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# A Structural Analysis for the Autofrettage of FGM Cylindrical Vessels

[Morteza Ashhadi, Hamid EkhteraeiTousi]

*Abstract*— **Recent technologies provide new opportunities for the construction of heterogeneous vessels with adaptable material compositions called Functionally Graded Materials (FGM). In order to benefit the damage tolerance advantages of internal compressive residual stresses, one may organize an autofrettage process in the FGM vessel. The process consists of a high internal pressure loading and unloading cycle. In this paper the autofrettage of an FGM vessel with nonlinear distribution of heterogeneity is studied. Using the semi-analytic technique known as Variable Material Properties (VMP), residual fields of stresses are derived. To validate the analytical technique, the solution of a typical problem is compared with the Finite Element results of the ABAQUS software. Based on the developed VMP solutions the effects of some parameters such as the loading pressure and volume fraction distribution pattern upon the level of compressive residual stresses are studied.**

**Keywords—Pressure Vessel, Residual Stresses, Kinematic Hardening, FGM, VMP**

## I. Introduction

Pressure vessels are used in different industries such as the food process, military, nuclear, oil and petrochemical industries for the compression of gases or liquids. This widespread demand encourages many researches working in the field of pressure vessels to increase the loading capacities of these structures. The process of autofrettage is one of the techniques that may employ to increase the loading capacity of pressure vessels. The process is extensively studied by Maning (1980) [1] and Chen (1990) [2], as well as Showin and Genglink [3]. Ouh [4] has studied the compression of an elastoplastic cylinder with strain hardening and Rolling and Jinlayyang [5] studied the compression of an elastic-perfectly plastic cylinder. The exact solution of thick walled cylinder under the internal pressures based on Tressca's criterion is done by Darijani et.al. [6]. Miligan et al. have also studied the Buchinger's effect. Among the recent studies of residual stresses in thick walled pressure vessels made of metal and metal-ceramic components one can refers to the works of Jahromi et.al. (2010) [8] in which using the semi analytic technique of Variable Material Properties (VMP) they studied the residual stresses in a composite cylindrical vessel.

Morteza Ashhdi
Ferdowsi University of Mashhad
Mashhad, I. R. Iran

Hamid EkhteraeiTousi
Ferdowsi University of Mashhad
Mashhad, I. R. Iran

The main groups of the recently developed pressure vessels are the pressure vessels made of Functionally Graded Materials (FGMs). FGMs are heterogeneous materials in which the distribution of mechanical properties can be adjusted locally to prepare much desirable structural behaviors. In connection with the subject of this paper, it can be remembered that, non-uniform distributions of material properties can be controlled to affect the distributions of residual stresses.

The autofrettage process is a procedure in which a pressure vessel is pressurized to a level much higher than its yielding capacity. By relieving the pressure, a bond of compressive residual stresses will form at the inner layers of the vessel. Once the vessel is under the action of excessive or fluctuating internal pressures the compressive zone helps the material to withstand the static or dynamic loadings and increase the working life of the structure. In fact the compressive stresses keep the micro cracks closed and postpone the fracture. In this study the residual stresses aroused in the autofrettage of an FGM vessel are studied. To perform this analysis the technique of Variable Material Properties (VMP) is utilized. In this technique applicable to axisymmetric plane stress or plane strain problems it is assumed that a cylindrical heterogeneous structure is composed of several nested thin cylindrical homogenous layers. Therefore the study of the heterogeneous structure is converted into the successive analysis of several interconnected homogeneous parts. Especially in elastoplastic studies, it creates an opportunity to transform the elastoplastic analysis to its quasi elastic counterpart which simplifies the analysis even furthermore. The other aspects of the analysis performed in this paper include the assumption of kinematic hardening for the flow rule and the verification of the results by ABAQUS simulations. Different figures showing the distribution of tangential components of stress in the radial direction are provided and the influences of different factors are discussed.

## II. Mathematical modeling

Any solution strategy is in need of a proper mathematical model. Usually the problem of residual stress analysis in a cylindrical pressure vessel under internal pressure is assumed to be an axisymmetric problem. In this case the equilibrium equation is as follows [9].

$$\frac{d(hr\sigma_r)}{dr} - h\sigma_\theta = 0 \qquad (1)$$

In which $\sigma_r$ and $\sigma_\theta$ are the radial and tangential components of stress respectively. Moreover $r$ is the radial coordinate.

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In elastic region the constitutive relations are provided as "2".

$$\varepsilon_{ij}^{e} = \frac{1+\nu}{E}\sigma_{ij} - \frac{\nu}{E}\sigma_{kk}\delta_{ij} \tag{2}$$

In which E and $\nu$ are the elastic modulus and Poisson's ratio respectively and $\delta_{ij}$ is Kroneker's delta. In each point total strain is a summation of the elastic and plastic parts. That is,

$$\varepsilon_{ij} = \varepsilon_{ij}^{e} + \varepsilon_{ij}^{p} \tag{3}$$

The Henky's flow rule theory or the general stress-strain relationship in plastic region is assumed to be effective. The rule is given as [10],

$$\varepsilon_{ij}^{p} = \phi S_{ij} \tag{4}$$

In which $S_{ij}$ is the deviatoric stress, defined as follows.

$$S_{ij} = \sigma_{ij} - \frac{1}{3}\sigma_{kk}\delta_{ij} \tag{5}$$

Besides $\phi$ is a scalar factor introduced in the following equation.

$$\phi = \frac{3\varepsilon_{eq}^{p}}{2\sigma_{eq}} \tag{6}$$

In which $\varepsilon_{eq}^{p}$ and $\sigma_{eq}$ are equivalent plastic strain and stress components. The vonMises criterion is selected as the failure criteria. Accordingly one can deduce,

$$\sigma_{eq} = \sqrt{\sigma_r^2 + \sigma_\theta^2 - \sigma_r\sigma_\theta} \, . \tag{7}$$

In axisymmetric plane stress conditions comparing "4" and "6" one obtains [11],

$$\varepsilon_{ij}^{e} = \frac{1+\nu_{eff}}{E_{eff}}\sigma_{ij} - \frac{\nu_{eff}}{E_{eff}}\sigma_{kk}\delta_{ij} \tag{8}$$

In which $\nu_{eff}$ and $E_{eff}$ are the effective Poisson's ratio and Young's modulus which using "8" and "2" can be obtained as,

$$E_{eff} = \frac{3E}{3+2E\phi} \tag{9}$$

$$\nu_{eff} = \frac{3\nu + E\phi}{3+2E\phi} \tag{10}$$

The technique of VMP suggests a successive approach for the solution of nonlinear, non-isotropic axisymmetric one dimensional plane strain or plane stress problems. In this approach the vessel is divided into several cylinders or layers of small thickness in each layer the properties of material are uniform. According to this technique, primarily the elastic solution of a cylindrical element must be obtained. To do this as in Fig. 1 the cross section of a thin uniform thickness cylinder with internal radius $r_i$ and external radius $r_0$ is considered. At the first step the elastic solution of this single layer is investigated. In this case the equilibrium equation of stresses in terms of the displacement component is appeared as follows [12],

$$\frac{d^2u}{dr^2} + \frac{1}{r}\frac{du}{dr} - \frac{u}{r^2} = 0 \tag{11}$$

Which its solution for a single vessel of the composite structure is given as,

$$u(r) = (q_1)r + (q_2)\frac{1}{r} \tag{12}$$

In which $q_1$ and $q_2$ are found to be [12],

$$q_1 = \frac{1-\nu}{E(r_1^2 - r_2^2)}(\sigma_1 r_1^2 - \sigma_2 r_2^2) \tag{13}$$

$$q_2 = \frac{(1+\nu)r_1^2 r_2^2}{E(r_1^2 - r_2^2)}(\sigma_1 - \sigma_2)) \tag{14}$$

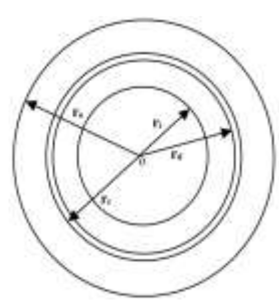

Figure 1. Cross sectional view of cylindrical vessel and thin annular element

The above equations can be united in a matrix form as below:

$$\{u\} = [C]\{P\} \tag{15}$$

The system of equations in "15" is resulted from the combination of several equilibrium equations for different elements. As the subdivided elements are in contact, the traction between them or the radial components of interfacial stresses are equal. Similarly the interfacial displacement and force of the adjacent elements are related as follows:

$$P_2^i = P_1^{i+1} \tag{16}$$

$$u^i(r_2) = u^{i+1}(r_1) \tag{17}$$

In which $i$ is the element number. Combining the known boundary conditions and the linear system of equations in "15" the displacements and accordingly the inter element radial forces are obtained. Then using "8" the stress components can be obtained.

In VMP method once an element enters plastic region, the effective amounts of $E_{eff}$ and $v_{eff}$ are replaced with elastic modulus and Poisson's ratio in "9" and "10". As in Fig.2 a procedure known as imaging method is used to transform the multi-axial stress-stain variations into its corresponding uniaxial amounts shown on a uniaxial stress-strain curve. Besides in this method it is assumed that the evolution of the sequential equivalent uniaxial solution imitates the uniaxial stress-strain behavior of the material.

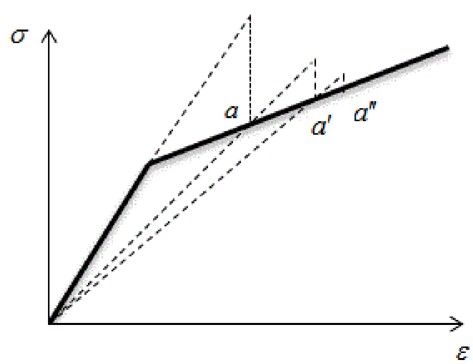


Figure 2. The procedure of imaging method application

## III. Homogenization

Here the word homogenization refers to the mathematical procedure in which overall mechanical or physical properties of a mixed material compound is related to the properties of its constituents. Using Fig. 3 the properties of a functionally graded composite of metal and ceramic phases is obtained by using of the modified mixture rule provided in [13]. That is,

$$E_{comp} = \left[(1-f)\left(\frac{q+E_c}{q+E_m}\right)+f\right]^{-1}\left[(1-f)E_m\left(\frac{q+E_c}{q+E_m}\right)+fE_c\right] \tag{18}$$

$$H_{comp} = \left[(1-f)\left(\frac{q+E_c}{q+H_m}\right)+f\right]^{-1}\left[(1-f)H_m\left(\frac{q+E_c}{q+H_m}\right)+fE_c\right] \tag{19}$$

$$\sigma_y^{comp} = \sigma_y^m\left[(1-f)+\left(\frac{q+E_m}{q+E_c}\right)\frac{E_c}{E_m}f\right] \tag{20}$$

In this study it is assumed that the bounding of phases is complete. Ceramic particles behave elastically and withstand any kind of breakdown. Metallic constituents behave elastic-plastically and follow the bilinear type of stress-strain curves. In "18-20" $E_m$ is the modulus of elasticity of the metallic phase, $E_c$ is the elasticity of the ceramic phase, $E_{comp}$ is the overall elasticity of the metal-ceramic compound, $H_m$ is the tangent modulus of the metallic phase and $H_{comp}$ is the tangent modulus of the ceramic-metal compound which yet indicates a linear hardening behavior. Moreover $\sigma_y^m$ is the yield stress of the metallic phase and $\sigma_y^{comp}$ is the yield stress of the metal-ceramic compound. $q$ is the stress to strain transmission ratio which is based on the experimental data [13]. Besides $f$ is the ceramic volume fraction at any point, obtained by using the following distribution function.

$$f(r) = f_0\left(\frac{r-a}{b-a}\right)^m \;; m>0,\; 0\le f_0 \le 1 \tag{21}$$

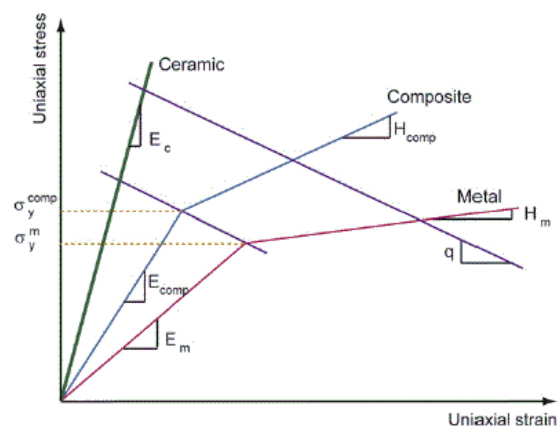


Figure 3. A mixture rule of FGM compounds [14]

## IV. Results and discussions

In order to evaluate the accuracy of the solution algorithm its exemplary results are compared with the results obtained by using ABAQUS software. In ABAQUS simulations to reduce the extent of calculations, the two-dimensional kind of nonlinear plane strain elements are used. The vessel is divided into several layers each one comprised of uniform material properties. The consequence of this assessment is provided in the following.

In line with the results represented by Gu et.al. [15], the mechanical properties of the metallic phase used in the functionally graded material are assumed to be $E_m = 56GPa$, $H_m = 12GPa$, $\sigma_y^m = 106MPa$ and $\nu = 0.25$. Similarly for the ceramic phase the amounts of $E_c = 80GPa$ and $\nu = 0.25$ are used. As in [15] a value of $17.2GPa$ is selected for the parameter $q$ in "18-20". The analyses show that for the vessel with continuously varying material properties when the number of layers exceeds to more than 20 strips the results of ABAQU*S* solutions come close to the VMP solutions. Fig. 4 shows the results when $p = 100MPa$, $f_o = 0.25$, $n$=1 and hardening is kinematic. These include the residual radial, tangential and von Mises stresses. As it can be seen the results are very close together.

Now we are going to focus on different aspects of the autofrettage of FGM vessels. One important point is that one should never expect that higher autofrettage pressures essentially results in higher fields of compressive stresses. In fact close similar to the homogeneous vessels in this heterogeneous vessels whenever internal pressure is high, by reducing the pressure as can be seen in Fig. 5 a fringe of compressive stress field will shape inside the vessel. Nevertheless if in unloading stage, the vessel goes into the reverse yielding then higher pressures may also result in less internal compressive stresses.

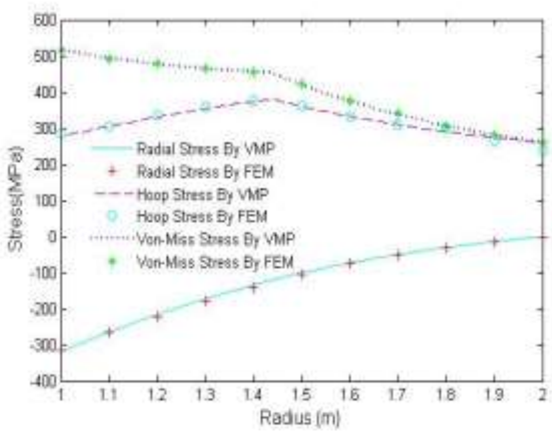


Figure 4. a comparison between the VMP and FEM solutions

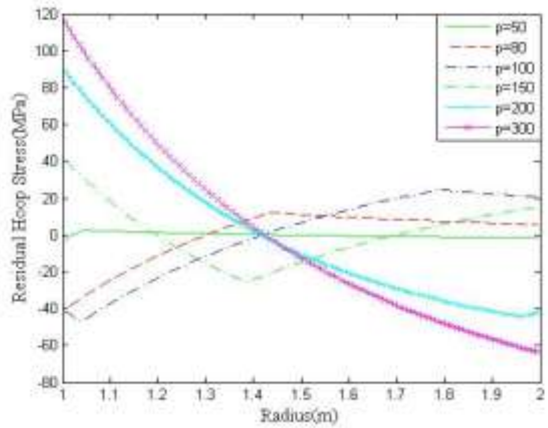


Figure 5. Residual tangential stress in the autofrettage of an FGM cylindrical vessel

The effect of reinforcement factor $f_0$ for a FGM vessel of elastic-plastic material with isotropic hardening is seen in Fig. 6, (Autofrettage pressure equals: $p = 300MPa$). It is seen that by increasing $f_0$ the residual field inside the vessel is more inclined toward the compressive stresses. Especially at $f_0$ =1 the best condition is attainable. In other words the highest level of compressive residual stresses can be seen at $f_0$ =1.

Fig. (7) depicts the effect of volume fraction exponent used in "21" with isotropic hardening assumption on the field of residual stresses. It is seen that by increasing $m$ the residual fields of stress inside the vessel tend to be more compressive but this trend is reversed when $m$ exceeds the unity (i.e., $m > 1$).

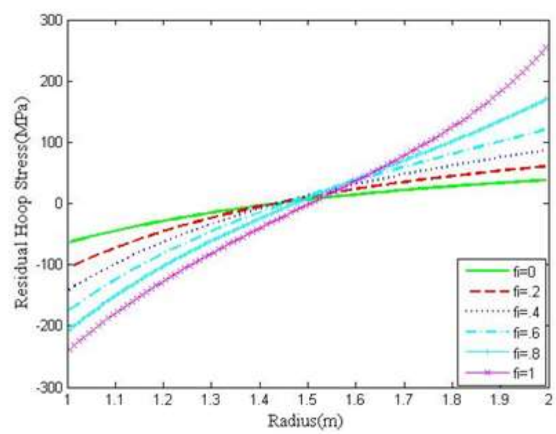


Figure 6. Residual tangential strss at different reinforcement factors and p=300Mpa

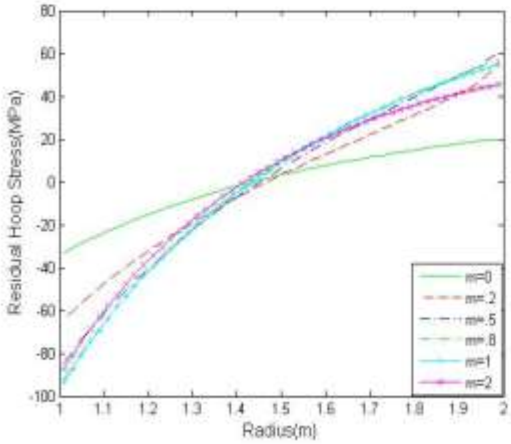


Figure 7. Residual tangential stress at different volume fractions at p=300Mpa

# V. Conclusion

In this study the fields of residual stresses in FGM cylindrical vessels are studied. To perform an axisymmetric elastoplastic analysis the technique of variable Material Properties is used. Primarily to ensure that the analytical procedure can really work, the results of a typical analysis has been performed both with VMP technique and ABAQUS software. After comparing the results, the efficiency of the VMP analytical procedure is assessed. In the provided analyses the residual stresses are found according to different isotropic and kinematic models of hardening. The derived fields of stresses are compared. The effects of autofrettage on the radial and tangential stresses are discussed. Moreover the effect of reinforcement factor in the FGM vessel which shows the percentage of ceramic in the ceramic-metal mixture inside the vessel is studied. The effects of the exponent of power function of volume fraction distribution on the residual stresses are studied too. The graphs can help researchers to select proper combinations of volume fraction, reinforcement factor and autofrettage pressure to obtain desirable level of compressive residual stress field inside the vessel.

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Morteza Ashhadi (1985), M.Sc., (is currently a graduate student of Aerospace at the Mechanical Engineering Department of Ferdowsi University of Mashhad; He is pursuing his researches in the fields of Heterogeneous Pressure Vessels and Residual Stress Analyses.

Hamid EkhteraeiTousi, (1961), Ph.D., is currently an Associate Professor at Ferdowsi University of Mashhad in Iran. He is the author of many English or Persian publications in the field of Computational Structural Mechanics, Plasticity and Creep of materials, many of them accessible at: (http://ekhteraee.profcms.um.ac.ir/).