International Journal of Advancements in Mechanical and Aeronautical Engineering– IJAMAE
Volume 2: Issue 1 [ISSN : 2372-4153]

Publication Date : 30 April, 2015

Gyroscope's Torques and Motions

Ryspek Usubamatov

***Abstract*—Gyroscope devices are primary unit in navigation and control systems that have wide application in aviation, space, ships and other industries. The main property of the gyroscope device is maintaining its axis. This gyroscope peculiarity is represented in term gyroscope effects, which mathematical models formulated on the law of kinetic energy conservation and the rate change in the angular momentum of a spinning rotor. However, the nature of gyroscope effects is more complex and known mathematical models do not match the actual forces and motions in the gyroscope devices. Recent investigations in area of gyroscope theory demonstrate that in the gyroscope are acting four dynamical components simultaneously and interdependently, namely: the centrifugal, inertial and Coriolis forces and the change angular momentum in the spinning rotor. The applied torque to the spinning rotor generates the resistance torque based on centrifugal and Coriolis forces. The torque generated by the inertial forces and by the change of the angular momentum on the spinning rotor resulting in the precession torque of a gyroscope. The action of the resistance and precession torques is combined in case of solution the gyroscope problem. The new mathematical principles for the gyroscope motions are tested and the results practically validated. This paper represents the physical principles of motions in gyroscope devices.**

***Keywords*—gyroscope theory**

I. Introduction

Gyroscopic effects are relayed in many engineering calculations of rotating parts and gyroscope properties enable function the numerous gyroscope devices in aviation, space and other industries. There are many publications regarding the gyroscope theory as well as many approaches and mathematical solutions that describe the gyroscope properties [1-3]. There are many valuable publications dedicated to the gyroscope effects and applications in engineering [4-5]. All publications in the area of gyroscope theory describe gyroscope effects in terms of conservation of kinetic energy and the change in the angular momentum. However, the nature of gyroscope effects is more complex and known theory and mathematical models do not match practice of gyroscope devices [6-8]. This unaccountable situation with gyroscope effects spawned terms like gyroscope resistance, gyroscope couple and fantastic properties like non inertial, non gravitational, etc. This is the reason that the gyroscope theory still attracts many researchers who continue to discover new properties among gyroscope devices [9-12].

Ryspek Usubamatov
University Malaysia Perlis
Malaysia

The recent investigations of physical principles of gyroscope motions demonstrate that four classical forces acting in the spinning rotor generate gyroscope effects. Conducted analyses show that pseudo centrifugal, inertial and Coriolis forces and the rate change in the angular momentum of the spinning rotor are basis of the all gyroscope effects [13-15]. The external torque applied to the gyroscope generates the torques that based on mentioned forces and motions. In turn, the centrifugal and Coriolis forces generate the resistance torque that counteract on inclination of the rotor's location. The inertial forces and the rate change of the angular momentum of the spinning rotor generate the precession torques. All these torques are interrelated and acted simultaneously. The action of the resistance and precession torques is combined and represents the gyroscope effects. A new mathematical model based on these four physical components describe accurately the gyroscope motions and validated by tests. This new fundamental principles in the gyroscope theory enable to solve gyroscope problems and close the term gyroscope mystery.

II. Methodology

The spinning rotor experiences a radial acceleration and centrifugal forces of the mass-elements, which act strictly perpendicular to the axis of the spinning rotor. In case of declination of the spinning rotor, the plane of rotating centrifugal forces creates a contracting moment. However, the plane of rotating centrifugal forces declines with defined angular velocity of precession and at the same time resists on action of the external torque applied. The mathematical model of the resistance torque that generated by the centrifugal forces of a spinning rotor is represented by the following equation [15].

$$T_{ct} = 2(\pi/3)^2 J\omega\omega_p \tag{1}$$

where T_{ct} is a resistance torque generated by the centrifugal forces, $J = (mR^2/2)$ is the rotor's mass moment of inertia about the rotor's axis, m is the rotor's mass, R is the external radius of the rotor, ω_p is an angular velocity of resistance precession of a spinning rotor.

In the uniform circular motion, the tangential velocity direction of the rotor's mass elements changes continuously. In the case of acting of the external torque that is applied to a gyroscope, the inclination of the spinning rotor leads to change the tangential velocity of mass elements in axial direction. This change is causing of the axial acceleration as well as the inertial forces of mass elements that are acting perpendicular to the plane of the centrifugal forces of the spinning rotor. The inertial forces are causing the angular torque and the angular velocity of the rotor precession. The mathematical model of the precession torque generated by the inertial forces is represented by the following equation [15]:

$$T_{in} = 2(\pi/3)^2 J\omega\omega_p \qquad (2)$$

where T_{in} is a precession torque generated by the inertial forces, ω_{p} is an angular velocity of inertial precession of a spinning rotor and other parameters are as specified above.

The analysis of (2) demonstrates the equation of the precession torque of the axial inertial forces of the spinning rotor is the same as (1), i.e., precession and resistance torques are generated by the same rotating masses, which accelerations are directed perpendicular to each other. The external torque applied to the spinning rotor causes its angular velocity of inertial precession about the axis, which is perpendicular to the rotor's axis and perpendicular to the axis of the resistance precession.

The inclination of the spinning rotor that leads to change the tangential velocity's direction of a mass element generates an acceleration as well as the Coriolis force of mass elements. This Coriolis forces are acting perpendicular to the plane of the spinning rotor and causing the resistance torque action at the same direction as the torque of centrifugal forces. The mathematical model of the resistance torque generated by the Coriolis forces is represented by the following equation [15]:

$$T_{cr} = (8/9)J\omega\omega_p \qquad (3)$$

where T_{cr} is a resistance torque generated by the Coriolis forces, other parameters are as specified above.

The rate change in the angular momentum of the spinning rotor generates the torque, which acts on two directions as precession and resistance torques in sequence of motions and represented by the well-known equation [1-4]:

$$T_{a.m} = J\omega\omega_p \qquad (4)$$

where $T_{\text{a.m}}$ is a torque generated by the rate change in the angular momentum, other parameters are as specified above.

The defined toques based on the centrifugal, inertial, Coriolis forces and the change in the angular momentum are acting simultaneously on the spinning rotor. Analysis of the acting forces in the gyroscope enables to state that actual gyroscope effects have more complex nature than represented by known publications. Table 1 represents the four different torques generated by the internal pseudo forces of the spinning rotor under the action of the external torque applied to the gyroscope. Simultaneous action of this torques should be considered at new mathematical models of gyroscope motions. The external torque applied to the gyroscope leads to angular velocities of precessions by two axis and activates simultaneously the four torques mentioned above. However, the action of this torques applied to different axes of the spinning rotor which perpendicular with each other's. The torque generated by the centrifugal and Corioils forces is represented the resistance torques. The torque generated by the inertial forces and by the change of the angular momentum is represented the precession torque. The values of this torques are represented in Table 1 and demonstrates that centrifugal, inertial and Coriolis forces of the spinning rotor are really active physical components. The change in the angular momentum of the spinning rotor is one component among others and plays not first role at gyroscope physics.

TABLE 1. INTERNAL TORQUES ACTING IN GYROSCOPE

Torque generated by:	Equation	Percentage, %
centrifugal forces	$T_{ct} = 2\left(\frac{\pi}{3}\right)^2 J\omega\omega_p$	34.95
inertial forces		34.95
Coriolis forces	$T_{cr} = (8/9)J\omega\omega_p$	14.16
change in the angular momentum	$T_{am} = J\omega\omega_p$	15.94
Total		100

III. Mathematical Model for Gyroscope Motions

The external torque applied to the gyroscope leads to an angular velocity of precessions and activates the centrifugal, inertial, Coriolis forces and the rate change in the angular momentum of the spinning rotor. The analysis of forces acting in a gyroscope shows that all of them come into action simultaneously. The new equations for the resistance and precession torques (Table 1) demonstrate dependency on the mass moment of inertia and angular velocity of the spinning rotor, as well as on the angular velocity of its precessions. This situation should be analyzed from the standpoint of interrelationships and sequences of acting forces in a gyroscope. The external torque applied to a gyroscope generates the net of the internal torques and turn motions of the spinning rotor about the axis *ox* and *oy*. These motions have the defined links. For accuracy in formulating of initial motions, the action of torques is accepted at counter clockwise direction as positive and clockwise direction as negative.

Consider the sequence of the gyroscope gimbals' torques and motions from starting condition in the system of axes $\sum oxyz$ (Fig. 1). The spinning rotor of the angular velocity ω_z, located symmetrically regarding its pivots on the gimbal, 1 which rotates on pivots in the gimbal 2 and last one rotates on pivots in the frame 3 (Fig. 1(*a*)). At starting condition, the inclination of the frame 3 about the axis *ox* that represented the applied torque *T*, leads to change the rotor axis *oz* location on planes *yoz* and *xoz*, (represented by oz^*), (Fig. 1(*a*)). These changes generate the following resistance and precession torques simultaneously acting on gimbals (Fig. 1(*b*)):

a) the resistance torques generated by the centrifugal $T_{ct.x}$ (1) and Coriolis forces $T_{\text{cr.x}}$ (3) about the axis *ox* act at clockwise direction

$$T_r = 2\left(\frac{\pi}{3}\right)^2 J\omega\omega_x + \frac{8}{9}J\omega\omega_x = \left[2\left(\frac{\pi}{3}\right)^2 + \frac{8}{9}\right]J\omega\omega_x \qquad (5)$$

b) the precession torques of the inertial forces $T_{in.x}$ and change of the angular momentum of rotating mass $T_{am.x}$ about the axis *oy* act at counter clockwise direction and turn the gimbal 2 at the plane *xoz*

$$T_p = 2\left(\frac{\pi}{3}\right)^2 J\omega\omega_x + J\omega\omega_x = \left[2\left(\frac{\pi}{3}\right)^2 + 1\right] J\omega\omega_x \qquad (6)$$

c) in turn, the precession torques of the inertial forces $T_{in.x}$ (2) and change of the angular momentum $T_{am.x}$ (4) about axis *oy*, generate the torques of the inertial forces $T_{in.y}$ and change of the angular momentum $T_{am.y}$ about the axis *ox* respectively and turning the gimbal 1 about the axis *ox* starting to shift at clockwise direction.

The sequence of the torques action and following motions of the gyroscope gimbals are as follow.

- The action of the applied torque T starts to incline the spinning rotor at counter clockwise direction. However, the resistance torques of the centrifugal $T_{ct.x}$ and Coriolis forces $T_{cr.x}$ are keeping the axis *oz* of the rotor in initial position at the plane *yoz*.
- The precession torques of the inertial forces $T_{in.x}$ and rate change of the angular momentum of rotating mass $T_{am.x}$ are acting and turning the gimbal 2 at counter clockwise direction about the axis *oy*. In turn, the change of the rotor's location at the plane *xoz* leads to activation the resistance torques of the centrifugal $T_{ct.y}$ and Coriolis forces $T_{cr.y}$ that are keeping the axis *oz* of the rotor at initial position on the plane *xoz*.
- The precession torques of the inertial forces $T_{in.y}$ and change of the angular momentum of rotating mass $T_{am.y}$ are acting and turning the gimbal 1 at clockwise direction about the axis *ox*.

The motions of the gimbals 2 and 1 are keeping the axis *oz* of the spinning rotor at initial position in space, but the system of axes $\sum oxyz$ is changed the space orientation. In case of the turn the gyroscope about the axis *ox* on 90^0 the new location of the gimbals is represented in Fig. 1(*c*). The rotation of these gimbals are causing by action of the summed action of the inertial forces T_{in} and change of the angular momentum of rotating mass T_{am} about the axis of consideration. The magnitudes of these summed torques about the axis *oy* and *ox* are the same, i.e., the angular velocities of gimbal's motions about the axis *oy* and *ox* are also same. This statement can be represented by the following equation:

$$\left[2\left(\frac{\pi}{3}\right)^2 + 1\right] J\omega\omega_y = \left[2\left(\frac{\pi}{3}\right)^2 + 1\right] J\omega\omega_x \text{ or } \omega_y = \omega_x \qquad (7)$$

Equations (8) represent external and internal pseudo torques acting in the gyroscope. The components of the second equation of (8) are the result of the action the external torque T that represents the peculiarity of gyroscope equations. All internal torques are representing the internal energy of the spinning rotor generated by the external torque. Hence, combining of two equations gives the following equation of the total torques acting in the gyroscope:

$$T - T_{ct.x} - T_{cr.x} - T_{in.y} - T_{am.y} + T_{in.x} + T_{am.x} - T_{ct.y} - T_{cr.y} = 0 \qquad (9)$$

Substituting expressions of the torques ((1), (2), (3) and (4)) into (9) and transforming give the following equations:

$$T - \left[2\left(\frac{\pi}{3}\right)^2 + \frac{8}{9}\right] J\omega\omega_x - \left[2\left(\frac{\pi}{3}\right)^2 + 1\right] J\omega\omega_y + \left[2\left(\frac{\pi}{3}\right)^2 + 1\right] J\omega\omega_x - \left[2\left(\frac{\pi}{3}\right)^2 + \frac{8}{9}\right] J\omega\omega_y = 0 \qquad (10)$$

Substituting (7) $\omega_y = \omega_x$ into (10) and transforming yield the following equation

$$\omega_x = \frac{T}{2\left[2\left(\frac{\pi}{3}\right)^2 + \frac{8}{9}\right] J\omega} \qquad (11)$$

where all parameters are as specified above.

Analysis of (11) shows the gimbals' motions of the gyroscope about the axis *ox* and *oy* have right dependency on the magnitudes of the external torque applied T and inversely proportional on the angular velocity of the rotor's spinning and its mass moment of inertia.

The external torque applied to the gyroscope leads to an angular velocity of gimbal's motions and activates the centrifugal, inertial, and Coriolis forces and the changes of the angular momentum of the spinning rotor about the axis of consideration. The formulated mathematical models for the resistance and precession torques that acting by the axis of consideration are resulting on the gimbals motions. The angular velocities of the gimbals precessions are equal that enable to keep the spinning rotor's orientation in space.

IV. Working Example

The gyroscope, which the rotor is carried in a casing mounted on bearings, is arranged in such a way that its axis is horizontal, but free to take up any direction (Fig. 1). The gyroscope is tilted through the angle in *yoz* plane. The rotor is a uniform solid disc with d = 0.05 m in diameter weighing m = 0.15 kg and running at ω = 900 rad/s. The torque applied T = 0.04 Nm. Here is how to define the gimbal's angular velocity of motion ω_x.

The angular velocity of the gimbal's motion based on the principles mentioned above is represented by (11). Substituting initial data of the gyroscope into (11) and transforming yield the following result:

$$\omega_x = \frac{T}{2\left[2\left(\frac{\pi}{3}\right)^2 + \frac{8}{9}\right]J\omega} =$$

$$\frac{0.04}{2\left[2\left(\frac{\pi}{3}\right)^2 + \frac{8}{9}\right]\times 0.000046875\times 900} = 0.1538 rad/s$$

where $J = md^2/8 = 0.15\times 0.05^2/8 = 0.000046875$ kgm^2 is the rotor's mass moment of inertia; other parameters are as specified above.

The resistance and precession torques generated by the centrifugal, Corilos and inertial forces and the rate change of angular momentum for the gimbals, decelerates the angular velocity of gimbals motions.

V. Results and Discussion

The external torque applied to the gyroscope leads to an angular velocity of precessions and generates the torques based on action of the centrifugal, inertial, Coriolis forces and changes in the angular momentum of the spinning rotor. The new approach demonstrates that the centrifugal and Coriolis forces of the spinning rotor resist any inclination of the rotor's axis and generate the resistance torque. The axial inertial forces and change of the angular momentum of the spinning rotor generate the precession torque. The action of this torques is combined in the gyroscope that depends on the motion of consideration. The action of this torques is formulated by the mathematical models for the accepted systems of axes. The magnitudes of the four torques depend on the mass, radius and angular velocity of the spinning rotor, as well as on the angular velocity of its precession.

The new analytical approach to gyroscopic problems demonstrates that centrifugal, inertial and Coriolis forces of the mass elements for the spinning rotor are really active physical components that act simultaneously with change in the angular momentum.

VI. Conclusion

The gyroscope theory in classical mechanics is one of the most complex and intricate in terms of analytical solutions. The known mathematical models of the gyroscope theory are mainly based on the actions of the external torque applied and the change in the angular momentum of the spinning rotor. This approach leads to many assumptions and simplifications in case of the unexplainable motions of the gyroscope devices. However, nature of the gyroscope motions is more complex than represented in the known gyroscope theories. Practically, gyroscope effects demonstrate that the centrifugal, inertial and Coriolis forces in the spinning rotor play a critical role. The change in the angular momentum of the spinning rotor is not a main component of the acting forces. All these forces act simultaneously, reciprocally and gyroscope effects dependent on all of them. The forces are acting in the gyroscope are well known in classical mechanics which actions never have described analytically in gyroscope physics. Presentation of the action of this forces enables clearly understand physical process that resulting in the gyroscope devices. Hence, the artificial properties that attributed to the gyroscope are removing forever.

New mathematical model of gyroscope effects describe clearly the known properties and will thus be useful for modelling the behaviour of the gyroscopic devices. Formulated analytical models of the gyroscope effects based on the principles of the centrifugal, inertial and Coriolis forces' and the rate change of the angular momentum of the spinning rotor give correct results. The new fundamental principles for the gyroscopic effects represent the basis of the new gyroscope theory and new challenges for future studies of the gyroscopic devices.

Acknowledgment

The author would like to thank the University Malaysia Perlis for granting this research and express respect to governing body for their excellent strategic policy in scientific research.

References

[1] M.N. Armenise, C. Ciminelli, F. Dell'Olio, V.M.N. Passaro, Advances in Gyroscope Technologies, Springer-Verlag Berlin and Heidelberg GmbH & Co. KG, Berlin, 2010.
[2] D.R. Gregory Classical Mechanics, Cambridge University Press, New York, 2006.
[3] R.F. Deimel, Mechanics of the Gyroscope, Dover Publications Inc. New York, 2003.
[4] G. Greenhill, Report on Gyroscopic Theory, General Books LLC, London, 2010.
[5] M.D. Ardema. Analytical Dynamics. Theory and Application. Academic/Plenum Publishers, New York, 2005.
[6] E. Butikov, Inertial rotation of a rigid body, Eur. J. Phys. 27, (2006), pp. 913 – 922.
[7] T.J. Quinn, A. Picard, The mass of spinning rotors: no dependence on speed or sense of rotation, Nature 343, France, (1990), pp. 732 – 735, doi: 10.1038/343732a0.
[8] H. Weinberg, Gyro Mechanical Performance: the most important parameter, Technical Article MS-2158, Analog Devices, Norwood, MA, 2011, pp. 1-5.
[9] Y. Ohshima, Analysis and testing of gyroscope performance for the gravity probe B relativity mission, Stanford University, CA, 2000.
[10] L. Zyga, Gyroscope's unexplained acceleration may be due to modified inertia, PhysOrg.com, July 26, 2011, 4.6 / 5, (28), 51.
[11] J.E. Faller, W.J. Hollander, P.G. Nelson, M.P. McHugh, Gyroscope-weighing experiment with a null result. Physical Review Letters 64, APS Journal, Vol. 64. No. 8, 1990, pp. 825–826.
[12] Z. Zhang, J. Sun, K. Wu, Error analysis and test study of fiber optic gyroscope north-finder. Advanced Sensor Systems and Applications II, Proceedings of SPIE, Vol. 5634, Cardiff, 2005, pp. 611- 618.
[13] R. Usubamatov, Z.M. Zain, Y.N. Zaiazmin, M.S. Hussin Centrifugal forces are the cause of gyroscope stabilization, Research Bulletin of the Australian Institute of High Energetic Materials, April, 2011, pp. 226-233.
[14] R. Usubamatov, Analysis of inertial forces acting on gyroscope precession, International Review of Aerospace Engineering, Vol. 4, No 2, 2011, pp. 103-108.
[15] R. Usubamatov, Mathematical Model for Gyroscope Effects, Proceedings of International Conference on Mathematics, Engineering and Industrial Application, ICoMEIA 2014, Paper ID-71, May Penang, Malaysia, 2014.

About Author:

Image

Dr.-Eng. Ryspek Usubamatov is currently with University Malaysia Perlis as a Professor in the School of Manufacturing Engineering. He graduated as Professional Engineer in Mechanical, Manufacturing and Industrial Engineering, and Ph.D. in Automation of Manufacturing Processes from Bauman Moscow State Technical University. He is Doctor of Technical Sciences, Habil. His current area of interest is Productivity Theory for Industrial Engineering, Metal Cutting Manufacturing Processes, Gyroscope Theory and Inventions. He has received more than 60 patents of inventions in engineering, published 7 books, 30 brochures and more than 300 papers.