

## Reliability and Flexural Behavior of NVC and SCC Rectangular Reinforced Concrete Beams

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### ABSTRACT

The objective of this paper is to experimentally study and analyze the flexural behavior of normal vibrated concrete NVC and self compacting concrete SCC beams subjected to three point test loading and four point test loading respectively. Experimental data of the beam center point deflection compares well with the finite element structural model center point deflection of beams obtained from a nonlinear analysis. Results from both experimental data of beam testing and finite element modeling verify the assessment of the developed reliability index  $\beta$ . The flexural behavior and performance of reinforced Self-Compacting Concrete beams based on experimental data showed that SCC beams are reliable and could be used instead of NVC beams in reinforced concrete structures.

Keywords: NVC Concrete beams, SCC Concrete beams, Experimental testing, Finite element, Nonlinear analysis, Reliability index.

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### INTRODUCTION

Concrete structures are widely used almost all over the world and concrete structural members such as beams exist in buildings and other structures in different forms. Different methods have been utilized to study the reliability and response of structural components such as the reliability index  $\beta$ , a measure of safety and reliability, finite element analysis, and experimental testing [1 and 2]. Experimental testing has been widely used to analyze individual elements and the effects of concrete strength under loading. Concrete beams of different mixes have to be analyzed for safety, stability, deformation, and crack formation using the ultimate-strength (USD) design method under the provisions of the ACI building design code [3, 4, 5 and 6]. Experimental testing is extremely time consuming and costly, but finite element analysis is much faster and much less expensive. Finite element structural models are used to simulate reinforced concrete beams and study their response to various load stages [7]. The objective of this paper is to experimentally study and analyze the flexural behavior of normal vibrated concrete NVC and self compacting concrete SCC beams subjected to three point loading and four point loading respectively. Self compacting concrete SCC is the type of concrete that it can flow around the reinforcement and fill the formwork without any mechanical consolidation [8 and 9]. The

experimental load-deflection data will be compared with the finite element non-linear load-deflection results of the simulated beams representing the actual beams [10]. The reliability of the beams will be assessed by the reliability index approach. The reliability index  $\beta$  measures the level

### Reliability Theoretical Formulation

The beam fails when the resistance of the beam is less than the action caused by the applied load. The beam resistance is measured by the design moment strength  $M_c$  and the beam action is measured by the external bending moment  $M_e$ , Fig. 1.

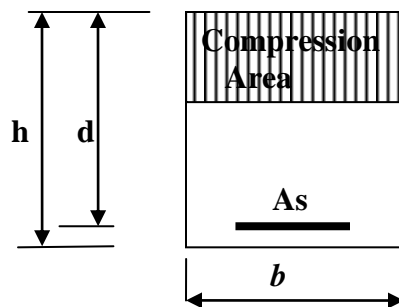


Fig. 1 Cross Sections of Rectangular Reinforced Concrete Beam

The beam limit state function is given by:

$$G(As, f'c, fy, Me) = Mc - Me \quad (1)$$

where

$Mc = \text{Design Moment Strength}$

$Me = \text{External bending moment}$

$As = \text{Area of tension steel}$

$fy = \text{Specified yield strength of nonprestressed reinforcing}$

$f'c = \text{Specified compression strength of the concrete}$

The triangular beam limit state function is given by the following equations:

$$GF(As, f'c, fy, Me) = \varphi \mu_{As} \mu_{fy} \left( d - \frac{1}{2} \frac{\mu_{As} \mu_{fy}}{\mu_{f'c} 0.85 b} \right) - \mu_{Me} \quad (2)$$

where

$\varphi = \text{Bending reduction factor}$

$b = \text{Width of the beam cross section}$

$d = \text{Effective depth}$

$h = \text{Total depth of the beam cross section}$

$\mu_{fy} = \text{Mean value of } fy$

$\mu_{f^c}$  = Mean value of  $f^c$

$\mu_{As}$  = Mean value of  $As$

$\mu_{Me}$  = Mean value of  $Me$

Because the limit state function is nonlinear, we can apply the Taylor series expansion to linearizing the nonlinear function and obtain an approximate answer [12]. The Taylor expansion about the mean value yields the following linear function:

$$G(As, f^c, fy, Me) = \left( \varphi \mu_{As} \mu_{fy} \left( \left( d - \frac{1}{2} \right) \right. \right. \\ \left. \left. + (As - \mu_{As}) \frac{dG}{dAs} + (fy - \mu_{fy}) \frac{dG}{dfy} + (Me - \mu_{Me}) \frac{dG}{dMe} \right) \right) \quad (3)$$

The reliability index  $\beta$  of the linear function is given by the following equation:

$$\beta = \frac{G(As, f^c, fy, Me)}{\sqrt{(\sigma_{AS} a1)^2 + (\sigma_{fy} a2)^2 + (\sigma_{f^c} a3)^2}} \quad (4)$$

where

$$a1 = \frac{\partial G}{\partial As} \left( \left( \varphi As fy \left( \left( d - \frac{1}{2} \frac{\mu_{As} \mu_{fy}}{\mu_{f^c} 0.85 b} \right) \right) \right) \right) \quad (5)$$

$$a2 = \frac{\partial G}{\partial fy} \left( \left( \varphi As fy \left( \left( d - \frac{1}{2} \frac{\mu_{As} \mu_{fy}}{\mu_{f^c} 0.85 b} \right) \right) \right) - Me \right) \quad (6)$$

$$a3 = \frac{\partial G}{\partial f^c} \left( \left( \varphi As fy \left( \left( d - \frac{1}{2} \frac{\mu_{As} \mu_{fy}}{\mu_{f^c} 0.85 b} \right) \right) \right) - Me \right) \quad (7)$$

$$a4 = \frac{\partial G}{\partial Me} \left( \left( \varphi As fy \left( \left( d - \frac{1}{2} \frac{\mu_{As} \mu_{fy}}{\mu_{f^c} 0.85 b} \right) \right) \right) - Me \right) \quad (8)$$

$\sigma_{As}$  = Standard Deviation of  $As$

$\sigma_{fy}$  = Standard Deviation of  $fy$

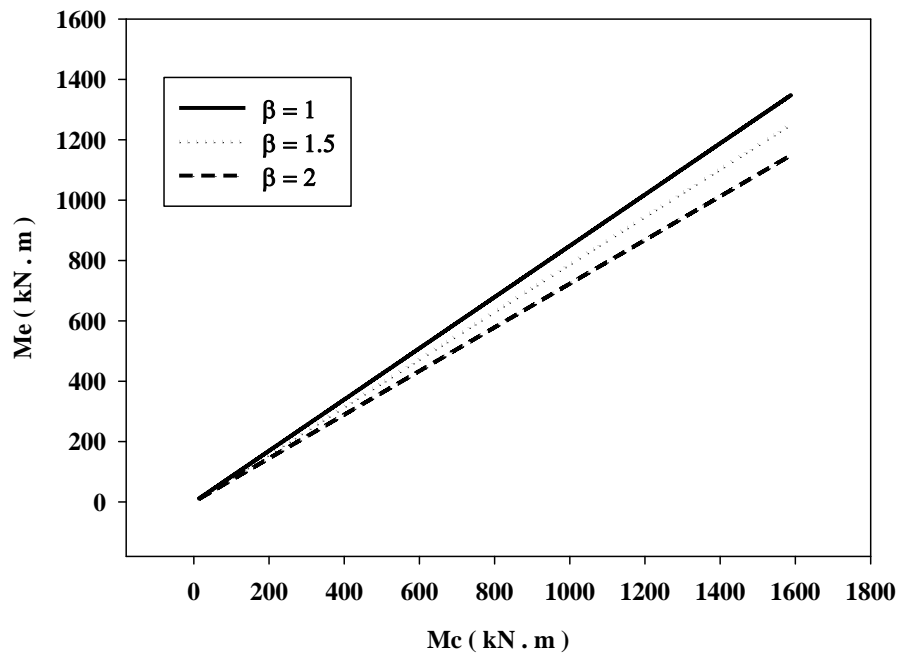
$\sigma_{f^c}$  = Standard Deviation of  $f^c$

$\sigma_{Me}$  = Standard Deviation of  $Me$

The standard deviation  $\sigma$  is the product of the mean value  $\mu$  and the coefficient of variation  $V$ . The formulation allows the estimation of the reliability of triangular reinforced concrete beams when subjected to flexural loads. The reliability index  $\beta$  is calculated for triangular reinforced concrete beams based on their resistance to applied loads, (Table 1, Fig. 2).

**Table 1 Rectangular Beams**

Me kN.m	Beam Data						Mc kN.m	$\beta$
	f <sup>c</sup> MPa	f <sub>y</sub> MPa	b mm	h mm	d mm	As mm <sup>2</sup>		
17	420	30	200	200	170	226	13.7	1
79.5			200	500	450	628.4	100	1.5
363			300	980	930	1500	504	2
745			400	1250	1200	2000	876	1
692			400	1250	1200	2000	876	1.5
1150			500	1560	1600	2700	1588	2
1350			500	1560	1600	2700	1588	1



**Fig. 2 Reliability Index of Rectangular Beams**

### Experimental Testing

Four rectangular reinforced concrete beams made of normal vibrated concrete NVC with yield strengths of 550 MPa and compression strengths of concrete of 30 MPa, and two rectangular reinforced concrete beams made of self compacting concrete SCC with yield strengths of 500 MPa and compression strengths of concrete of 55 MPa. All beams were tested in Qatar University to analyze and study the behavior of the beams under real flexure loads applied at the beam center point and at third point for the NVC beams and SCC beams respectively from an Instron HDX150 testing machine, Figs. 3 and 4, Table 2,[13 and 14].



**Fig. 3 Instron HDX150 Static Universal Testing System**

**Table 2 Experimental Beams**

Beams	f <sub>c</sub> MPa	F <sub>y</sub> MPa	Length m	b mm	h mm	A <sub>s</sub> mm <sup>2</sup>
B1-NVC	30	550	2	200	200	402.2
B2-NVC				200	200	307.8
B3-NVC				200	250	402.2
B4-NVC				200	250	226
SB1-SCC	55	500	2.25	200	250	402.2
SB2-SCC				200	250	628.4



**SCC Beams**



**NVC Beams**

**Fig. 4 Beams Testing Setup**

The testing machine applies an increasing load until the beam collapses and provides a data set of time, flexural loads, flexural stress, flexural strain and displacement, (Fig.5).

Time sec	Extension mm	Strain 1 %	Load N	Flexure stress MPa	Flexure extension mm	Flexure strain %	Flexure load N	Displacement (Strain 1) mm	Corrected Position mm
0	-2.004	7.30E-05	-6173.72	1.11127	-0.003	-6.96E-05	6173.721	0.000146	-2.004
5.356	-2.023	7.26E-05	-6674.07	1.201332	0.016	0.000371	6674.067	0.000145	-2.023
11.088	-2.047	0.000105	-7174.16	1.291349	0.04	0.000928	7174.162	0.000211	-2.047
16.534	-2.069	8.49E-05	-7675.09	1.381516	0.062	0.001438	7675.089	0.00017	-2.069
22.012	-2.086	0.0001	-8175.22	1.471539	0.079	0.001832	8175.219	0.0002	-2.086
27.708	-2.107	0.000164	-8675.61	1.561609	0.1	0.002319	8675.606	0.000327	-2.107
33.116	-2.134	1.89E-05	-9175.9	1.651662	0.127	0.002945	9175.901	3.78E-05	-2.134
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

1830.484	-69.07	0.003731	-144413	25.99434	67.06299	1.555084	144413	0.007462	-69.07
1830.66	-69.632	0.003802	-144913	26.08435	67.62499	1.568116	144913.1	0.007604	-69.632
1830.844	-70.216	0.003817	-145419	26.1754	68.20899	1.581658	145418.9	0.007635	-70.216
1831.064	-70.916	0.003853	-145919	26.26544	68.90899	1.59789	145919.1	0.007706	-70.916
1831.464	-72.19	0.003814	-146424	26.35636	70.18299	1.627432	146424.2	0.007628	-72.19
1831.484	-72.256	0.003793	-146400	26.352	70.24899	1.628962	146400	0.007587	-72.256

**Fig. 5 Rectangular Beam Machine Flexural Data**

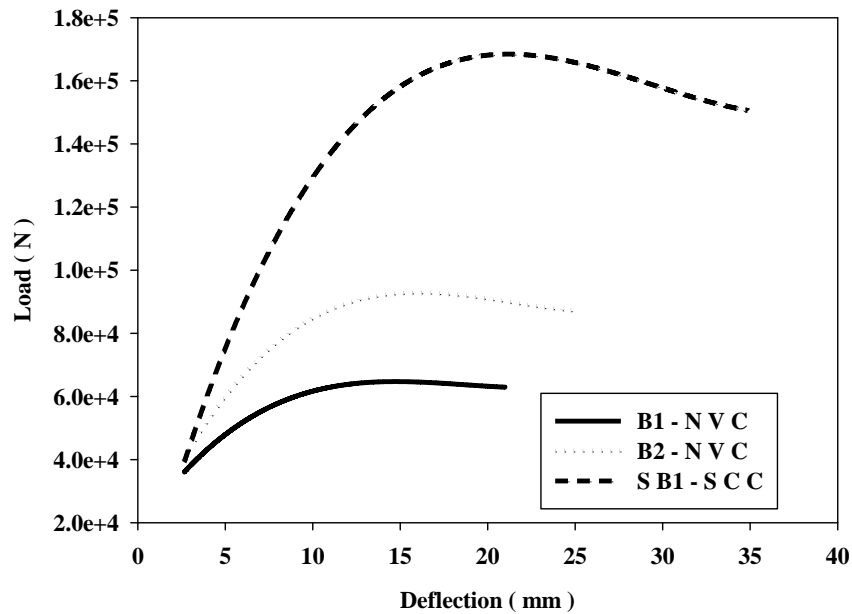
The beam deflection under applied loads is obtained from the machine flexural testing data, Figs. 5 and 6, Table 3.



**Fig. 5 Crack Patterns and Collapse Loads of SCC Beam**

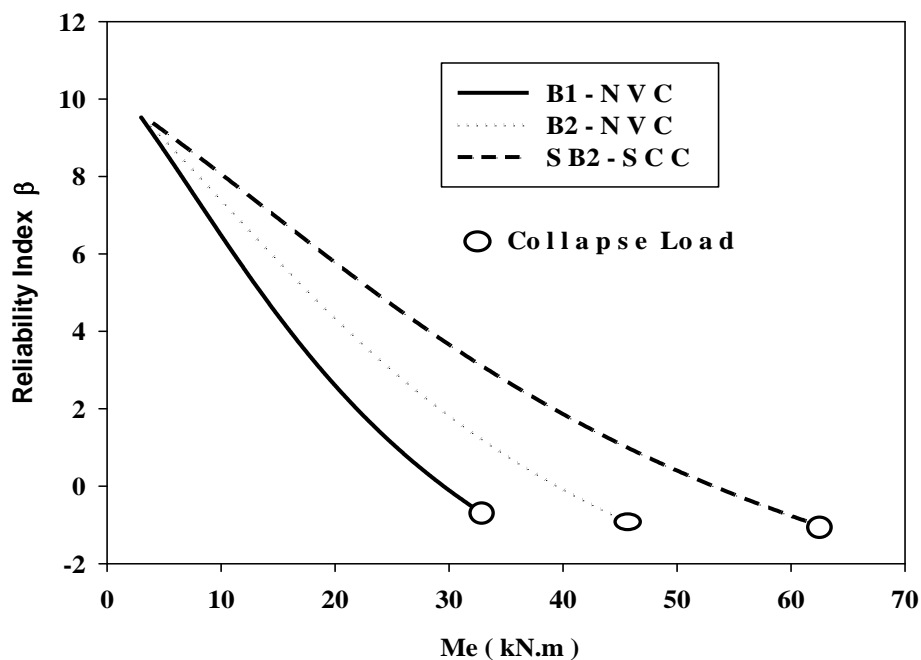
**Table 3 Collapse Load Deflection**

<b>Beam</b>	<b>B1-NVC</b>	<b>B2-NVC</b>	<b>B3-NVC</b>	<b>B4-NVC</b>	<b>SB1-SCC</b>	<b>SB2-SCC</b>
<b>Collapse Load kN</b>	<b>64.6</b>	<b>52.2</b>	<b>91.2</b>	<b>51</b>	<b>167</b>	<b>169</b>
<b>Deflection mm</b>	<b>13.8</b>	<b>17.3</b>	<b>15.2</b>	<b>30</b>	<b>22.7</b>	<b>28.2</b>



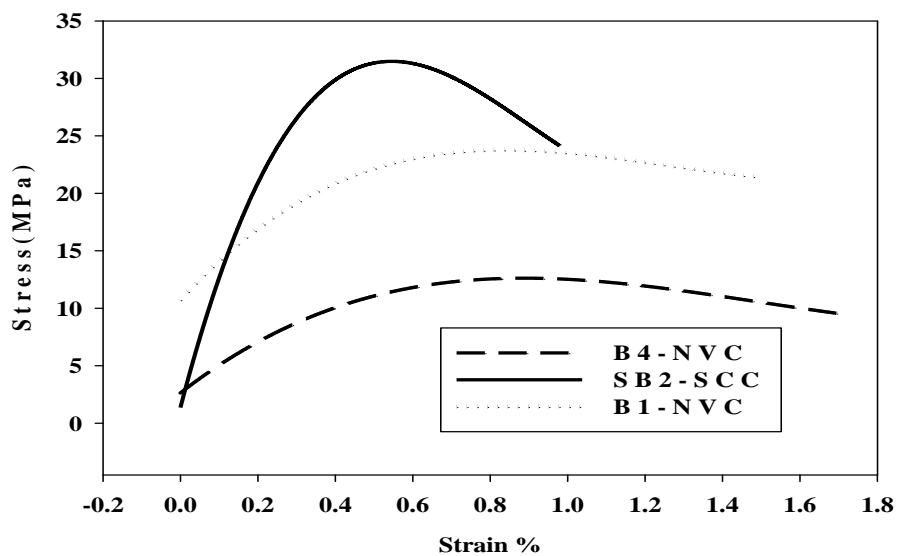
**Fig. 6 Load – Deflection Response**

The simple beam moment formulas  $\frac{PL}{4}$  for NVC beams and  $\frac{PL}{6}$  for SCC beams were used to compute the external moment  $M_e$ . Equation 4 was used to determine the reliability index  $\beta$  of the experimental beams. The reliability index  $\beta$  had negative values of collapse loads, (Fig.7).



**Fig. 7 Reliability Index of the Experimental Beams**

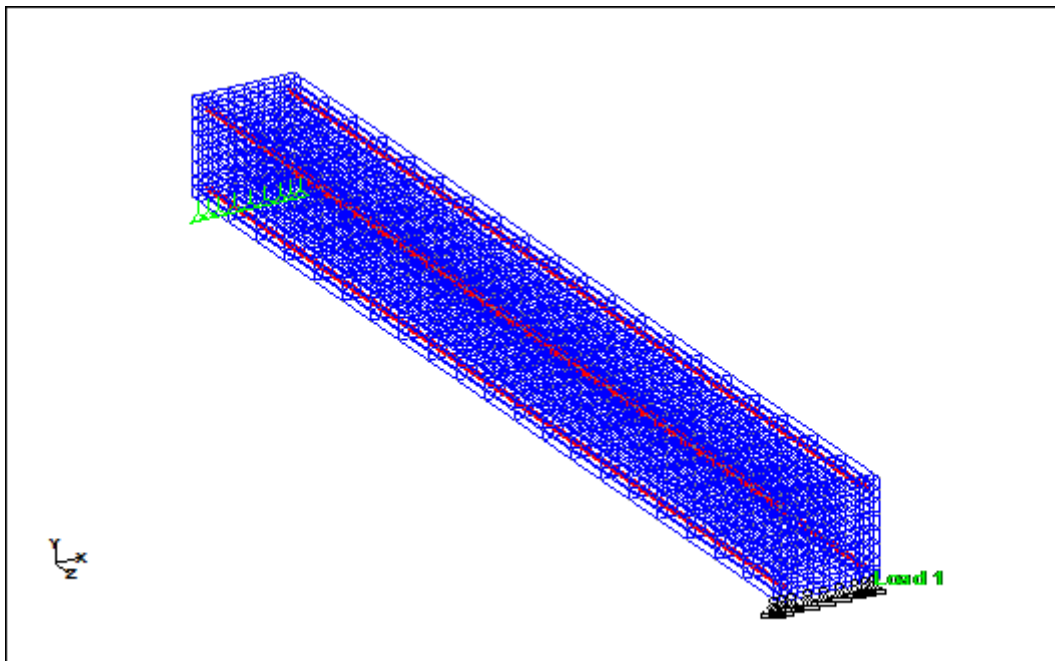
Stress and strain of NVC and SCC beams are within normal and accepted limits of stress and strain, (Fig.8).



**Fig. 8 Stress - strain curves of the Experimental Beams**

#### FINITE ELEMENT ANALYSIS

A nonlinear finite element analysis was used to simulate the experimental beams, Fig. 9, [15, 16 and 17].



**Fig. 9 Finite Element Structural Model**



The nonlinear vertical displacements  $\delta_{VFE}$  of the simulated beams compared well with the experimental vertical displacements  $\delta_{VEXP}$ . For the nonlinear analysis the horizontal displacement  $\delta_{ZFE}$  is limited to zero mm, and the buckling displacement  $\delta_{HFE}$  equals zero, (Table 7).

**Table 4 Nonlinear Beam Displacements**

Beam	Collapse Load kN	Finite Element – Nonlinear Displacement ( mm )			Experimental Displacement ( mm )
		$\delta_{HFE}$	$\delta_{VFE}$	$\delta_{ZFE}$	$\delta_{VEXP}$
<b>B1-NVC</b>	<b>64.6</b>	<b>0</b>	<b>15.65</b>	<b>0</b>	<b>13.8</b>
<b>B3-NVC</b>	<b>91.2</b>	<b>0</b>	<b>17</b>	<b>0</b>	<b>15.2</b>
<b>SB1-SCC</b>	<b>167</b>	<b>0</b>	<b>26.6</b>	<b>0</b>	<b>22.7</b>

## RESULTS AND DISCUSSION

Reinforced concrete beams were tested experimentally, analyzed analytically by a finite element method and the reliability of the beams was assessed using the reliability index approach. The developed reliability index  $\beta$  for the rectangular beams shows that for  $\beta$  equals 1, 1.5 and 2 the safety percentages are 17%, 28% and 38% respectively. The experimental load-deflection response of the beams showed that NVC beams and SCC beams have the same type of response. The design moment strength  $M_c$  computed based on experimental collapse loads of the beams were 29.5kN.m, 23.3kN.m and 52.0kN.m for the B1-NVC, B2-NVC, and SB2-SCC respectively. The design moment strength  $M_c$  computed by the ACI design code formulation were 32.3kN.m, 26kN.m and 63.4kN.m for the B1-NVC, B2-NVC, and SB2-SCC respectively. Reliability analysis of the experimental data predicts a low reliability index  $\beta$  of -0.5, -0.7 and -0.9 for the B1-NVC, B2-NVC, and SB2-

SCC respectively, at the collapse load. Stress and strain of NVC and SCC beams are within normal and accepted limits of stress and strain but NVC beams are more ductile because of lower compressive strength of concrete  $f'_c$ . Finite element beam displacement  $\delta_{VFE}$  at the collapse load is larger than the actual experimental displacement  $\delta_{VEXP}$  by 13%, 12%, and 17% for the B1-NVC, B3-NVC, and SB1-SCC respectively. These percentages indicate that the finite element nonlinear analysis is safe and close to the actual experimental displacement provided that the horizontal displacement  $\delta_{HFE}$  is limited to 0.0 mm and no buckling displacement  $\delta_{ZFE}$  is allowed.

## CONCLUSIONS

Experimental testing of simply supported rectangular beams was conducted to study and analyze the behavior of NVC and SCC beams. Experimental data of the beams center point deflection compares well with the finite element structural model center point deflection of beams using a nonlinear analysis. The experimental load-deflection response of the beams

showed that both types NVC and SCC cracked and collapsed in similar patterns. The design moment strengths  $M_c$  computed by the ACI design code formulation were safe and close to the design moment strengths  $M_c$  computed based on the experimental collapse loads of the beams. The experimental data of the beam testing verified the assessment of the developed reliability index  $\beta$  for reinforced concrete beams. The flexural behavior and performance of reinforced Self-Compacting Concrete beams based on experimental data showed that SCC beams are reliable and could be used instead of NVC beams in reinforced concrete structures.

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