

# Platoon Simulation of Vehicle Robots According to Vehicle following Model

E. Kita, K. Asahina, C. Ushida, Y. Wakita, T. Tamaki

**Abstract**—Vehicle platoon is one of important techniques both for improving the traffic safety and for increasing the traffic network capacity. Vehicle platoon system allows vehicles to drive at the same velocity and to accelerate or brake simultaneously. In this study, we will present the velocity control algorithm of the vehicles in the vehicle platoon and then, apply it to the velocity control of the vehicles in the vehicle platoon of vehicle robots. Vehicle robot can observe the distance by the ultrasonic sensor and the velocity data through Bluetooth communication. The experimental result is compared with the numerical simulation in order to confirm the validity of the model. The experimental result agrees relatively well with the computer simulation.

**Keywords**—Vehicle platoon, Multi-leader vehicles model, LEGO Mindstorm.

## I. Introduction

Intelligent Transportation Systems (ITS) are advance applications of Information and Communication Technology (ICT) for road traffic systems. Development of the ITS can improve the safety and efficiency of the road traffic system.

Vehicle platoon is one of the promising systems in the ITS. Vehicle platoon system allows many vehicles to drive at the same velocity and to accelerate or brake simultaneously. Platoon system decreases the distances between vehicles using electronic and mechanical systems and therefore, it can increase the capacity of road networks.

The velocity control algorithm of vehicles in vehicle platoon is presented in this study. The algorithm is defined according to the vehicle following model [1-6]. The vehicle following model controls the vehicle velocity or acceleration according to the information from the nearest leader vehicle such as the velocity, position and so on. Chandler model controls the vehicle acceleration according to the velocity difference of the vehicle and its nearest leader vehicle [1]. Bexelius extended Chandler model to multi-leader vehicles following model [5]. In this study, Bexelius model is applied to the vehicle velocity control in the vehicle platoon. Vehicle velocity is represented with the linear combination of the velocity differences between the vehicle and its leader vehicles with unknown parameters. The total sensitivity for the leader vehicles is maximized to determine the unknown parameters.

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The model is used for vehicle platoon simulation of robots. Vehicle robot, which is made of LEGO Mindstorm NXT, has the ultrasonic sensor and the Bluetooth communication. Each vehicle can obtain the distance information from the nearest leader vehicle by the sonic sensor and the velocity data from the other vehicle through the Bluetooth communication. In the numerical results, the experimental results are compared with the numerical simulation.

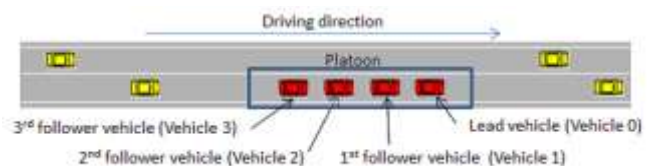


Figure 1: Vehicle platoon

## II. Velocity Control Model

### A. Vehicle Following Model

In this study, the platoon of four vehicles is considered as an example. The first vehicle is named as the lead vehicle of the platoon and the other vehicles are as first, second, third and fourth follower vehicles (Fig.1).

Chandler-type multi-leader vehicle following model is defined as follows:

$$\ddot{x}_n(t + \Delta t) = \sum_{j=1}^m a_j (\dot{x}_{n-j}(t) - \dot{x}_n(t)) \quad (1)$$

where the notation  $x_n(t)$  denotes the  $n$ th vehicle position at the time  $t$  and the notation  $a_j > 0$  denotes the sensitivity from  $n$ th vehicle to  $(n - j)$ th vehicle. The parameter  $m$  is the number of the leader vehicles for the  $n$ th vehicle. The upper dot ( $\ddot{\phantom{x}}$ ) and ( $\dot{\phantom{x}}$ ) denote the first- and second-derivatives with respect to the time, respectively.

In this model, the vehicle acceleration  $\ddot{x}_n$  is controlled according to the velocity difference between the vehicle and the leader vehicle  $\dot{x}_{n-j}(t) - \dot{x}_n(t)$ . Chandler model [1] takes  $m = 1$  and Bexelius model [5] and Wakita model [6] take  $m > 1$ .

### B. Stability Analysis

In equation (1), we will consider as the stable state that all vehicles move at the same velocity  $v_0$ .

Let  $y_n$  be a small deviation from the steady state velocity;

$$\dot{x}_n = v_0 + y_n \quad (2)$$

Substituting equation (2) to equation (1), we have

$$\dot{y}_n(t + \Delta t) = \sum_{j=1}^m a_j (y_{n-j}(t) - y_n(t)). \quad (3)$$

Fourier series of  $y_n$  is given as

$$y_k(n, t) = \exp(i\alpha_k n + zt) \quad (4)$$

$$\alpha_k = \frac{2\pi}{N} k, k = 0, 1, 2, \dots, N - 1,$$

where  $N$  and  $i$  denote total number of vehicles and the imaginary unit, respectively.

From equations (3) and (4), we have

$$e^{\Delta t z} z - \sum_{j=1}^m a_j (e^{j i \alpha_k} - 1) = 0 \quad (5)$$

Applying Taylor series expansion  $e^{\Delta t z} \cong 1 + \Delta t z$  to equation (5), we have

$$\Delta t z^2 + z - \sum_{j=1}^m a_j (e^{j i \alpha_k} - 1) = 0 \quad (6)$$

On a critical curve, the real part of the imaginary number  $z$  is zero and therefore,  $z = iv$ . Substituting it to equation (6), we have

$$\Delta t = \frac{\sum_{j=1}^m a_j (1 - \cos(j\alpha_k))}{\left\{ \sum_{j=1}^m \sin(j\alpha_k) \right\}^2} \quad (7)$$

### C. First Follower Vehicle

The first follower vehicle is the vehicle just behind the lead vehicle. It has the lead vehicle as only one leader vehicle.

Substituting  $m = 1$  to equation (7), we have

$$\Delta t = \frac{1}{2a_1 \cos^2(\alpha_k/2)} \quad (8)$$

Substituting  $\alpha_k = 0$  to equation (8), we have the stability condition

$$a_1 \leq \frac{1}{2\Delta t}. \quad (9)$$

Taking  $\Delta t = 1$  at the above equation, we have

$$0 \leq a_1 \leq \frac{1}{2} \quad (10)$$

This result shows that, in this case, the maximum sensitivity with respect to one leader vehicle is  $b_1 = a_1 = 0.5$ .

### D. Second Follower Vehicle

The second follower vehicle has two leader vehicles. Substituting  $m = 2$  to equation (7), we have

$$\Delta t = \frac{a_1 + 2a_2(1 + \cos \alpha_k)}{2(a_1 + 2a_2 \cos \alpha_k)^2 \cos^2(\alpha_k/2)} \quad (11)$$

Substituting  $\alpha_k = 0$  to equation (11), we have the stability condition

$$\frac{(a_1 + 2a_2)^2}{a_1 + 4a_2} \leq \frac{1}{2\Delta t}. \quad (12)$$

Taking  $\Delta t = 1$  at the above equation, we have

$$0 \leq a_2 \leq \frac{(1 - 2a_1) + \sqrt{(1 - 2a_1)^2}}{4} \quad (13)$$

The total sensitivity for all leader vehicles is defined as follows.

$$b_2 = a_1 + a_2 \leq \frac{(1 + 2a_1) + \sqrt{(1 - 2a_1)^2}}{4} \quad (14)$$

The total sensitivity  $b_2$  takes the maximum value  $(b_2)_{max} = \frac{9}{16}$  at  $a_1 = \frac{3}{8}$  and  $a_2 = 3/16$ .

### E. Third Follower Vehicle

The third follower vehicle has three leader vehicles. Substituting  $m = 3$  to equation (7), we have

$$\Delta t = \frac{a_1 + 2a_2(1 + \cos \alpha_k) + a_3(4 \cos^2 \alpha_k + 4 \cos \alpha_k + 1)}{2(a_1 + 2a_2 \cos \alpha_k + a_3(4 \cos^2 \alpha_k - 1))^2 \cos^2(\alpha_k/2)} \quad (15)$$

Substituting  $\alpha_k = 0$  to equation (15) and taking  $\Delta t = 1$  at the above equation, we have

$$0 \leq a_3 \leq \frac{(3 - 4a_1 - 8a_2) + \sqrt{(9 - 16(a_1 + a_2))}}{4} \quad (16)$$

The total sensitivity for all leader vehicles is defined as follows.

$$b_3 = a_1 + a_2 + a_3 \leq \frac{(3 + 4a_1 + 8a_2) + \sqrt{(9 - 16(a_1 + a_2))}}{4} \quad (17)$$

The total sensitivity  $b_3$  takes the maximum value  $(b_3)_{max} = \frac{2}{3}$  at  $a_1 = \frac{1}{2}$ ,  $a_2 = 0$  and  $a_3 = \frac{3}{16}$ .

## III. Simulation

### A. Velocity Control of Robots

Assume the positions of the lead, the first follower, the second follower and the third follower vehicles as  $x_0, x_1, x_2$  and  $x_3$ , respectively.

Since the first follower vehicle has only one leader vehicle, it is considered that the first follower vehicle follows the single-leader vehicle following model:

$$\ddot{x}_1(t + \Delta t) = \frac{1}{2}(\ddot{x}_0(t) - \ddot{x}_1(t)) \quad (18)$$

According to the above results, the velocity control models of the second and the third follower vehicles are respectively given as follows.

$$\ddot{x}_2(t + \Delta t) = \frac{3}{8}(\ddot{x}_1(t) - \ddot{x}_2(t)) + \frac{3}{16}(\ddot{x}_0(t) - \ddot{x}_2(t)) \quad (19)$$

$$\ddot{x}_3(t + \Delta t) = \frac{1}{2}(\ddot{x}_2(t) - \ddot{x}_3(t)) + \frac{3}{16}(\ddot{x}_0(t) - \ddot{x}_3(t)) \quad (20)$$

The velocity differences to far leader vehicles are approximated with the finite difference as follows

$$\dot{x}_{j-1}(t) - \dot{x}_j(t) \equiv \Delta \dot{x}_j \cong \frac{\Delta x_j(t) - \Delta x_j(t - \Delta t)}{\Delta t} \quad (21)$$

where  $\Delta x_j \equiv x_{j-1}(t) - x_j(t)$  and  $j = 1, 2$ .

### B. Vehicle Platoon

LEGO Mindstorm NXT is designed to a vehicle robot. The vehicle platoon is composed of four vehicle robots. A vehicle has the ultrasonic sensor and the Bluetooth communication. The distance  $\Delta x_j$  in equation (21) is estimated by the

ultrasonic sensor. The lead vehicle velocity  $\dot{x}_0(t)$  is obtained from the lead vehicle through the Bluetooth communication.

### C. Vehicle Platoon Simulation

Velocity control process of the first follower vehicle is summarized as follows.

1. Initialize time-step as follows.  $t = 0$ .
2. Estimate the vehicle head distance from the nearest leader vehicle  $\Delta x_1(t)$  by ultrasonic sensor..
3. Increment time-step as follows.  $t = t + \Delta t$ .
4. Estimate the vehicle head distance from the nearest leader vehicle  $\Delta x_j(t)$  by ultrasonic sensor..
5. Calculate the acceleration by equation (18).
6. Update the velocity by
 
$$\dot{x}_i(t) + \Delta t = \dot{x}_i(t) + \ddot{x}_2(t + \Delta t) \cdot \Delta t \quad (22)$$
7. Return step 3.

Velocity control process of the second and the third follower vehicles is summarized as follows.

8. Initialize time-step as follows.  $t = 0$ .
9. Estimate the vehicle head distance from the nearest leader vehicle  $\Delta x_j(t)$  by ultrasonic sensor..
10. Increment time-step as follows.  $t = t + \Delta t$ .
11. Estimate the vehicle head distance from the nearest leader vehicle  $\Delta x_j(t)$  by ultrasonic sensor..
12. Obtain the lead vehicle velocity  $\dot{x}_0(t)$  by Bluetooth communication.
13. Calculate the acceleration by equation (19) or (20).
14. Update the velocity by
 
$$\dot{x}_i(t) + \Delta t = \dot{x}_i(t) + \ddot{x}_2(t + \Delta t) \cdot \Delta t \quad (23)$$
15. Return step 3.

## iv. Experimental Result

The vehicle platoon experiment is performed by four LEGO Mindstorm NXT robots. A lead vehicle robot is programmed to start at the velocity = 18 cm/s and slow down and speed up during the simulation. The velocity history of the lead vehicle robot is shown in Fig.2. The velocity history of the third follower vehicle robot is shown in Fig.3. The velocity fluctuation of the third follower vehicle is bigger than the other vehicles. Computer simulation result is also shown in the figure. The experimental result agrees relatively well with the computer simulation. Velocity fluctuation of the experimental result is bigger than that of the simulation result. This is because of the delay time in the velocity control and the observation error of the sensors in the robot experiment.

## v. Conclusion

The mathematical model of vehicle velocity control of the vehicle platoon was shown in this study. The model is defined according to Chandler model, which is one of popular vehicle following models. Stability analysis of the model is performed to determine the model parameters. The simulation model was applied for the vehicle platoon experiment of LEGO Mindstorm robots. The experimental result was compared with the computer simulation. The experimental result agrees relatively well with the computer simulation. Velocity fluctuation of the experimental result is bigger than that of the

simulation result. This is because of the delay time in the velocity control and the observation error of the sensors in the robot experiment.

## Acknowledgment

This work was supported by JSPS KAKENHI Grant Number 24560157.

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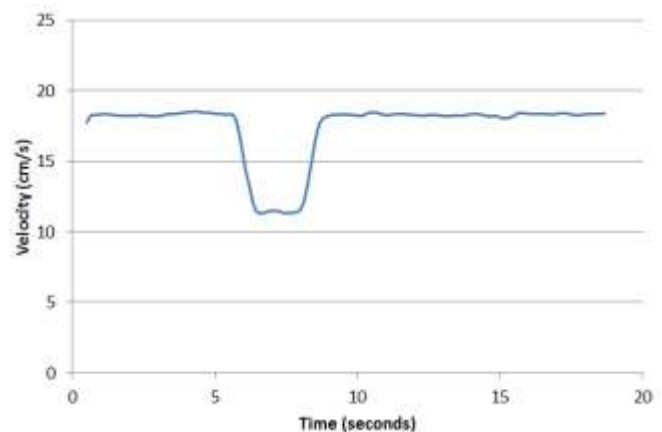


Figure 2 Velocity history of lead vehicle robot

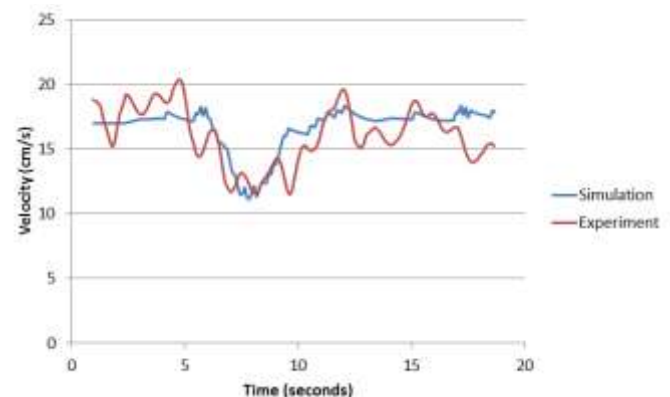


Figure 3 Velocity history of third follower vehicle robot