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Study of Pavement Maintenance and Rehabilitation Activities Using Belief-Function Theory

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***Abstract*—Road development in Taiwan has progressed to the maintenance and management stages of its life cycle. Therefore, it is necessary to introduce information technology approaches to assist in pavement management activities. Belief-function theory (BFT), otherwise known as the Dempster-Shafer theory of evidence, has been advocated by many as a method of representing uncertain, incomplete, and imprecise evidence of a system's knowledge base. BFT is still scarce in applications for the construction industry and even pavement management until now. This paper applied BFT to pavement management, in particular to the pavement maintenance and rehabilitation (M&R) activities. One case study was conducted using BFT to understand whether M&R activities need to be implemented for a pavement section. This study completed a preliminary study of BFT to promote efficiency in pavement management.**

***Keywords*—pavement, maintenance and rehabilitation, belief-function theory, Dempster's rule of combination**

I. Introduction

The belief-function theory (BFT) is used as a non-Bayes' theorem of quantifying subjective judgments based on mathematical probability. In comparison to the Bayes' theorem, which is based on assessing probabilities directly to retrieve the answer to the hypotheses of interest, a BFT approach evaluates probabilities for hypothesis-related questions to further investigate the implications of these probabilities for the hypotheses of interest. Despite the fact that antecedents of BFT can be traced back to the seventeenth and eighteenth centuries, the evolutions of the theory toward its present form is due to the work of A. P. Dempster in the 1960s and the book of G. Shafer in the 1976s, *A Mathematical Theory of Evidence* [1]. BFT, for this reason, is also referred to as the "Dempster-Shafer theory" [2]. This work presents a case study on pavement maintenance and rehabilitation (M&R) activities using the BFT. Chapter 2 provides an introduction to the basics of BFT and principal equations including Dempster's rule of combination. Chapter 3 then elaborates BFT on a case study on pavement M&R activities.

II. Belief-Function Theory (BFT)

A. *Basic Concept*

Bayes' theorem (also known as the subjective probability theorem) assumes $P(H)+P(\bar{H})=1$, where H represents the hypothesis to be evaluated, and $\bar{H}$ denotes its alternative hypothesis. As such, a subjective belief $P(H)$ claiming that H is true implies that the subjective belief that H is false would be $P(\bar{H})=1-P(H)$. However, it is impossible to distinguish if the subjective belief $P(\bar{H})$ is due to disbelief with sufficient evidence or a lack of belief such that the hypothesis cannot be inferred. The BFT method is proposed because of the ignorance issue that Bayes' theorem cannot deal with and the hypothesis of closure characteristics (i.e. $P(H)+P(\bar{H})=1$).

The "belief" in BFT represents the degree of support or justification of the statement, whereas the "belief" in the Bayes' theorem represents the likelihood or the truth of the statement. With regard to an inference, the supportive level for a hypothesis is more vital than the truth level for a hypothesis. Thus, BFT in comparison to the Bayes' theorem is more applicable in making decisions in the engineering field. BFT has four main differences from the Bayes' theorem as follows:

- BFT allows unknown or incomplete probabilities. When prior probabilities or conditional probabilities cannot be retrieved, Dempster's rule of combination is still able to effectively integrate the obtained evidences;
- BFT is unnecessary to conform to $P(H)+P(\bar{H})=1$ as the Bayes' theorem does. Therefore, an increase in the degree of belief to one hypothesis does not have an impact on the degree of belief to its alternative hypothesis. For example, an increase in the degree of belief to H does not imply a decrease in the degree of belief to $\bar{H}$;
- BFT is able to handle uncertain situations. Therefore, "0" in the BFT does not represent impossibilities (a probability of 0%) but merely a lack of evidence;
- BFT is more generalized compared to the Bayes' theorem [1]. In the case that the belief can be assigned to every single element by basic probability assignment (BPA) functions, the result obtained from the Dempster's rule of combination and the Bayes' theorem would be the same.

Based on the aforementioned comparisons, BFT is not required to provide all the answers to the hypotheses with regard to probabilities, but only part of the answers. In other words, BFT is used to assess the closeness between evidence and the hypothesis rather than the hypothesis and the truth.

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B. *Principal Equations*

1) Basic Probability Assignment Function (BPA Function)

Let Θ be the frame of discernment, and A be one of its subsets. Consider a function $m: 2^{\Theta} \to [0,1]$, where the power set 2^{Θ} represents all the subsets in the frame of discernment. The m-function is a BPA function if the following two conditions are satisfied:

$$m(\phi)=0 \quad \text{where } \phi \text{ is the null set and } \sum_{A\subset\Theta} m(A)=1 \tag{1}$$

The BPA function is also referred to as the m-function, the values of which are called m-values. If $m(A)>0$, the A is called the focal element of the belief function in Θ. The probability in the Bayes' theorem is assigned to some particular elements in Θ, while the m-value represents the probability assignment to each subset in Θ. An m-value can be obtained in two ways. It can be assigned directly by the decision maker based on his subjective judgments, or from its compatibility relationships with some other frame of discernment with its BPA functions known. The BPA function $m(\bullet)$ for the unknown frame of discernment T can be determined by the compatibility relationship with other frame of discernment S with its BPA function known, is referred to as the vacuously extension.

2) Belief Functions

The belief function $Bel: 2^{\Theta} \to [0,1]$ is defined as follows:

$$Bel(A)=\sum_{B\subseteq A} m(B) \quad , \quad \text{where} \quad Bel(\phi)=0 \quad , \quad Bel(\Theta)=1 \tag{2}$$

The degree of belief to A, represented by $Bel(A)$, is equal to the sum of m-values for all the subsets of A (B represents all the subsets of A).

3) Plausibility Functions

The plausibility function $Pl: 2^{\Theta} \to [0,1]$ is defined as

$$Pl(A)=\sum_{A\cap B\neq\phi} m(B)=1-Bel(\sim A) \quad , \quad Pl(\phi)=0 \quad , \quad Pl(\Theta)=1 \tag{3}$$

$Pl(A)$ is equal to one minus the belief function of the complement of A (or equivalently $\sim A$). Consequently, $Pl(A)$ represents the maximum potential degree of belief to the hypothesis A. Due to the fact that $Bel(A)+Bel(\sim A)\le 1$, the relationship between the belief function and the plausibility function must satisfy $Pl(A)\ge Bel(A)$.

4) The vital characteristics of the BPA function, belief function, and plausibility function

- m-value of the null set is zero, or equivalently $m(\phi)=0$
- The sum of m-values for all the subsets in the frame of discernment is equal to 1, or equivalently $\sum_{A\subset\Theta} m(A)=1$
- The values of the belief function and the plausibility function for the null set are both equal to zero, or equivalently $Bel(\phi)=0$, $Pl(\phi)=0$
- The values of the belief function and the plausibility function for the frame of discernment are both equal to 1, or equivalently $Bel(\Theta)=1$, $Pl(\Theta)=1$
- The sum of the degree of belief to hypothesis A and to its complementary set (represented by $\sim A$) is always equal to or less than 1, or equivalently $Bel(A)+Bel(\sim A)\le 1$
- The value of BPA function is always equal to or less than the value of the belief function, which is then always equal to or less than the value of the plausibility function, or equivalently $m(A)\le Bel(A)\le Pl(A)$
- Both the belief function and the plausibility function are increasing functions, if $B\subseteq A$ then $Bel(B)\le Bel(A)$ and $Pl(B)\le Pl(A)$
- If A is a singleton, then the values of the BPA function, the belief function, and the plausibility function are equal, or equivalently $m(A)=Bel(A)=Pl(A)$

5) Dempster's rule of combination

BPA functions and belief functions are used to provide the degree of belief to a hypothesis. In reality however, where multiple sources of evidence might exist. Dempster's rule of combination is able to combine the belief function for independent sources of evidence to obtain the overall degree of belief for the hypothesis of interest by calculating the orthogonal sum. The rule can deal with various independent evidences including conflicting evidences, which would make mathematical computation more complicated. As a result, BFT is usually implemented by means of computers and applied in the field of artificial intelligence.

The rule is defined as follows. Assume two independent sources of evidence exist for the same frame of discernment Θ. The corresponding belief functions are Bel_1 and Bel_2, respectively, with the value of its BPA function being m_1 and m_2. Let $A_{11}, A_{12}, ..., A_{1i}$ and $A_{21}, A_{22}, ..., A_{2j}$ be the focal elements, respectively. If $0<\sum_{i,j;A_{1i}\cap A_{2j}\neq\phi} m_1(A_{1i})m_2(A_{2j})\le 1$, define function $m: 2^{\Theta} \to [0,1]$ as follows: $m(A)$

$$= \frac{\sum_{i,j;A_{1i}\cap A_{2j}=A} m_1(A_{1i})m_2(A_{2j})}{1-\sum_{i,j;A_{1i}\cap A_{2j}\neq\phi} m_1(A_{1i})m_2(A_{2j})} = \frac{1}{K}\left\{\sum_{i,j;A_{1i}\cap A_{2j}=A} m_1(A_{1i})m_2(A_{2j})\right\}$$

$$= \frac{\sum_{i,j;A_{1i}\cap A_{2j}=A} m_1(A_{1i})m_2(A_{2j})}{\sum_{i,j;A_{1i}\cap A_{2j}\neq\phi} m_1(A_{1i})m_2(A_{2j})}, \quad K \equiv 1-\sum_{i,j;A_{1i}\cap A_{2j}\neq\phi} m_1(A_{1i})m_2(A_{2j}) \quad (4)$$

Equation (4) formulates the Dempster's rule of combination. The combination of two independent sources of evidence $(m_1 \oplus m_2)$ to obtain a new belief function is illustrated in Fig. 1 [1]. The shaded area represents the value that corresponds to the combined evidence. As there would be some null set after combination due to conflicts between evidence, the area that is taken by those null sets should be removed. The denominator of Equation (4) is called normalization constant. Assume that the normalization constant is represented by K, which is in the interval of 0 and 1. K being closer to 0 indicates that the two sources of evidence are more conflicting to each other, while K being equal to 0 indicates that it is not possible to combine those two sources of evidence and to compute the orthogonal sum. In other words, the rule is not applicable when K is equal to zero. The approach of combining multiple sources of evidence to obtain an overall degree of belief for a hypothesis is called marginalization.

III. The Application of BFT to Pavement M&R Activities

In a decision making process, in general, a lack of information or an inaccuracy in human semantic expressions might result in the decision makers not being able to make accurate judgments. BFT has a better performance over the traditional probability theory in dealing with the uncertainties of the problem. In addition, due to the fact that the constraints in belief functions are not as strict as in the Bayes' theorem, they can better deal with questions with high level of uncertainties. Though computation for belief functions would be complicated when handling complicated questions, this is not a barrier with the aid of computers [3-5]. In pavement management, pavement engineers normally make decisions on the M&R activities based on their subjective judgments or objective experiments and inspections. The degree of belief to the decisions on the M&R activities would be increased as more sources of evidence support it. This approach is consistent with the BFT, which is very suitable for applying to pavement management that is always complicated and changing over time, and helping decision makers towards the correct judgment.

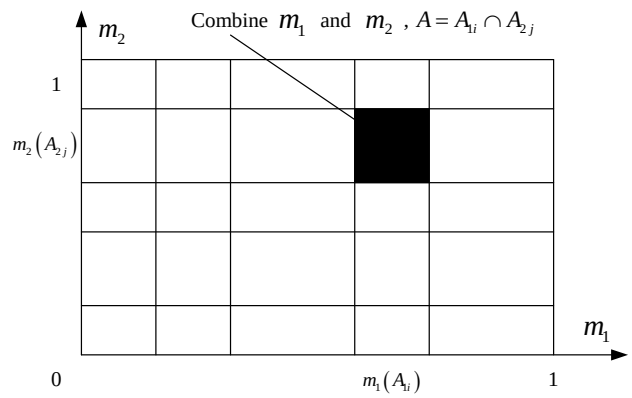

Figure 1. Illustration of Dempster's rule of combination [1].

A. Assumptions of Case Study

A case study is presented focusing on the issue of pavement deterioration by applying the BFT. Assume that pavement engineer (A) claims to the manager that according to his professional judgment, the condition of a particular pavement section would be deteriorated to such an extent in five years that M&R activities are necessary. A probability of 80% that the engineer's judgment is professional and correct is then deemed by the manager. In the case that the judgment from the engineer (A) is considered the only source of evidence, the degree of belief to the manager that the deterioration would require M&R activities in five years would be 80% as well. Belief functions can be used indirectly to assess the question of interest based on the degrees of belief to its related questions. In this case, the manager is able to evaluate the deterioration level of the pavement section in terms of the degrees of belief to the judgment of the engineer (A) being professional and correct.

Nevertheless, a 80% degree of belief does not imply that the manager has a 20% degree of belief that the section would not be deteriorated to such an extent that M&R activities would not be required in five years. This can be explained by the fact that the affirmative judgment made by the engineer (A) is the only source of evidence, whereas there is no evidence claiming that the section would not be deteriorated to such an extent. According to the BFT, the manager would have a 0% degree of belief that the section would not be deteriorated in five years by that much, where 0% does not indicate that there is 0% probability that the section would not be deteriorated to such an extent, but merely that the manager is not able to support such claim due to lack of evidence. This example demonstrates the difference between the BFT and the Bayes' theorem in that the Bayes' theorem is entailed by the axiom of additivity where probabilities of the two opposite statements must add to 1, where in BFT the axiom of additivity is not imposed (as in the example, the sum of degrees of belief is 80%+0% which is less than 80%). In addition, the Bayes' theorem is not able to distinguish ignorance and lack of evidence. 20% in the Bayes' theorem can only be interpreted that there is a 20% probability that the pavement section would not have deteriorated such that M&R activities would be required in five years, but cannot be interpreted as evidence

that the section would not be deteriorated. In BFT, a lack of evidence is represented as a 0% degree of belief.

Assume that the manager obtains from another pavement engineer (B) an affirmative judgment that the pavement section would be deteriorated in five years resulting in necessary M&R activities. The manager has an 70% degree of belief to engineer (B) being professional and reliable. In the case that the two sources of evidence are independent, the manager can combine the two sources of evidence according the multiplication of probabilities as follows:

- 0.8 * 0.7 = 0.56: probability that both evidences are reliable;
- 0.8 * 0.3 = 0.24: probability that engineer (A) is reliable but engineer (B) is unreliable;
- 0.2 * 0.7 = 0.14: probability that engineer (A) is unreliable but engineer (B) is reliable;
- 0.2 * 0.3 = 0.06: probability that both sets of evidence are unreliable.

Due to the fact that the manager would have evidence of the affirmative judgment as long as there is at least one evidence that is reliable, the manager would have a 94% (56% + 24% + 14%) degree of belief that the pavement section would be deteriorated in five years, after which M&R activities are required. However, the manager would still have a 0% degree of belief that the section would not be deteriorated to such an extent that M&R activities are necessary since no evidence has given a negative judgment that the section would not be deteriorated by such extent. This is a simple example of the Dempster's rule of combination.

B. *Principal Equations for The Case*

Let $S \equiv \{s_1, s_2\}$ be the frame of discernment, where s_1 and s_2 represents engineer (A) being reliable and unreliable, respectively. Let $T \equiv \{t_1, t_2\}$ be the second frame of discernment, where t_1 refers to the situation that the pavement section would require M&R activities in five years due to deterioration, while t_2 denotes the situation that the pavement section would not require M&R activities in five years. It is known previously that $m[\{S_1\}] = 0.8$ and $m[\{S_2\}] = 0.2$. The $m(\bullet)$ of the BPA function for the frame T with unknown probability assignment can be obtained in terms of the compatibility relationship of the frame T and the frame S, the probability assignment of which is known. Let sCt represent the situation where the element s in the frame S is compatible with the element t in the frame T. The value of s_1Ct_1, s_2Ct_1, and s_2Ct_2 can be determined from the definition of frames S and T and the judgment of t_1 made by engineer (A). Consequently, $m[\{t_1\}] = m[\{s_1\}] = 0.8$, $m[\{t_1, t_2\}] = m[\{s_2\}] = 0.2$, and $m[\{t_2\}] = 0$. The reason why the probability assignment for $\{t_2\}$ is equal to zero is that the manager has no evidence to support the statement that the section would not be deteriorated to such an extent that M&R activities would be required.

Considering the judgment made by engineer (A) as the only evidence, the manager would have a degree of belief represented by $Bel(\{t_1\})$ that the pavement section would be deteriorated in five years such that M&R activities are required, where $Bel(\{t_1\}) = m(\{t_1\}) = 0.8$. Since there is no evidence from the pavement engineer claiming that the section would not require any A&R activities in five years, its degree of belief is represented by $Bel(\{t_2\})$, where $Bel(\{t_2\}) = m(\{t_2\}) = 0$. The degree of level for the entire frame of discernment is denoted by $Bel(\{t_1, t_2\}) = Bel(\{T\}) =$

$$m(\{t_1\}) + m(\{t_2\}) + m(\{t_1, t_2\}) = 0.8 + 0 + 0.2 = 1.0.$$

Since $Bel(\{t_1\}) = 0.8$ and $Bel(\{t_2\}) = 0$, therefore $Pl(\{t_1\}) = 1 - Bel(\{t_2\}) = 1$ and $Pl(\{t_2\}) = 1 - Bel(\{t_1\}) = 0.2$. The reason why $Pl(\{t_1\})$ is equal to 1 is that the manager does not have any evidence claiming that M&R activities would not be required, resulting in a 100% plausibility that M&R activities would be required in five years. $Pl(\{t_2\})$ equal to 0.2 represents a maximum probability of 0.2 that no M&R activities would be required even though there is no evidence that supports that statement.

C. *Dempster's Rule of Combination for The Case*

Let the frame T be equal to $\{t_1, t_2\}$, where t_1 represents the statement that M&R activities would be required in five years for the pavement section, and t_2 refers to the statement that no M&R activities would be necessary. Let $S_1 \equiv \{s_{11}, s_{12}\}$ and $S_2 \equiv \{s_{21}, s_{22}\}$ be the frame of discernment that corresponds to engineer (A) and (B) being professional and reliable, respectively, where $m(\{s_{11}\})$ and $m(\{s_{12}\})$ represents the probability of engineer (A) being reliable and unreliable and $m(\{s_{21}\})$ and $m(\{s_{22}\})$ represents the probability of engineer (B) being reliable and unreliable. Assume that $m(\{s_{11}\}) = 0.8$, $m(\{s_{12}\}) = 0.2$, $m(\{s_{21}\}) = 0.7$, and $m(\{s_{22}\}) = 0.3$.

- [Scenario 1] Both evidences give affirmative judgments

If both engineer (A) and (B) claim that the M&R activities would be required in five years for the pavement section, its degree of belief represented by $Bel(\{t_1\})$ can be determined based on Dempster's rule of combination. $(s_{11}, s_{21})Ct_1$,

$(s_{11}, s_{22})Ct_1$, $(s_{12}, s_{21})Ct_1$, $(s_{12}, s_{22})Ct_2$, $(s_{12}, s_{22})Ct_1$ can be obtained from the compatibility relationships between the frame T and S_1 as well as S_2. Therefore,

$$\begin{aligned} Bel(\{t_1\}) &= m(\{s_{11}\}) \times m(\{s_{21}\}) + m(\{s_{11}\}) \times m(\{s_{22}\}) \\ &\quad + m(\{s_{12}\}) \times m(\{s_{21}\}) \\ &= 0.8 \times 0.7 + 0.8 \times 0.3 + 0.2 \times 0.7 = 0.94 \end{aligned}$$

$Bel(\{t_2\}) = m(\{t_2\}) = 0$ (Both evidences give affirmative judgments)

$Bel(\{t_1, t_2\}) = m(\{t_1, t_2\}) = 0.2 \times 0.3 = 0.06$

- [Scenario 2] Both evidences give negative judgments

If both engineer (A) and engineer (B) make a negative judgment that M&R activities would not be necessary for the pavement section in five years, then

$Bel(\{t_1\}) = m(\{t_1\}) = 0$ (Due to lack of evidence for $\{t_1\}$)

$Bel(\{t_2\}) = m(\{t_2\}) = 0.94$ (Same as how $m(\{t_1\})$ has been calculated in Scenario 1)

$Bel(\{t_1, t_2\}) = m(\{t_1, t_2\}) = 0.2 \times 0.3 = 0.06$

- [Scenario 3] Both engineers make affirmative judgment, while another source gives negative evidence (conflicting evidence)

When two or more sources of evidence are handled by Dempster's rule of combination, two of them are first combined to obtain the degree of belief for $(Bel_1 \oplus Bel_2)$, which is then combined with the next one to obtain $(Bel_1 \oplus Bel_2) \oplus Bel_3$, and so on. The manager would have a 94% degree of belief based on the judgment made by engineer (A) and (B) that M&R activities would be required in five years. In the case that the manager receives another independent source of evidence from a pavement contractor, who has made a judgment based on his years of experience in pavement M&R activities that no M&R activities would be required in five years for the pavement section. Assuming that the manager considers a 60% probability that the contractor's judgment is professional and reliable, for the frame of discernment S_3, represented by $\{s_{31}, s_{32}\}$, it would be known that $m(\{s_{31}\}) = 0.6$ (from $s_{31}Ct_2$, $s_{32}Ct_1$, and $s_{32}Ct_2$, which are compatible with $\{t_2\}$), and $m(\{s_{32}\}) = 0.4$ (from compatibility with $\{t_1, t_2\}$).

It has been shown previously by combining the judgments from engineer (A) and (B) that $Bel_1 \oplus Bel_2(\{t_1\}) = m(\{t_1\}) = 0.94$, and $Bel_1 \oplus Bel_2(\{t_1, t_2\}) = m(\{t_1, t_2\}) = 0.06$. Considering the judgment from the pavement contractor as the third source of evidence, the overall degree of belief is then obtained by Dempster's rule of combination as follows:

- The degree of belief that M&R activities would be required in five years:
$$Bel_1 \oplus Bel_2 \oplus Bel_3(\{t_1\}) = m(\{t_1\}) = \frac{0.94 \times 0.4}{1 - 0.94 \times 0.6} = 0.86$$
- The degree of belief that no M&R activities would be required in five years:
$$Bel_1 \oplus Bel_2 \oplus Bel_3(\{t_2\}) = m(\{t_2\}) = \frac{0.06 \times 0.6}{1 - 0.94 \times 0.6} = 0.08$$

The degree of belief that both engineers' judgments are reliable (i.e., M&R activities would be required in five years) and simultaneously the contractor's judgment is reliable (i.e., M&R activities would not be required in five years) can be calculated by $0.94 \times 0.6 = 0.564$. However, this case is conflicting and thus not possible, which, consequently, would be a null set. According to Equation (4), the normalization factor (K) must be applied, which is equal to $1 - 0.94 \times 0.6 = 0.436$.

IV. CONCLUSIONS

In recent years, several vigorous approaches of knowledge management (KM) and technology information (IT) have been applied to the life cycle analyses of construction and their competence in promoting management and decision-making performance and prolonging the life-spans of various constructions has been confirmed. Road development in Taiwan has progressed to the maintenance and management stages of its life cycle. It is truly necessary to introduce KM and IT approaches to assist in pavement management activities. BFT are still scarce in their applications to construction and even pavement management until now. This paper applied BFT to pavement management, in particular for pavement M&R activities. One case study was conducted using BFT to understand whether M&R activities need to be implemented for a pavement section. This study completed a preliminary study of BFT to promote efficiency of pavement management.

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