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Advanced approach to moment capacity assement of FRP reinforced concrete structural member

Bond-slip relation in FRP reinforced concrete and its link to member curvature

[Jan Zatloukal, Jindrich Fornusek, Petr Konvalinka]

Abstract— **The response of FRP (Fiber Reinforced Polymer) reinforced concrete beam has been the topic of previous research, because of use of modern FRP composite materials in the building industry as concrete reinforcement. The behavior of FRP reinforced member is different from the one reinforced with regular steel reinforcement. This difference is caused by order of magnitude different moduli of elasticity of the respective materials and results in the fact that conventional design methods used for years in the field of reinforced concrete structures give poor results if used with FRP reinforced structural members. Results of conventional methods tend to overestimate load capacity of the member and underestimate deformations – both resulting in unsafe predictions. This paper points to formulating easy to use and comprehensible method of predicting moment capacity of FRP reinforced concrete beams subjected to bending loading, utilizing the bond-slip relation of the FRP reinforcement and validation of the proposed method via set of experiments.**

***Keywords*—FRP, composite reinforcement, bond-slip relation, moment capacity reduction**

I. Introduction

Since the proliferation of FRP (Fiber Reinforced Polymer) composite materials as concrete reinforcement is mostly restricted by their still high price, there are also several technical aspects restricting their wider use. Besides their fragility, unclear long-term durability, partial flammability in case of carbon fiber RP, by nature orthotropic mechanical behavior and physical properties (namely by order of magnitude different coefficient of thermal expansion respective to the fiber orientation), one of the factors is also their difficulty to describe behavior of FRP reinforced structural members in calculation. This paper focuses on formulation of easy to use and comprehensible method of evaluation and prediction of the moment capacity of FRP reinforced concrete beams.

In available literature, the formulae recommended for load bearing capacity prediction and design are based on empirical approach, mostly resulting from statistically processed experimental data [1,2]. Our work, on the contrary, derives theoretical model of behavior of FRP reinforced concrete section under flexural load and uses this model to formulate very easy to use design formula.

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II. Analytical model

A. *Moment-curvature relation*

In the case of continuous centerline of beam, including continuity of derivatives (smooth curve without breaking points), the internal forces can be put into relation with the geometry of the centerline curve using relations of theory of elasticity of continuous beams.

For the simplest case of bending (assuming only linear elastic state of the material), the formula governing the relation between acting bending moment M and curvature κ of the centerline can be written in the form of differential equation of bending

$$-M = EI\kappa = EIw'', \tag{1}$$

where I is the moment of area of the cross section relative to the axis of acting moment, E is the Young's modulus and w'' denotes the second derivative of transversal coordinate, perpendicular to the acting moment direction.

The curvature κ can be defined as reciprocal value to the radius of curvature ρ of the beam center line, i.e. $\kappa = 1/\rho$. The relation of curvature and the centerline $w(x)$ is defined by the formula

$$\kappa = w''/(1+w'^2)^{3/2}. \tag{2}$$

In engineering applications, where the slope of the centerline can be assumed small, we can approximate $w' \approx 0$ and as result the curvature from the previous equation will be equal to the second derivative alone, i.e. $\kappa \approx w''$. The moment-curvature relation can be used to describe elasto-plastic behavior of the cross section, in terms of defining the point of elastic limit and plastic limit of moment capacity in case of ductile material being used as reinforcement.

B. *Reduction of tensile capacity of FRP reinforcement due to member curvature*

In the case of FRP reinforcement one particular problem appears. It is not present in ductile material (as steel) and is specific problem of brittle FRP material. The bar failure is driven by fiber rupture, in which case the load carried by single fiber has to be distributed among other fibers throughout the reinforcement cross section. In case this increment causes rupture of other fiber, chain reaction of fiber rupture will occur and the bar will fail in brittle manner. As a result, a bar subjected to tensile loading as in reinforced concrete tensile zone, may fail even before reaching its ultimate stress in axial tension simply by introducing slight bending of the bar, resulting in additional tensile stressing of outlaying fibers of the FRP bar. The load carrying capacity of the FRP has to be reduced with increasing curvature of the FRP reinforced member. As was mentioned before, this reduction of load carrying capacity is strictly specific to FRP and similar materials, ductile materials are able to redistribute the load throughout reinforcement cross section utilizing the yielding and plastic capacity of the material.

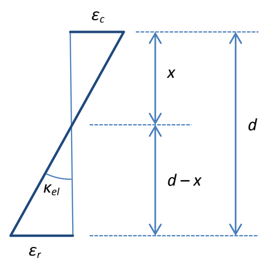

Figure 1. Curvature of elastic section.

The curvature κ_{el}, based on the basic assumptions of theory of elasticity as sectional planarity, can be calculated, assuming the beam is subjected to pure bending, simply using the strain in compressive (ε_c) fiber and reinforcement (ε_r) of the cross section of effective height d, see Fig. 1:

$$\kappa_{el} = (\varepsilon_c + \varepsilon_r)/d. \quad (3)$$

The reinforcement centerline obviously needs to copy the centerline of the entire member, as it is embedded in it. The curvature κ of the structural member thus induces additional bending moment M_r in the FRP reinforcement bar with magnitude of $M_r = E_r I_r \kappa$, where E_r is the modulus of elasticity of the FRP reinforcement and I_r is the moment of area of the reinforcement bar with diameter Ø and W_r is its sectional modulus. The additional stress in reinforcement $\sigma_{r.add}$ induced by the constrained rebar curvature is

$$\sigma_{r.add} = M_r/W_r = M_r/(2I_r/Ø). \quad (4)$$

By substituting the additional bending moment in rebar into (4) and simplifying, we get the relation for additional stress in the rebar $\sigma_{r.add}$ as function of structural member curvature κ:

$$\sigma_{r.add} = E_r Ø\kappa/2. \quad (5)$$

Thus, the total stress in the most stressed fiber of the FRP reinforcement bar can take the form of failure criteria for reinforcement:

$$\sigma_{r.tot} = \sigma_r + \sigma_{r.add} = \sigma_r + E_r Ø\kappa/2 \leq f_r. \quad (6)$$

where σ_r is the stress in reinforcement calculated by conventional means of section evaluation and f_r is the reinforcement tensile strength.

It should be noted, that for small curvatures the reduction is virtually insignificant, in the order of less than 1 % of load bearing capacity, but with increase of reinforcement ratio and deflection (and thus curvature) at peak loading the reduction can bring down the load bearing capacity of the reinforced section by significant 15 % or more.

C. Moment-rotation relation

The previous paragraph, describing moment-curvature relation, assumed that the centerline of the flexed beam is continuous including the derivatives, i.e. the centerline curve is smooth. The concept of plastic hinge however implies the formation of discontinuity in the centerline derivative, forming a breaking point in the centerline curve. As long as the curvature at such point is infinite (radius of curvature is equal to zero), the moment-curvature concept is not of use in this case. The rigid body rotation model, providing relation between bending moment and the rotation angle of the two rigid parts is used instead. It is to be noted that the product of moment and rotation angle of the two rigid parts can be interpreted as energy dissipated in the plastic hinge.

The rigid-body rotation model is especially suited for applications, where the strain is concentrated into limited area, typical for plastic hinges. In the theory of reinforced concrete beams such plastic hinge is formed by reinforcement yielding. The assumption is that the reinforcement yields in ideal elasto-plastic manner, i.e. no hardening of the reinforcement is taken into account. By such assumptions, the rotation capacity of plastic hinge in steel reinforced concrete beam is limited by the ultimate compressive strain of concrete ε_{cu}. As the strain in the plasticized reinforcement increases, so does the strain in compressive area, ultimately leading to concrete crushing failure of the compressive zone.

Formulating the constitutive relation of flexural behavior of reinforced section has one more advantage, as commercially available software usually includes user-definable plastic hinge model, and thus such relation can be adopted for use in various environments without requiring separate single purpose software.

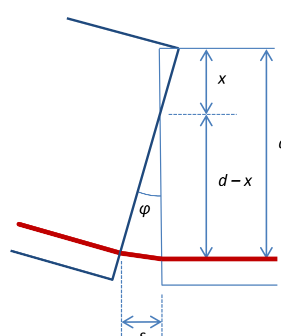

Figure 2. Rigid body rotation angle definition.

It is possible to formulate moment-rotation relation utilizing the reinforcement slippage model, as described in [5,6]. When crack developed in reinforced beam and reinforcement slipping due to axial force is assumed, the point on the reinforcement and point in the concrete matrix, which were coincident before the loading are now slipped apart by distance $s(F)$. The rotation angle φ between the two crack faces can now be written as (see Fig. 2):

$$\varphi \approx \tan\varphi = s(F)/(d - x). \quad (7)$$

This assumption should be valid for any crack surface, with exception of the crack localized at peak bending moment, as reinforcement slippage in such case occurs on both crack surfaces. The total rotation angle 2φ should be considered on such cracks. In cracks in the area of continuously increasing bending moment, only one crack surface (the one facing the moment maximum) is subjected to reinforcement slippage.

D. Reinforcement slippage

The bond-slip relation applied to the face of reinforcement in concrete can be arbitrary; in this case a bond-slip relation formulated by Eligehausen et al. [3] and modified by Cosenza et al.[4] for use with FRP bars will be used. This relation allows for solving analytically for slip, normal stress and bond stress along the reinforcement bar under tension together with exact solution of the stress development length.

The differential equation governing the bond problem of rebar is derived from equilibrium of rebar and consideration of linear elastic behavior of rebar. In this section, the ξ coordinate will be used to describe the longitudinal dimension along rebar of diameter Ø and modulus of elasticity E_r, s will be the slippage (i.e. the distance between a point on rebar and in matrix, that were coincident before load was applied) and τ is the bond contact shear stress. The equation governing the bond-slip relation is formulated as:

$$d^2s/d\xi^2 - 4\tau(\xi)/(E_r Ø) = 0. \quad (8)$$

For the relation governing the bond-slip stress, the model introduced in [4] specifically for FRP bars is used. This model puts in relation bond stress and slip between rebar and concrete, thus introducing the relation in form $\tau = \tau(s)$. Such constitutive law is given piecewise by

$$\tau(s) = \tau_1(s/s_1)^{\alpha};\ 0 \le s < s_1 \quad (9)$$

$$\tau(s) = \tau_1 - \tau_1 p(s/s_1 - 1);\ s_1 \le s < s_3 \quad (10)$$

$$\tau(s) = \tau_3;\ \ s \ge s_3, \quad (11)$$

where τ_1 is peak bond stress, τ_3 is residual bond stress, s_1 is peak bond stress slippage, s_3 is threshold slippage for residual bond stress, α is parameter describing ascending branch of the relation and p is parameter describing the softening branch, see Fig. 3. The values for these parameters given by [4] for FRP bars are in Table 1:

TABLE I. FRP BOND-SLIP MODEL PARAMETERS

Outer surface characteristic	Bond-slip model parameters				
	α	*p*	*s₁ [mm]*	*τ₁[mm]*	*τ₃[mm]*
Smooth	0.145	1.87	0.26	1.19	0.99
Ribbed	0.283	14.88	1.23	11.61	7.79
Grain covered	0.067	3.11	0.13	12.05	3.17

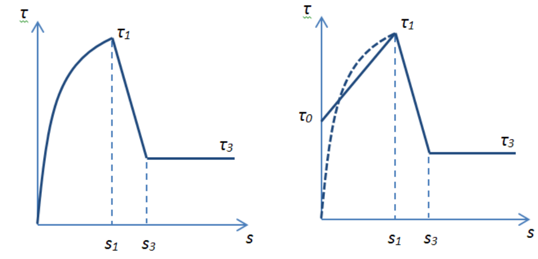

Figure 3. Bond-slip stress (left) and its simplification (right).

The exponential model described by equation (9) proved to be difficult to solve analytically, so simplification was applied, transforming the exponential model to linear one, by introducing one additional parameter τ_0. The simplification was performed in such manner, that the maximum bond stress τ_1 is preserved and the potential energy of the deformation is retained as well, i.e. the area under the curve is kept equal. Equation (9) will be changed to form

$$\tau(s) = (\tau_1 - \tau_0)(s/s_1) + \tau_0;\ 0 \le s < s_1 \quad (12)$$

where $\tau_0 = \tau_1(1-\alpha)/(1+\alpha)$. For detailed derivation see [5].

In order to find function describing the slippage in relation to longitudinal coordinate along rebar $s(\xi)$, we need to solve the differential equation (8) with different formulas for $\tau(s(\xi))$ in the respective intervals. The solution exists, but its complexity is beyond extent of this paper. Important is, that such solution gives relation between slippage and position along the reinforcement bar $s(\xi)$, which using the bond-slip constitutive law can be transformed to bond stress distribution $\tau(\xi)$ along the bar. Such relation allows solving for equilibrium of total bond force with tensile load of the reinforcement and calculation of development length λ as bi-linear function of axial force in reinforcement. Finally, it is possible to formulate relation between axial force F_r in the reinforcing bar and slippage at the face of the crack, $s(F_r)$ using bi-quadratic simplified relation with five parameters. It is possible to tabulate these five parameters for various rebar diameters and FRP materials or include such relation in to engineering software.

E. Moment-curvature and moment-rotation compatibility problem

The moment-curvature relation is based on theory of elasticity and assumes elastic behavior of beam under flexion and moment-rotation describes the behavior of inelastic hinge formed upon reaching of given limit load. The compatibility issue occurs in case we need to superpose the two states, i.e. to model crack opening in already deflected beam. It is not possible to simply combine the two models, simply because of the dimension of the variables – the rotation is denominated in angular units, i.e. is dimensionless and curvature is defined as reciprocal of radius of osculating circle or second derivative of centerline deflection curve, with dimension of reciprocal length.

The curvature can be calculated utilizing sectional dimensions and strain in compressive and tensile fiber, as seen in equation (3). The rotation after section cracking can be calculated using the reinforcement slippage and neutral axis location as seen in equation (7).

In order to be able to combine the two variables we will introduce the quantity with dimension of reciprocal length, replacing the rotation. As this quantity has the same dimension as curvature, we will call this quantity pseudo-curvature and denote κ_{ps}. The smooth curved centerline should be replaced with polygonal chain of lines of finite length s_m (crack distance) and angle at each apex φ. We can now define a circle of radius r, coincident with every single apex of the polygonal chain, see Fig. 4. The reciprocal of radius r is the pseudo-curvature κ_{ps} of the polygonal chain and it value is $\kappa_{ps} = \varphi/s_m$.

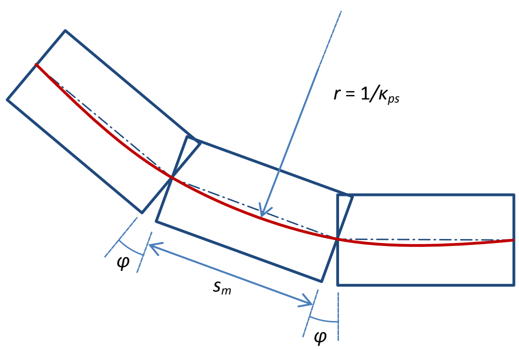

Figure 4. Polygonal chain of rigid bodies, pseudo-curvature definition.

The pseudo-curvature κ_{ps} resulting from previous assumption of rigid body rotation can now be combined with curvature κ_{el}, obtained from elastic calculation and their sum κ_{tot} can be used as the curvature in the reinforcement failure criteria (6).

III. Calculation of the moment capacity

As has been stated before, the notorious design formula of evaluating the moment capacity of the reinforced concrete beam, presented in Eurocode 2 is unsafe to use with FRP reinforcement. The proposed reduction coefficient obviously has to be related to the reinforcement ratio, as for slightly reinforced sections the reduction in moment capacity induced by member curvature is negligible and for sections with high reinforcement ration, even at the threshold of concrete crushing failure, the reduction may reach levels higher than 15 %. Such high reduction is caused by very high curvatures the FRP reinforced members display at failure point, as the Young’s modulus of FRP reinforcement is by order of magnitude lower than the one of steel, resulting in much lower stiffness of the structural member (the fact which in engineering practice leads to Serviceability Limit State based design of FRP reinforced members).

The proposed formula for reduction coefficient C_{red} was based upon investigation made on theoretical models and verified using series of experiments, using cross sections with various reinforcement ratios ρ, ranging from 0.1 % up to 1.5 %, where the failure is driven by concrete crushing. Reinforcement ratio ρ is calculated as the ratio of sectional area of reinforcement A_r relative to effective sectional area $b\times d$, where b is the width of compressive zone and d is the distance of reinforcement form the compressive fiber of the section. As the reduction was found to be strongly non-linear, a function with more than one parameter was required to approximate it. Logarithmic expression with two parameters was found to fit the results well. The formula was proposed in the following form:

$$C_{red} = a(\ln \rho + b), \qquad (13)$$

where a and b are arbitrary constants and for the time have been found to be a = 0.075 and b = 2 and the value of C_{red} represents the relative amount of moment capacity that is lost due to member curvature. For reinforcement ratios ρ lower than 0.15 %, the reduction should be considered zero. It should be a topic of further research whether the reduction formula would give better results, if formulated in form of other function, for example in the form of bi-parametric square root function. The resulting evaluation algorithm based on Eurocode 2 takes the form of:

$$x = (A_r f_r)(0.8 b \alpha f_c), \qquad (14)$$

$$M_R = (1 - C_{red}) A_r f_r (d - 0.4x), \qquad (15)$$

where M_R is the moment capacity, x is the neutral axis coordinate relative to compressive fiber of the cross section, A_r is sectional reinforcement area, f_r is tensile strength of reinforcement, b is width of the compressive area, α is coefficient (usually 0.85 or 1.0) and f_c is concrete compressive strength.

IV. Experimental results and conclusion

The theoretical models, described in previous paragraphs, were compared to experimental results on medium scale test specimens. The experimental program was conducted in the laboratories of the Experimental Centre of the Faculty of Civil Engineering of the Czech Technical University in Prague during the spring of 2011, as part of bachelor thesis of Filip Vogel [7,8]. Experimental setup was slightly unorthodox due to the fact that the primary purpose of the experiment was to investigate the moment redistribution ability of the FRP reinforced beams and the moment capacity measurement (for this paper) was just a by-product of the investigation. The experiments were conducted on FRP reinforced concrete continuous beams with identical outer dimensions (180×130 mm cross section, 4.0 m length) differed in their reinforcement ratio. The reinforcement in all cases was symmetrical for upper and lower surface of the beam, as positive and negative bending moments were anticipated on the continuous beam. The main three specimens used GFRP reinforcement of different diameters: 2Ø6, 2Ø8 and 2Ø10 respectively as upper and lower reinforcement.

The length of the test specimens (4.0 m) was limited by the dimensions of the laboratory equipment and the layout of the test had to be chosen in order to make best use possible of the 4.0 m long specimen. The requirement was for the test to represent at least once statically indeterminate structure, in order to be able to measure hypothetical moment redistribution upon reaching desired plastic hinge. Acting force F and all support reactions A, B, C were measured, together with deflections. The experiment layout is in Fig. 5.

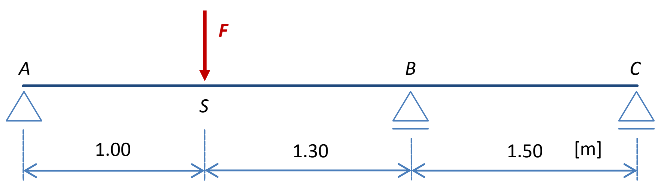

Figure 5. Experiment layout.

All the test specimens underwent destructive test with the results of the measured ultimate moment capacities and their comparison to predicted values presented in comprehensible form in Table 2.

TABLE II. COMPARISON OF RESULTS

	GFRP reinforcement layout					
	2Ø4	**2Ø6**	**2Ø8**	**2Ø10**	**2Ø12**	**2Ø14**
ρ [%]	0.12	0.28	0.50	0.78	1.13	1.55
M_R (EC2) [kNm]	2.55	5.63	9.76	14.77	20.45	26.56
C_{red} [%]	0.0	5.4	9.7	13.1	15.9	18.3
M_R (red.) [kNm]	2.55	5.33	8.81	12.83	17.20	21.71
M measured	N/A	5.35	9.23	12.27	N/A	N/A

As can be seen from the table, the reduced moment capacity results in the line MR (red.) provide much safer prediction than conventional design formula in line MR (EC2), compared to the actual measured moment capacity M measured. Still, the proposed moment capacity reduction formula (Eq. 8) is quite simple and comprehensible.

Acknowledgment

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