for sellers. Then in each iteration it checks all the possible combinations of goods available at a particular range and then checks the availability and increases the value by ϵ. The algorithm for this auction is shown below.

Iterative Combinatorial Double Auction Mechanism for Combinatorial exchanges (ICDAM).

1. Collect the bids and asks from the agents. Let $R(S_t)$ be the Reserve price and $W(B_t)$ be the total value of the bundle.
2. Start with initial values taking reserve price to be 0 for sellers sub problem and taking maximum value for buyers sub problem.
3. Solve the sellers sub problem according to the equation below to maximize the revenue of the goods.

$$R(S_t) \leq V^* < W(B_t)$$
$$V^* = \max \sum_{j \in N} \sum_{S \subseteq G} \hat{v}_i^b \, y(S,i)$$
$$s.t \sum_{k \in S} \sum_{i \in M} a(S,k) y(S,i) \leq A_k \;\; \forall k, \forall S \subseteq G$$
$$y(S,j) = 0,1 \;\; \forall S \subseteq G, \forall j \in N$$

4. Similarly solve the buyers sub problem according to the equation below to minimize the procurement cost.

$$R(S_t) \leq V^* < W(B_t)$$
$$V^* = \min \sum_{i \in M} \sum_{S \subseteq G} \hat{v}_j^s \, y(S,j)$$
$$s.t \sum_{k \in S} \sum_{j \in N} a(S,k) y(S,j) \geq D_k \;\; \forall k, \forall S \subseteq G$$
$$y(S,j) = 0,1 \;\; \forall S \subseteq G, \forall j \in M$$

5. If the supply and demand constraints are satisfied according to the equation below

$$\sum_{k \in S} \sum_{i \in N} a(S,k) y(S,j) \geq \sum_{k \in S} \sum_{i \in M} a(S,k) y(S,i)$$

Then end the auction.

6. Update the prices by ϵ. Go to step 3.

Here step 3 and step 4 will match with the seller and buyer groups. Step 5 is the exit condition which states that if the payment of buyer bundle and seller bundle matches the given constraints then check whether total availability of goods for seller bundle is greater than buyer bundle then end the auction. It is shown in detail in the following example for the scenario shown in Figure 1. For readability all existing cases are shown whereas in reality there will be no repetition of similar cases.

Buyers Sub Problem			Sellers Sub Problem		
ϵ	Agents	Availability	ϵ	Agents	Availability
30	1,2,3	2A,2B,3C	10	2	B
25	1,2,3	2A,2B,3C		3	B,C
20	1,3	2A,B,2C	15	1	A,C
	2,3	A,2B,2C		2	B,C
15	1	A,B,C		3	A,B,C
	2	B	20	1,3	A,2B,2C
	3	B,C		2,3	2B,C

Example 1 Showing the agents and availability of goods taking $\epsilon = 5$.

The algorithm that can be easily vulnerable to manipulation since the final payments for the agents does not depend upon their individual valuation of goods hence it is not a feasible solution. The following algorithm takes in to account the face value of each good and fixes some base value for each of them. Then it finally separates the original combinatorial auction into simple double auction where we apply the concept for McAfee for winner determination [9].

Truthful Combinatorial Double Auction Mechanism for Combinatorial Exchanges (TCDAM).

1. Take bid vector and sort them in non-increasing order for each type of goods (without considering combinations). Take ask vector and sort them in non-decreasing order for each type of goods (without considering combinations).
2. Find the winning price for each goods according to McAfee Double auction and save the result.
   Let $p_{eq,i}^w \;\; \forall i \in S$ be the result of price generated for buyers according to McAfee Double auction and let $p_{eq,i}^s \;\; \forall i \in S$ be the price generated for sellers according to McAfee Double Auction.
3. For each combination, if $\hat{v}_i^w$ is greater than $\sum_{i \epsilon S} p_{eq,i}^w$ then set the prices for each of the auction proportionally to $p_{eq,i}^w$.

i.e. If $\hat{v}_i^w \geq \sum_{i\epsilon S} p_{eq,i}^w$

then $\hat{v}_{i,j}^w = \hat{v}_{i,j}^w * (\frac{p_{eq,j}^w}{p_{eq,j}^w})$

4. Similarly for sellers, if $\hat{v}_i^S$ is greater than $\sum_{i\epsilon S} p_{eq,i}^S$ then set the prices for each of the auction proportionally to $p_{eq,i}^S$.

i.e. If $\hat{v}_i^s \geq \sum_{i\epsilon S} p_{eq,i}^s$

then $\hat{v}_{i,j}^s = \hat{v}_{i,j}^s * (\frac{p_{eq,j}^s}{p_{eq,j}^s})$

5. Update buyers list and sellers list with the items of the combinations which satisfy the equations in step 3 and step 4 and then repeat step 2.
6. If there is a change in prices after step 5 then repeat from step 3.
7. Return prices $p_{eq,i}^S$ $\forall\, i \,\epsilon\, S$ and declare winners.

Here step 3 and step 4 takes the winning prices generated in step 2 and divides the combinations separately in to individual goods thus by reducing combinatorial auctions into simple double auctions. Then it generates the winner according to McAfee Double Auction Mechanism (as shown in Table 1).

A. Justification of TCDAM: If $\boldsymbol{\hat{v}_i^w < \sum_{i\epsilon S} p_{eq,i}^w}$ then $\boldsymbol{\sum_{j=1}^{n} \hat{v}_{i,j}^w < p_{eq,i}^w}$. i.e. if the overall valuation of goods for a buyer agent is less than the total equilibrium price then the transaction is not possible. Similarly for a seller if $\boldsymbol{\hat{v}_i^s \geq \sum_{i\epsilon S} p_{eq,i}^s}$ transaction is not possible. Hence the TCDAM takes only the winning bids to the next round by eliminating bids which are lower than equilibrium price.

B. Algorithm TCDAM is Truthful: The payment of agent 'i', $\boldsymbol{p_i^w}$ is independent of his valuation $\boldsymbol{\hat{v}_i^w}$. If the agent misreports his true valuation he cannot gain anything since the payment of winners remain same for all(as shown in Table 1). Hence truth-telling is the best strategy for the agents in this mechanism.

C. Social Efficiency: The utility of agent $\boldsymbol{u_i^a}$ is greater than 0 since $\boldsymbol{\hat{v}_i^a > p_i^a}$(since payment of the agent is less than his valuation) and from Eq 1 $\boldsymbol{u_i^a}$ > 0. Hence buyer/seller satisfaction is achieved.

V. Comparisons of Proposed Schemes and Related Works

In this paper alternative solutions are proposed in selling tickets in combinations of items where a single agent can give demand for combination of goods in static environment. All the alternative solutions were proposed from the view point that, whether the existing schemes of selling the goods is efficient in a competitive environment. The alternative solutions were proposed with the robust theory of mechanism design as the participating agents in the market have their own private information that are not known to others. Both Truthful and non-truthful mechanisms were proposed in the current paper. What should be the optimal reservation price is not mathematically justified.

	ICDAM	TCDAM
Truthfulness	Not Satisfied	Satisfied
Budget	Not Balanced	Balanced
Social Efficiency	Not Satisfied	Satisfied

However when the equilibrium price or base value for a particular good is not achieved then ICDAM is applied instead of TCDAM. Combinatorial auctions are notoriously difficult to solve from a computational point of view ([1]) due to the exponential growth of the number of combinations [2]. The combinatorial auction problem can be modeled as a set packing problem (SPP) [2-5, 16-17] our first algorithm can also be said as SPP. Sandholm mentions that determining the winners so as to maximize revenue in combinatorial auction is NP-complete [6-8]. In particular, they present the mathematical formulation of combinatorial double auctions and show that a general combinatorial double auction can be reduced to a combinatorial single-sided auction, which is a multi-dimensional knapsack problem, a problem known to be a NP-hard in computational complexity.

VI. Conclusions and Future Works

In this paper alternative solutions are proposed in selling goods in combinations of items where a single agent can give demand for multiple combinations in static environment. All the alternative solutions were proposed from the view point that, whether the double auction scheme of selling the goods is efficient in a competitive environment. The alternative solutions were proposed with the robust theory of mechanism design as the participating agents in the market have their own private information that is not known to others. Both DSIC and non-DSIC mechanisms were proposed in the current paper. What should be the optimal reservation price is not mathematically justified. Considering the quality factor for the resale goods in the auction and for divisible goods like spectrum allocation for secondary users in combinatorial domain will be an interesting future work.

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