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Nonlinear Model for Lead Rubber Bearings Including Axial Load Effects and Large Deformations

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***Abstract*— Several nonlinear models have been developed to simulate the behavior of single isolation bearings. However, these models neglect certain aspects of bearing response behavior. For instance, rubber bearings have been observed to decrease in both elastic and post-elastic stiffness with increasing axial load, and soften in the vertical direction at large deformations. In this paper a macro-element model for lead rubber bearings is developed. The proposed model includes axial load effects and accounts for large deformations. The nonlinearity in the model is modeled via the Bouc-Wen model of hysteresis. Response obtained using the numerical model is shown to be in good agreement with observed behavior in test results on single lead-rubber bearings. A stability analysis on a Lead-Rubber Bearing is then conducted to investigate the accuracy of the proposed macro-element on capturing the effect of axial loads.**

***Keywords*— Lead-Rubber, Macro-element, Axial Load, Large deformations, Bouc-Wen**

I. Introduction

Several factors are often considered in the selection and design of seismic isolation devices. The selection of the appropriate seismic isolator is based on some requirements ranging from the lateral and vertical stiffnesses, cost benefits to the durability. The design of isolation systems takes into consideration the stability of individual isolators and that of the structure as one block.

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Studies and tests performed on single elastomeric bearings [1,2,3] demonstrated that bearing devices show a general reduction of stiffness with increasing axial loads. Rubber bearings have also been shown to soften in the vertical direction at large deformations [4].

Based on the fact that most isolation bearings are inherently nonlinear, especially Lead-Rubber Bearing (LRB) due to the yielding of lead core, a nonlinear extension of the linear two-spring model [1] was proposed by Ryan *et al.* [4], this model includes a linear vertical spring to account for the axial load effects, and as an option a model for lead strength degradation was incorporated to the *Coupled Nonlinear Variable Strength Model* [4]. However, this model addresses only small rotations and the determination of its nominal parameters are based on judgment.

The objective of this paper is to develop a macro-element model for LRB that includes the effect of axial loads and accounts for large deformations. This model can be easily incorporated into available structural analysis software.

II. Macro-Element Formulation

The proposed macro-element model of the LRB is shown in Fig. 1. The total potential energy of the model with reference to Fig. 1 is expressed as follows:

$$\Pi = k_\theta \theta^2/2 + k_{b0} s^2/2 + k_{bz0} v^2/2 - P u_{bz} - f_b u_b \quad (1)$$

where u_b and u_{bz} are the horizontal and vertical top displacements, respectively, they depend on the bearing height, h_b, and the spring deformations:

$$u_b = (h_b - v)\sin(\theta) + s\cos(\theta) \quad (2\text{–a})$$

$$u_{bz} = h_b(1-\cos(\theta)) + s\sin(\theta) + v\cos(\theta) \quad (2\text{–b})$$

Assuming linear material behavior, the nominal shear stiffness of the bearing (shear spring) is [5]:

$$k_{b0} = GA/t_r = GA_s/h_b \quad (3)$$

Similarly the nominal vertical stiffness of the bearing (vertical spring) is [5]:

$$k_{bz0} = E_c A/t_r = E_c A_s/h_b \quad (4)$$

where G = shear modulus; A = cross-sectional area; t_r = total thickness of the rubber layers; $A_s = A(h_b/t_r)$ = modified area, and E_c = instantaneous compression modulus of the rubber-steel composite bearing. The rotational stiffness $k_\theta = P_E h_b$, where $P_E = (\pi^2/h_b^2)EI_s$ is the Euler buckling load [5]. Here $EI_s = (E_c I h_b)/3t_r$ is the bending stiffness of a multilayer bearing [5], and I = conventional moment of inertia.

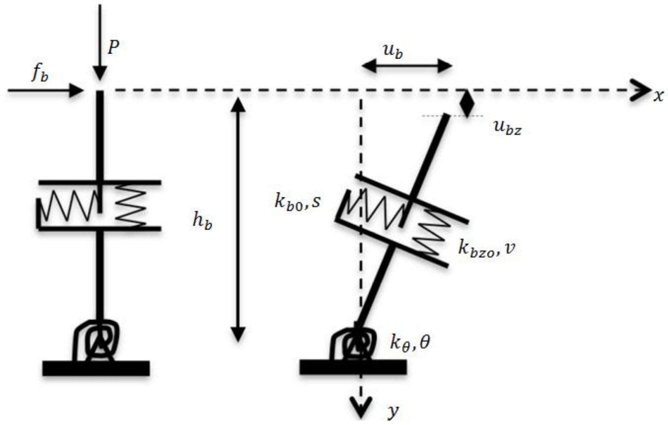

Figure 1. Proposed Macro-element model of Lead-Rubber Bearings

Imposing the stationary of Π with respect to s,v and θ yield the following equilibrium equations:

$$\begin{aligned} g1 &= k_{b0}s - f_b\cos(\theta) - P\sin(\theta) = 0 \\ g2 &= k_{bz0}v + f_b\sin(\theta) - P\cos(\theta) = 0 \\ g3 &= k_\theta\theta - P\left[(h_b - v)\sin(\theta) + s\cos(\theta)\right] \\ &\quad - f_b\left[(h_b - v)\cos(\theta) - s\sin(\theta)\right] = 0 \end{aligned} \tag{5}$$

The system of governing equations includes additionally to the equilibrium equations the kinematic equations (Eq. 2), and it can be recast in a root-finding form:

$$g = \begin{Bmatrix} g1 \\ g2 \\ g3 \\ g4 \\ g5 \end{Bmatrix} = \begin{pmatrix} k_{b0}s - f_b\cos(\theta) - P\sin(\theta) \\ k_{bz0}v + f_b\sin(\theta) - P\cos(\theta) \\ k_\theta\theta - P\left[(h_b - v)\sin(\theta) + s\cos(\theta)\right] \\ -f_b\left[(h_b - v)\cos(\theta) - s\sin(\theta)\right] \\ u_b - \left[(h_b - v)\sin(\theta) + s\cos(\theta)\right] \\ u_{bz} - \left[h_b(1 - \cos(\theta)) + s\sin(\theta) + v\cos(\theta)\right] \end{pmatrix} \tag{6}$$

The above system represents a system of five nonlinear equations in five unknowns: f_b, P, s, θ, and v, it can be solved by Newton's method, i.e., find $x = \langle f_b, P, s, \theta, v\rangle^T$ to satisfy g(x) = 0. To consider a different constitutive model for the shear spring, the term $k_{b0}s$ in Eq. 6 has to be replaced by a general force $f_s(s)$. The force $f_s(s)$ mobilized in the shear spring may be expressed in the Bouc-Wen [6,7] fashion as a linear part and a hysteretic part:

$$f_s(s) = k_{b0}s + Qz \tag{7}$$

where Q = yield strength of the lead core; z = a hysteretic dimensionless controlling the nonlinear lateral behavior of the bearing. The latter is governed by the following differential equation with respect to time [6,7]:

$$\dot z = \frac{\dot x}{s_y}\left[A - |z|^n\left(\beta\,\mathrm{sgn}(\dot x z) + \gamma\right)\right] \tag{8}$$

In Eq. 8, A, n, β and γ are the parameters that control the shape of the hysteretic loop; s_y is the yield deformation of the shear spring, s_y can be estimated as 1 cm for LRB isolators [4]. The variable $\dot z$ can be discretized by the Backward-Euler scheme to cancel the time variable then the Newton scheme of the form $x^{m+1} = x^m - f(x^m)/f'(x^m)$ to solve a general nonlinear equation f(x)=0 may be used to solve for z^{i+1}. The reader is referred to [8] for the complete numerical solution procedure to obtain $f_s(s)$ and the algorithmically consistent tangent $\partial f_s(s)^{i+1}/\partial s^{i+1}$ needed in the Jacobian matrix, which in turn is needed to apply the Newton method to solve Eq. 6.

III. Verification with Experimental Results

To verify the accuracy of the proposed macro-element model in predicting the global LRB cyclic behavior; its predictions are compared with experimental results. Two types of LRB isolators are considered in the comparison, LRBnz and LRBcn; details about the properties of the devices can be found in [9] for LRBnz and [10] for LRBcn.

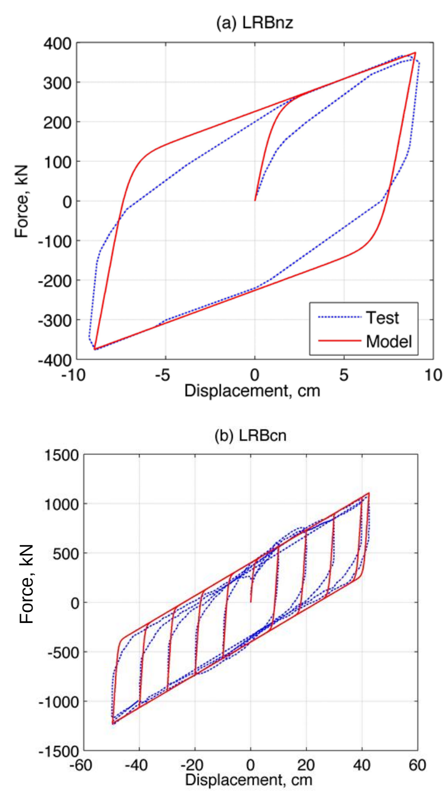

Figure 2. Comparison of experimental and numerical lateral force-displacement curves

The macro-element model is used to simulate LRBnz and LRBcn. In both numerical models the parameters of the Bouc-Wen [6,7] model of hysteresis were chosen as: $A = 1$, $n=2$, $\beta=0.9$, and $\gamma = 0.1$. Fig. 2 and 3 compare the experimental and numerical lateral force-displacement curves. The numerical model reproduces correctly the behavior of the LRBnz and the LRBcn, verifying the ability of the proposed macro-element model to simulate the behavior of LRB bearings.

A. *Axial Load Effects*

To investigate the axial load effects, the numerical model of LRBnz was subjected to different axial forces. Fig. 3(a) shows the lateral force-displacement curves for different axial loads. Both elastic stiffness and post-elastic stiffness decrease with increasing axial load. According to the linear two-spring model [5] the critical buckling load is approximated as $P_{cr} \approx \pm\sqrt{P_s P_E}$, where $P_s = GA_s$, applying this formula gives a critical load of 14832 kN. The critical load from the numerical model was estimated as 14750 kN, this proves both the accuracy of the macro-element model and the approximate formula provided by Kelly [5]. Fig. 3(b) shows the reduction in the elastic stiffness (k_{I0} represents the elastic stiffness at zero axial load) of the bearing and the equation that approximates that reduction as a function of the critical buckling load. To verify the accuracy of the proposed macro-element model in capturing the reduction of the post-elastic stiffness of the bearing (k_{b0}represents the post-elastic stiffness at zero axial load), Fig. 3(c) plots that reduction in stiffness and the equation proposed by Kelly [5].

The height reduction due to the horizontal displacement of the bearing, given by Eq. 2-b, as a function of axial load is shown in Fig. 3(d) for the LRBnz bearing. Comparison between the numerical and similar experimental and analytical results [11,12] indicates that useful estimates of the height reduction can be obtained.

IV. Conclusions

A nonlinear macro-element model for Lead-Rubber Bearings has been developed. The model includes axial load effects, large displacements and large rotations. The nonlinearity in shear stiffness of the bearing is included using the Bouc-Wen [6,7] model of hysteresis. The model has been shown to be capable to predict correctly the observed behavior in test results.

It was demonstrated also the accuracy of the proposed model to predict the instability of the bearing under axial loads and large displacements in a very satisfactory manner. The macro-element model can be incorporated easily in available open source software.

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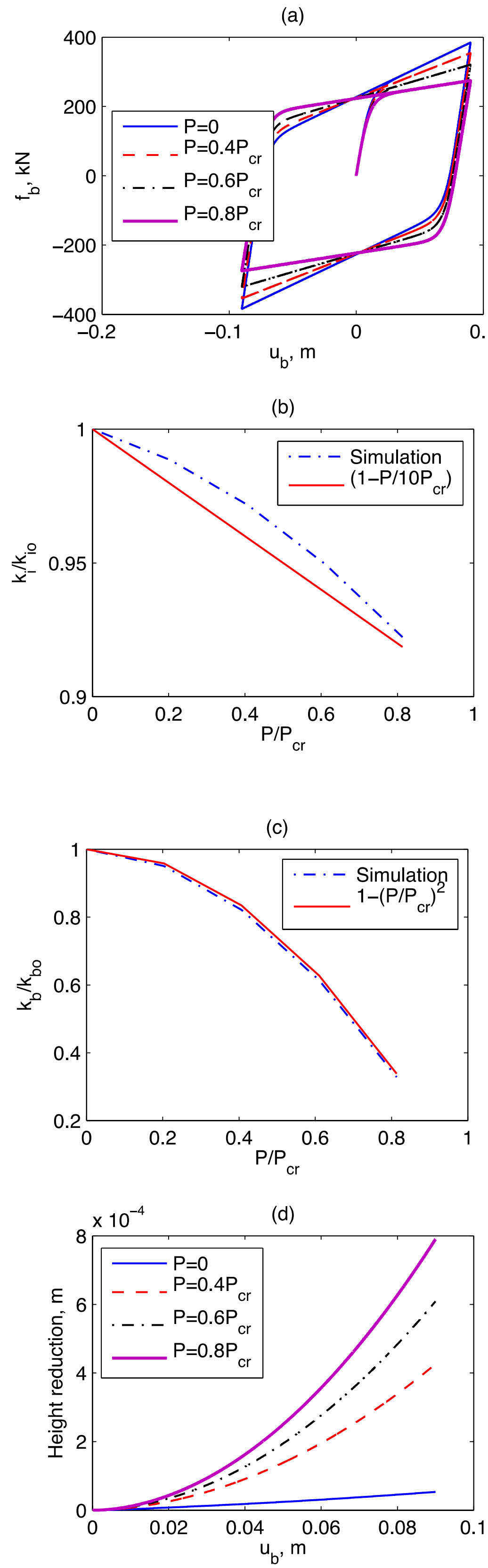

Figure 3. Axial load effects on a Lead Rubber Bearing

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