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Assessing Life Annuities' Biometric Risk Under The New Solvency Directive

[Sana Ben Salah* and Lotfi Belkacem]

Abstract— In this paper we propose to assess the biometric risk involved in a life annuity portfolio under the new solvency directive. In particular, a biometric scenario generator is developed to derive a distribution of the insurer's liabilities useful for the solvency capital estimation. Then, gender-specific implicit longevity shocks are deduced in a marginal nested simulation context consistent with the Solvency II general principle. These shocks that are specified by age and maturity are proved to be less conservative than the unique permanent standard longevity stress what causes significant capital savings.

Keywords—Biometric risk, Solvency Capital Requirement, Simulation, Mortality modeling.

I. Introduction

In the context of Solvency II, the Solvency Capital Requirement (SCR) is defined as the amount that the insurer must hold so as to remain solvent over the forthcoming 12 months at 95.5%. Within this framework, the SCR can be determined based on either the standard approach proposed by the Committee of European Insurance and Occupational Pension Supervisors (CEIOPS), or a more sophisticated internal model. Undertakings are also allowed to use partial internal models in which some risks are internally modeled while the rest is treated in line with the standard approach. Except for the first approach, the supervisor's prior approval is usually required. One of the most important risks threatening life insurers is the longevity risk, i.e. the risk that policy holders on average survive longer than expected. This systematic risk becomes increasingly striking with the lengthening of the human lifespan. Then, as pointed out by Börger (2010) and Plat (2011), for the insurers to be shielded from mortality decreasing and to adequately asses the longevity risk under the new solvency regime, a reliable stochastic mortality model must be established. According to the new solvency directive, the longevity risk is considered as a sub-module of the life underwriting risk. In this framework and according to the standard formula, the longevity SCR corresponds to the change in the net asset value (NAV= Market value of assets minus best estimate of liabilities) resulting from a one-off instantaneous longevity shock standing at 20%.

Sana Ben Salah, Lotfi Belkacem
LaREMFiQ-IHEC / University Of Sousse
Tunisia

This shock corresponds to a permanent decrease in mortality and is assumed to be consistent with the general principle of the new directive based on the 0.5% quantile of the Net Asset Value over a one-year horizon. Two relevant questions arise in this context: first this stress is not too conservative? And second would not be more appropriate to calibrate the longevity stress at a more granular level? This paper addresses these issues and proposes assessing the longevity risk over a one-year time horizon by generating mortality scenarios based on the stochastic Poisson log-bilinear mortality model. In this context, the standard formula capital requirement is compared to that obtained in a nested simulation-type methodology consistent with the one-year value-at-risk concept of the Solvency II directive. The remainder of this paper is organized as follows: The first section deals with stochastic mortality modeling, after presenting and fitting the mortality model used in this work, we focus on stochastic mortality projection to generate a set of prospective life tables useful for an accurate evaluation of the longevity risk within the Solvency II framework. The second section casts light on the adverse selection problem faced when dealing with experience mortality data. Afterwards, we emphasize on the longevity risk assessment under Solvency II and propose a methodology allowing calibrating experience shock coefficients specified by age, gender and maturity. The SCR obtained for a life annuity portfolio is then compared to its counterpart based on the standard formula approach. Finally, the paper ends with some concluding comments.

II. Notations and Data

In the sequel of this paper, $p_{x,t}$ denotes the probability for an x-aged individual in year t to be still alive at age x+1 and $q_{x,t} = 1 - p_{x,t}$ designates the corresponding death probability. The age specific death rate is expressed as $m_{x,t} = \frac{D_{x,t}}{E_{x,t}}$ where $D_{x,t}$ and $E_{x,t}$ denote respectively the number of deaths and the risk exposure recorded for age x during year *t*. Henceforth $\mu_{x,t}$ designates the age-specific time-varying force of mortality. Widely used in mortality modeling, this index measures the probability that an individual aged *x* dies between age *x* and $x + \Delta x$. The force of mortality, also called instantaneous death rate, is believed to be close to the age-specific death rate $m_{x,t}$ under the piecewise constant force of mortality assumption, that is: $\mu_{x+u,t+s} = \mu_{x,t}$, for $0 \le u < 1$ and $0 \le s < 1$. The models presented in this paper are fitted on the gender-specific French mortality data spanning the period 1970-2010 for the age range [0,100]. This datasets are supplied by the Human Mortality Database[1]. In addition to these national data, we resort to experience mortality data relative to a life annuity portfolio pertaining to a French life insurance company.

III. Mortality Modeling

Human mortality modeling is the subject of an abundant literature and numerous stochastic models have been introduced for mortality projection purposes. One of the most broadly used one is that proposed by Brouhns et al. (2002).

A. The Poisson Log-Bilinear Model

Aiming at extending the original Lee-Carter model while avoiding its weaknesses, Brouhns et al. (2002) introduced a new approach based on a Poisson regression model. The log-bilinear form is kept and the model is described as follows:

$$D_{x,t} \sim Poisson\left(E_{x,t}\mu_{x,t}\right) \text{ with } \mu_{x,t} = \exp\left(a_x + b_x k_t\right);\ \sum_{x_{\min}}^{x_{\max}} b_x = 1,\ \sum_{t_{\min}}^{t_{\max}} k_t = 0 \quad (1)$$

Where $\mu_{x,t}$ points out the observed force of mortality for age x and calendar year t, a_x describes the average pattern of mortality by age across years, k_t reflects the variations in the mortality level over time whereas b_x describes the deviations from the averaged pattern when k_t varies. t_{min} and t_{max} designate respectively the first and last observation years, x_{min} and x_{max} denote respectively the first and last ages taken into consideration. Finally $\varepsilon_{x,t}$ are error terms reflecting the specific features of an age x or year t not captured by the model. $\varepsilon_{x,t}$ are i.i.d. according to a normal distribution N $(0,\sigma^2)$. Model parameters are estimated using the maximum likelihood technique. After estimating parameters, the following step consists in projecting mortality into the future by extrapolating the mortality trend. We resort to the Box-Jenkins methodology to model the mortality index using an adequate ARIMA model what allows, afterwards, performing simulations of the future mortality trajectory.

B. Generating Prospective Lifetables

Considering the Poisson log-bilinear mortality model discussed above, we build S gender-specific dynamic mortality tables for the French overall population as split by gender. To this end, we generate S new matrices (S=5000) containing time-dependent age-specific death numbers from a Poisson distribution with parameter. Then, for each scenario s, we estimate the Poisson log-bilinear model and fit an ARIMA model for each series considering the initial model orders. Finally, we perform projections of the series what allows deducing time-dependent age-specific forces of mortality, these latter are used to infer projected death probabilities as follows :

$$\mu_{x,t}^{(s)} = \exp\left(a_x^{(s)} + b_x^{(s)} k_t^{(s)}\right);\ q_{x,t}^{(s)} = 1 - \exp\left(-\mu_{x,t}^{(s)}\right) \quad (2)$$

Thus, we obtain projected age-specific mortality rates for both genders what allows building projected life tables necessary for an accurate assessment of the longevity risk within the Solvency II framework. These tables are extrapolated up to age 130 using the methodology proposed by Denuit and Goderniaux (2005) (See Denuit and Quashie (2005)).

C. Experience Mortality Moseling :The Adverse Selection Problem

One important feature that characterizes the life insurance industry is that policyholders' mortality rates may deviate significantly from those of the overall population raising thus an adverse selection problem. It seems thus crucial to take this risk into consideration so as to accurately evaluate the company-specific longevity risk profile. Nevertheless, mortality data for experience portfolios are generally not sufficient to build a vigorous experience life table. Actuaries and demographers have a tradition of using reconciliation techniques in order to relate mortality features of the group under consideration to those of a reference population. Then, it is easy to deduce experience mortality projections from those of the general (national) population.

As the term indicates, a relational model is an expression relating a population's mortality patterns to that of another group. Ledermann and Breas (1959) made the first known contribution in this field. Various propositions have been suggested since then and numerous reviews have been written on the subject. The most famous relational model has been introduced in 1971 by Brass who suggests considering a linear relationship between the logits of the death rates of two populations. We resort here to this model to describe experience mortality features based on those of the overall French population. In this context, the age-dependent experience mortality rates are expressed as follows:

$$log\ it\left(q_{x,t}^{exp}\right) = \alpha + \beta\, log\ it\left(q_{x,t}^{nat}\right);\ log\ it(q) = log\left(\frac{q}{1-q}\right) \quad (3)$$

Table I presents the estimation results of the Brass relational model as fitted to the French general mortality data and the experience death rates.

TABLE I. BRASS PARAMETERS ESTIMATED ON THE GENERAL AND EXPERIENCE POPULATIONS

Genders	Gender-specific Brass Parameters[a]	
	α	**β**
Males	-0.418	1.068
Females	-0.975	1.091

a. *All parameters are significant at 1% risk level and R^2 (men) = 95.5%, R^2 (women) = 95.8%*

Based on these results, we deduce experience mortality projections from those of the general population.

[1] http://www.mortality.org

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IV. Calibrating Age-Specific Experience Longevity Shocks

In the fifth quantitative impact study, the longevity shock is set to 20%. This unique one-off shock is assumed to be consistent with the one-year VaR general principle of Solvency II. Aiming to carry out comparisons with the solvency II standard formula longevity shock, we propose to calibrate experience longevity stress coefficients specified by gender and age, based on projected experience death rates obtained with the Poisson mortality model and the Brass relational model.

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A. *Methodology*

In order to warrant coherence with the general solvency II principle, we consider for the first year an extreme longevity shock, and then deduce the next years' mortality levels conditionally to what happened in the first year. This simulation-based methodology is close to the marginal nested simulation technique where a unique risk is considered. Within the solvency II standard formula framework, the longevity SCR is determined based on the change in net asset value resulting from a 20% reduction in mortality :

$$SCR^{SF} = NAV_0 - (NAV_0 | longevity\ shock) \quad (4)$$

In accordance with the value at risk Solvency II general principle, this capital is assumed to reflect one-in-200-year event and be an approximation of:

$$SCR^{VaR} = \arg\min_x \left\{ P\left(NAV_0 - \frac{NAV_1}{1 + i(0,1)} > x \right) \leq 0.005 \right\} \quad (5)$$

Where i(0,1) denotes the annual risk-free interest rate at time zero for maturity one year. Noting that, on the one hand, in the context of computing the longevity capital requirement under Solvency II, changes in the death rates affect only the insurer's liabilities (we consider only the biometric risk then the NAV can only be affected by a change in mortality and this effect concerns only liabilities) and on the other hand the NAV is a monotonic function of the death rate, it is equivalent to consider the quantile of the NAV or that of the death rate. Thus, in the sequel of this paper, we consider a quantile of the annual death rate rather than that of the net asset value.

B. *Calibrating The Longevity Stress*

Having run 5000 simulations of the first year's death rate $q^{s\ nat}_{x,T+1}$, we resort to the Brass Model presented and fitted above to deduce the experience mortality scenarios $q^{s\ exp}_{x,T+1}$. The age-specific first year's shock coefficient is expressed as follows:

$$sc^{exp}_{x,T+1} = \left[q^{shock\ exp}_{x,T+1} - \frac{1}{S}\sum_{s=1}^{S} q^{s\ exp}_{x,T+1} \right] \Big/ \frac{1}{S}\sum_{s=1}^{S} q^{s\ exp}_{x,T+1} \quad (6)$$

$q^{shock\ exp}_{x,T+1}$ denotes the extreme first year experience scenario deduced from that relative to the overall population $q^{shock\ nat}_{x,T+1}$ corresponding to the 0.5% empirical quantile. T refers to the last available observation year (T+1 is thus the first projection year), S is the total number of scenarios (S = 5000), $q_{0.5\%}\left(q^{nat}_{x,T+1}\right)$ designates the 0.5% quantile of the projected death rate for age x and year (T+1) and $q^{s\ nat}_{x,T+1}$ is a death rate scenario. As shown in *Fig.1*, the first year experience shock coefficients are far from being equivalent to the standard formula longevity shock standing at -20%.

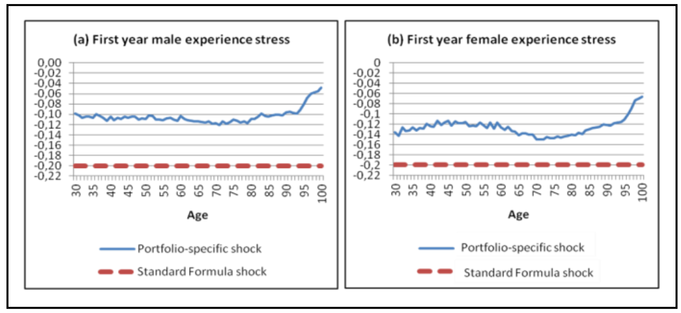

Figure 1. *First year experience longevity stress*

Beyond the first projection year, we run simulations of the future death rates conditionally to the extreme first year mortality deviation. The shock coefficients are determined as follows:

$$sc^{exp}_{x,T+j} = \left[\frac{1}{S}\sum_{s=1}^{S} q^{s\ shock\ exp}_{x,T+j} - \frac{1}{S}\sum_{s=1}^{S} q^{s\ exp}_{x,T+j} \right] \Big/ \frac{1}{S}\sum_{s=1}^{S} q^{s\ exp}_{x,T+j};\ j \geq 2 \quad (7)$$

Where $q^{s\ shock\ exp}_{x,T+j} = \log it^{-1}\left(\hat{\alpha} + \hat{\beta}\ \log it\left(q^{s\ shock\ nat}_{x,T+j}\right)\right);\ j \geq 2$ and $q^{s\ shock\ nat}_{x,T+j} = \left[q^{s\ nat}_{x,T+j} \middle| q_{0.5\%}\left(q^{nat}_{x,T+1}\right) \right]$ refers to the simulation s of the future projected death rate determined conditionally to the stressed first year scenario. As revealed in *Fig.2*, the experience longevity shock coefficients differ significantly from the constant standard formula stress not only in structure but also in magnitude. In fact, these coefficients that vary outstandingly with age and maturity are considerably less restrictive than the 20% standard formula stress.

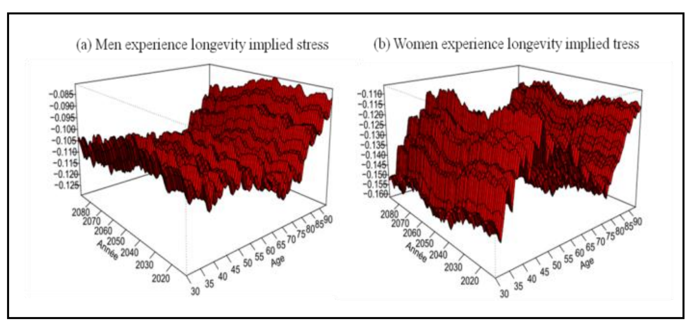

Figure 2. *Experience longevity stress specified by age and maturity for males (a) and females (b)*

V. SCR Estimation and Comparative Analysis

In order to assess the effect of using the methodology presented above on the longevity capital requirement for a life annuity portfolio, we calculate the longevity SCR based on the Standard Formula and the aforementioned experience simulation methodology. The life insurer's portfolio considered here is made of 256 single premium in payment immediate life annuity contracts with no discretionary benefits and no options or guarantees for surrender and death benefits. These contracts pay a fixed annuity amount yearly if the insured is still alive. The portfolio is supposed to be closed (run-off). The longevity SCR can thus be expressed as follows:

$$SCR^{ps} = \frac{((BEL_1 + CF_1)|longevity\,shock)}{1 + i(0,1)} - BEL_0 \qquad (8)$$

Where BEL_0 designates the insurer's liabilities at $t = 0$, BEL_1 denotes the company's best estimate liabilities at t=1 and CF_1 designates the first projection year best estimate payment[2] $i(t,T)$ is the annual interest rate for maturity T at time t ($t \leq T$) deduced from the QIS5 term structure for EU with 100% allowance for illiquidity premium. The quantity $((BEL_1 + CF_1)|longevity\,shock)$ is calculated based on the experience stressed death rates. This quantity is consistent with the 99.5% quantile of the insurer's liability distribution over one year. For an immediate life annuity contract paying a fixed amount ***a*** yearly, the best estimate liabilities at time t are:

$$BEL_t = \sum_{T>t} a\left[E\left[{}_T p_{x,t} | F_t\right] \Big/ \left(1 + i(t,T)\right)^{T-t}\right] \qquad (9)$$

Where F_t expresses the information available at time t and ${}_T p_{x,t}$ is the probability for an x-aged insured person at t to be still alive at the age $x+T$:

$${}_T p_{x,t} = \prod_{s=0}^{T-1} p_{x+s,t+s} \qquad (10)$$

$p_{x,t}$ denotes the one-year survival probabilities obtained from experience mortality projections. From the central and stressed experience death rates for both genders, we deduce 5000 scenarios of the survival probabilities what allows obtaining the insurer's liabilities distribution over one year. Table 2 reports the insurer's best estimate liabilities at time zero as well as the SCR obtained based on the liabilities distribution over one year (SCR^{ps}) and on the standard approach (SCR^{SF}). We can observe that SCR^{ps} is about 44% lower than its counterpart based on the standard formula indicating that the 20% shock overestimates the portfolio's longevity risk as compared to the one-year VaR-based approach. This deviation may be caused, in part, by the structure of the data as well as the assumptions surrounding mortality projections. In fact, the standard formula shock was calibrated on the unisex mortality data for nine populations (DE, FR, England & Wales, ES, IT, SE, PL, HU, CZ) spanning the period 1992-2006. In addition, annual mortality improvements for the aforementioned countries are assumed to follow a Normal distribution.

TABLE II. SCR COMPUTATION

(€)	BEL_0	SCR^{PS}	SCR^{SF}	$(SCR^{PS}- SCR^{SF})/SCR^{SF}$
Males	1020674	53996	102499	-47.3%
Females	5370109	255985	452698	-43.4%
Total	6390783	309981	555197	-44.1%

VI. Conclusion

This paper deals with the longevity risk assessment within the Solvency II framework. In this context, and according to the standard formula, the SCR is determined as the deviation in the net asset value due to a 20% permanent reduction in death rates for all ages. Although simple and easy to implement, this scenario-based approach comes under some criticism related mainly to the structure and magnitude of the longevity stress. Given that the goal is for the SCR to reflect the one-year 99.5% percentile of the forthcoming 12 months payments distribution, we have proposed a methodology allowing obtaining experience longevity shocks specified by gender, age and maturity. Based on the Poisson log-bilinear mortality model, we have generated 5000 gender-specific prospective mortality life tables for the overall French population via stochastic projection techniques. Afterwards, in order to address the anti selection problem, we resorted to a Brass-type relational model to perform experience mortality projections based on the French overall population mortality features. The insurer's liabilities distribution over one year has been obtained based on a marginal nested simulation technique consistent with the one-year value at risk Solvency II general principle. After computing the extreme first year mortality scenario, we have ran simulations for the subsequent periods conditional to what happened in the first year. Then, longevity shocks have been deduced from central and stressed death rates. These coefficients, specified by age, maturity and gender, are considerably less conservative than the standard formula stress standing at 20% reduction in mortality for all ages and maturities. This methodology is proved to produce significant capital savings for the longevity risk as compared to the standard formula approach.

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About Author (s):

Sana Ben Salah

PHD Student

Tunisia