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A Numerical Study of Cyclic Behaviour of NiTi Shape Memory Alloy Belleville Washers

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***Abstract*—This study aims to explore the potential of superelastic/austenitic NiTi shape memory alloy (SMA) Belleville washers for the applications of seismic resisting devices. Employing a sophisticated superelastic-plastic constitutive model for NiTi SMA, a numerical study is performed to investigate the self-recovery and damping properties of NiTi SMA Belleville washers under the action of cyclic compression. A spectrum of geometric configuration is considered, where the effects of varying cone angles and thicknesses are discussed in detail. The effect of friction when two or more washers are stacked in parallel is also studied. Based on the parametric study results, a preliminary design recommendation on NiTi SMA Belleville washers is proposed. The current numerical study can facilitate further developments of viable re-centring and energy dissipating seismic resisting devices such as base isolations and dampers.**

***Keywords*—NiTi shape memory alloy (SMA), Belleville washer, cyclic loading, recentring, seismic.**

I. Introduction

Innovative engineering technologies are often driven by the discovery of new materials. NiTi Shape Memory Alloys (SMAs) have found their promising engineering applications because of their ability, in austenite form, to undergo reversible deformations (up to 8% strain) and to dissipate energy when subjected to cyclic loading [1]. Over the last decade, engineers and researchers have been attempting to design various superelastic SMA-based seismic resisting components, such as base isolators, dampers, bracings, column bases, and connections, to mitigate the structural vibration and damage caused by earthquakes [2].

Among the common forms of SMA products, wire is the most widely used one. Larger scale SMA bars have also been gaining attention, especially in the field of civil engineering. Other typical forms include plates, sheets, and helical springs. In this study, another possible form of SMA-based component, namely, SMA Belleville washers, is investigated. The Belleville washer, also known as conical spring washer, is an well established mechanical component. It can sustain a large load with small installation space. Because of the annular shape, force transmission is even and concentric. Importantly, possible combinations (e.g. in parallel and in series) can further extend their applications.

When the Belleville washers are endowed with superelastic ability, which enables self-recovery and hysteretic damping, their application may be further expanded. The first study of SMA Belleville washers can be traced back to the late 1990s, where CuAlNiMnTi based Belleville washers were considered in electrical applications for the purpose of stabilisation of circuit [3]. The potential of NiTi based Belleville washers for the application of civil engineering was first investigated by Speicher et al. [4]. The washers showed good recentring ability with moderate energy dissipation. However, it was found that the load carrying capacity decreased with increasing deformation beyond the peak load, and as a result the washers tended to invert, which compromised their damping performance. Therefore, it was recommended in Speicher et al. [4] that different geometric shapes of the washers should be studied in order to identify the suitable ones where the load decreasing effect can be minimised. More recently, Maletta et al. [5] introduced a new thermomechanical process for producing SMA Belleville washers through disk cutting and a successive shape setting by heat treatment. It is worth noting that the thermomechanical properties of the alloy were found to change evidently after the heat treatment.

While the above-mentioned studies shed light on the potential of SMA-based Belleville washers for engineering applications, detailed parametric studies on such washers are generally lacking. More work is required to investigate the influences of various geometric properties and stacking combinations. An optimised design of SMA Belleville washers is also desirable. Towards this end, a parametric study is performed in this study to investigate the hysteretic response of austenitic SMA Belleville washers with varied cone angles and thicknesses. The general nonlinear finite element (FE) analysis package ABAQUS [6], which is capable of capturing both material and geometric nonlinearities, is employed. The built-in superelastic-plastic constitutive model is used to simulate the superelastic behaviour of NiTi SMA. The load-displacement, recentring, and energy dissipation characteristics of the washers are examined. The effect of friction when two or more washers are stacked in parallel is also studied. A preliminary design recommendation on geometric properties is finally proposed, which could contribute to further development of seismic resisting devices using SMA Belleville washers.

II. FE Models

A. *Geometric Configurations*

For a Belleville washer, the key geometric parameters include thickness *t*, cone angle θ, net cone height *H*, as well as the ratio of D_e/D_i and *H*/*t* as shown in Fig. 1. In this study, in

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order to keep the number of parameters in a manageable level, the net cone height H is kept constant, i.e. 5mm, and the other parameters are varied through changing t, D_e, and D_i. It is worth mentioning that some parameters are not independent, e.g. the D_e/D_i ratio and the cone angle (which are mutually dependent). The FE models are divided into 5 series, as shown in Fig. 1, and the same geometric property (except the thickness t) is considered for each series. A wide spectrum of the geometric parameters is covered, and the details of the geometric properties are given in Table 1. For the designation, S represents Series, and T represents thickness.

TABLE I. GEOMETRIC PROPERTIES OF SMA WASHERS

Series	Designation	t	D_e	D_i	θ (°)	D_e/D_i	H/t
1	S1-T2	2	40	22	29°	1.82	2.5
	S1-T3	3	40	22	29°	1.82	1.67
	S1-T4	4	40	22	29°	1.82	1.25
	S1-T5	5	40	22	29°	1.82	1.00
2	S2-T2	2	50	22	19.6°	2.27	2.5
	S2-T3	3	50	22	19.6°	2.27	1.67
	S2-T4	4	50	22	19.6°	2.27	1.25
	S2-T5	5	50	22	19.6°	2.27	1.00
3	S3-T2	2	50	28	24.5°	1.79	2.5
	S3-T3	3	50	28	24.5°	1.79	1.67
	S3-T4	4	50	28	24.5°	1.79	1.25
	S3-T5	5	50	28	24.5°	1.79	1.00
4	S4-T2	2	60	22	14.7°	2.73	2.5
	S4-T3	3	60	22	14.7°	2.73	1.67
	S4-T4	4	60	22	14.7°	2.73	1.25
	S4-T5	5	60	22	14.7°	2.73	1.00
5	S5-T2	2	70	22	11.8°	3.18	2.5
	S5-T3	3	70	22	11.8°	3.18	1.67
	S5-T4	4	70	22	11.8°	3.18	1.25
	S5-T5	5	70	22	11.8°	3.18	1.00

The units for t, D_e, and D_i are mm

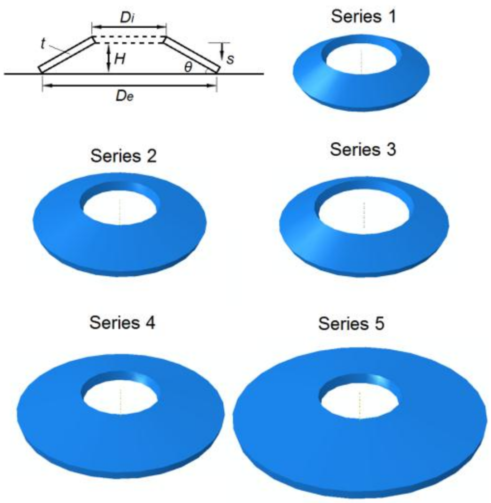


Figure 1. Geometric configurations of SMA washers.

B. *Material Modelling*

In ABAQUS [6], a built-in superelastic-plastic user-defined material model using the Auricchio's approach [7] is employed to simulate the NiTi SMA under isothermal conditions. Auricchio's model considers the Drucker-Prager type loading function, which is a robust solution applicable to general Finite Element (FE) tools such as ABAQUS. Key isothermal parameters for NiTi SMA in ABAQUS include transformation stresses (σ^{Ms}, σ^{Mf}, σ^{As}, and σ^{Af}), Young's Moduli (E^A and E^M), maximum transformation strain ε_L, and Poisson's Ratios (v^A and v^M =0.33). The superelastic-plastic material model allows plastic deformations to be developed when the stress level achieves σ^p beyond σ^{Mf}. In this study, an idealised stress-strain relationship for NiTi SMA is considered [1], where the key material parameters are marked in Fig. 2. It is assumed that when the stress exceeds 1000 MPa, permanent deformation can be induced. The same material model is applied to all the FE models in order to study the influences of the considered geometric parameters.

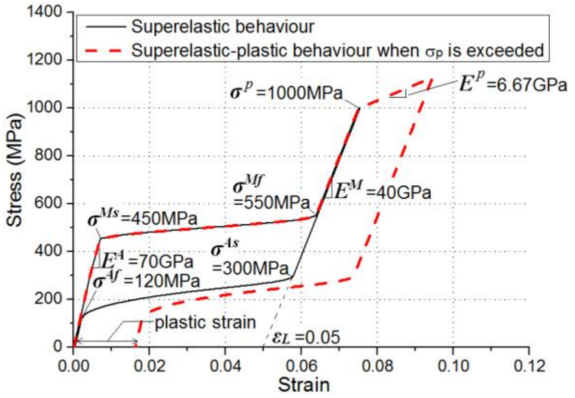


Figure 2. Stress-strain relationship of NiTi SMA.

C. *Finite Elements and Loading*

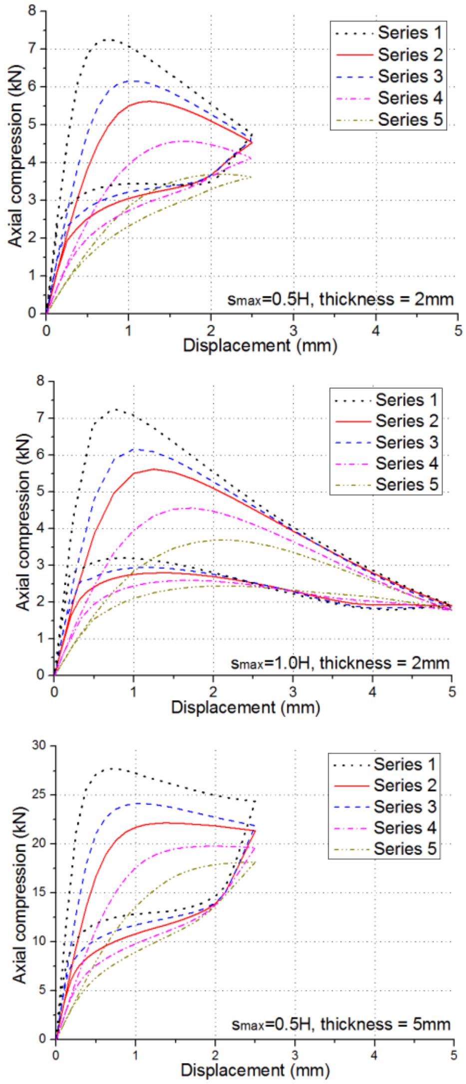
The SMA Belleville washers are modelled using normal 8-node linear brick elements C3D8 in ABAQUS. The mesh size is taken as 1 mm × 1 mm. A reference point, where its vertical displacement is 'coupled' with that of the upper circular edge of the washer, is established as the load applying point. The lower edge of the washer is restrained vertically but is free to move outward under compression. Each washer is subjected to two cycles of compressive load with the peak displacements s_{max} of 0.5H and 1.0H successively. In this study, H is consistently taken as 5 mm, so when s = 5 mm (1.0H), the washer becomes flattened.

III. Results of FE Study

A. *Load-displacement Response*

The typical load-displacement responses of the Belleville washers under the displacements of s_{max} = 0.5H and s_{max} = 1.0H are shown in Fig. 3. Considering the limit of the length of this manuscript, only the load-displacement curves of the 2 mm and 5 mm thickness washers are shown. For the washers of 2 mm thickness, 'duck head' shaped force-displacement curves are typically shown, especially for series 1, 2, and 3 where the cone angles θ are relatively large. The washers have stable initial stiffness, but the load starts to decrease beyond the peak value. A similar load-displacement shape was found in the tests performed by [4]. The maximum axial compression (peak load) varies between 3.5 kN and 7.4 kN, depending on the varying geometric configurations. It seems that a greater cone angle θ leads to higher initial stiffness and peak load. Upon unloading, the washers can return to its original shape without permanent deformation. It is also of interest to find that when the displacement s achieves $1.0H$, the axial loads of the five washers tend to meet at the same point, i.e. around 2 kN.

When the thickness of the washer increases to 5 mm, the drop off of the post-peak load becomes less significant, and thus more recognisable flag shaped load-displacement curves are featured. The peak loads are evidently increased when compared with those of the 2 mm thick washers. The peak load can reach up to 28 kN, and again, the initial stiffness and peak load are influenced by the cone angle. Finally, it is found that for washer series 1 (with 5 mm thickness), where the largest cone angle is considered, a residual displacement of around 0.2 mm is induced after the second cycle ($s_{max} = 1.0H$). This is due to the plastic deformation developed in the washer.



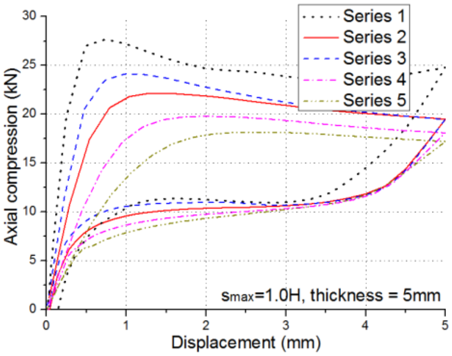


Figure 3. Load-displacement responses of selected SMA washers.

B. ***Stress Distribution***

The von Mises stress distributions of the washers S1-T2 and S5-T2 are shown in Fig. 4. The two selected models represent the two extreme cases of the considered cone angles in this study, i.e. 29° and 11.8°. The models are cut vertically in order to show the stress distributions more clearly. When the displacement reaches H, the high stress zone is mainly concentrated near the internal ring edge. In some cases another high stress zone is observed at a certain distance away from the inner ring edge, and this is probably due to a minor the bending effect of the washer when flattened, especially for the case of 'slender' cone, i.e. S5-T2. In general, the stress level in most of the area is less than the plastic stress $\sigma^{p} = 1000$MPa.

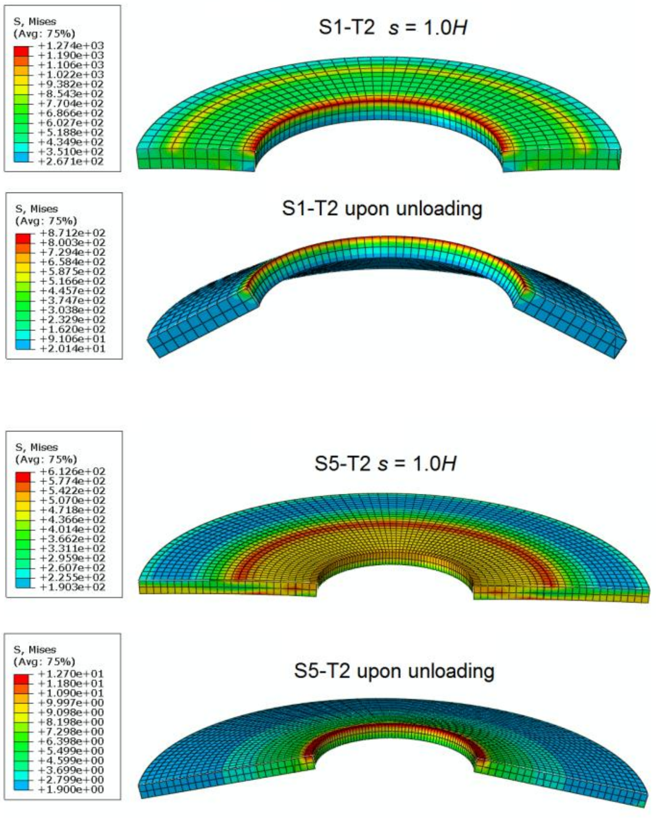


Figure 4. Typical Mises stress distributions.

Through the stress comparison between models S1-T2 and S5-T2, it is clearly found that a larger cone angle leads to a much higher stress level. When the displacement returns back to zero, most of the area shows no residual stress, except at the inner ring. The maximum residual stress of model S1-T2 is much higher than that of model S5-T2. The residual stress of the former achieves 871 MPa, while that of the latter is only 12.7 MPa. In both cases, the residual stress zone is concentrated within a very limited area. Finally, the stress level is also dependent on the thickness (which is not shown in Fig. 4): a thicker washer experiences a higher stress level. The high stress zone in thick washers can lead to permanent plastic deformations, e.g. model S1-T5 as previously shown in Fig. 3.

IV. Further Discussions

Normally the key hysteretic characteristics of a SMA-based seismic resisting device include stiffness, strength, energy dissipation, and recentring ability. In order to examine the feasibility of SMA Belleville washers in seismic engineering applications, this section further discusses the influence of the geometric configuration on the hysteretic responses of the washers.

A. Influence of Cone Angle θ

The cone angle considered in this study ranges from 11.8° to 29°. As previously discussed, the initial stiffness and peak load of the washers are directly related to the cone angle: increasing cone angle leads to increasing initial stiffness and peak load. However, the decrease of the load carrying capacity after reaching the peak load could be a critical issue that affects the reliability of the washers used as dampers. One concern is that the decreasing load path can lead to the ‘inverting’ effect of the washers. The inverting effect, which is a snap-through typed mechanism, can significantly deteriorate the energy dissipation and recentring abilities [4]. The significance of load decrease can be indicated by the F_s/F_{max} ratio, where F_s is the load carrying capacity at the displacement s ($s = 0.5H$ or $s = 1.0H$), and F_{max} is the peak load. $F_s/F_{max} = 1.0$ indicates no load decrease.

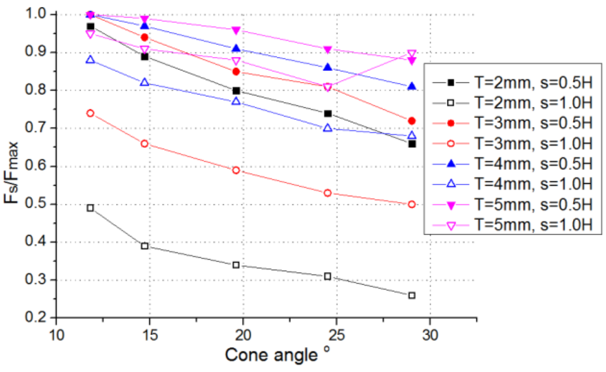


Figure 5. Influence of cone angle on F_s/F_{max} ratio.

Fig. 5 shows the F_s/F_{max} ratios of the considered washers with varying cone angles. It is clearly shown that the ratio generally decreases with the increasing cone angle. When the maximum displacement reaches 1.0H, the lowest ratio observed in this study is 0.25 for model S1-T2 ($\theta = 29°$). The results imply that high initial stiffness/peak load and high F_s/F_{max} ratio (low post peak load decrease) cannot be pursued simultaneously via only adjusting the cone angle. In other words, a balance may need to be considered when both stiffness/strength and F_s/F_{max} ratio are desirable in the design of SMA Belleville washers.

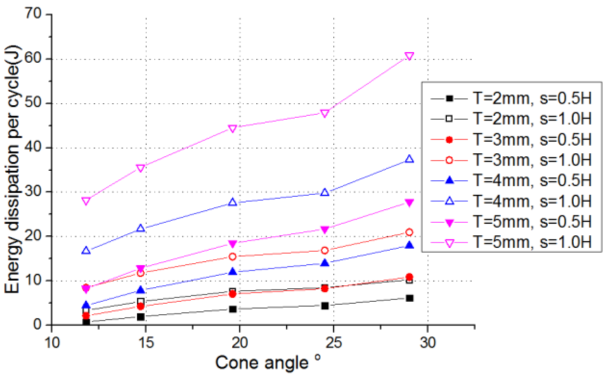


Figure 6. Influence of cone angle on energy dissipation.

Energy dissipation is another key factor considered for any potential seismic resisting device. The energy dissipation per cycle equals the enveloped area of the hysteretic curve. Fig. 6 shows the energy dissipation per cycle with varying cone angles. It is found that a larger cone angle leads to higher dissipated energy. This trend seems to be consistent with that of the peak load and initial stiffness. As indicated in Fig. 3, the higher dissipated energy is directly due to the increased load carrying capacity. As the unloading load-displacement path is not greatly affected, the total loop area (equals the dissipated energy) is thus increased.

B. Influence of H/t Ratio

The H/t ratio is an important parameter influencing the behaviour of normal Belleville washers [8]. Since the net height H is kept constant in this study, the H/t ratio is varied through changing the thickness. Fig. 7 shows that the F_s/F_{max} ratio decreases with increasing H/t ratio. This indicates that when H is fixed, decreasing the thickness t can lead to a more significant load decrease beyond the peak load. In other words, increasing the washer thickness is beneficial to the stability of the loading path. In this respect, increasing the washer thickness is desirable to avoid inverting (snap through). On the other hand, increasing the thickness also increases the stress level. This can cause more residual/permanent deformation, and thus compromise the recentring ability. It is worth mentioning that it is assumed in this study that the permanent deformation is only induced by the plastic deformation when the stress exceeds σ^p. In reality, however, as the number of load cycle increases, permanent deformation can still be gradually induced at the stages prior to σ^p. This type of permanent deformation cannot be simulated by the existing constitutive model; therefore, further physical tests are required to investigate the recentring ability of the SMA Belleville washers.

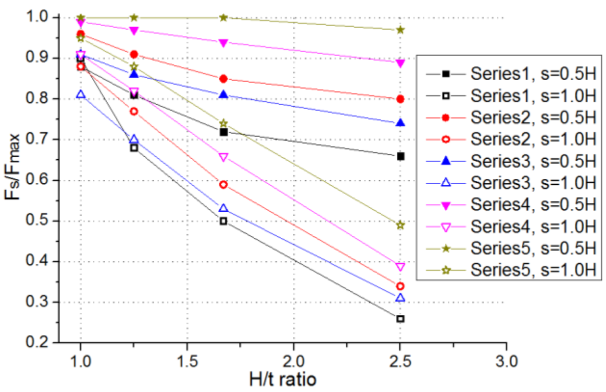


Figure 7. Influence of H/t ratio on F_s/F_{max} ratio.

C. *Influence of Friction When Stacked*

The load carrying capacity of Belleville washers can be increased when the washers are stacked in parallel. When the stacked washers are under compression, the effect of friction may influence the overall hysteretic curve. Fig. 8 shows the hysteretic curves of triple-stacked SMA Belleville washers (model S4-T3) under the maximum displacement of 1.0H. 'Hard contact' behaviour with no penetration in the normal direction is assumed for all contact faces between the washers. A Coulomb friction model is used with two values of coefficient of friction, namely, 0.0 and 0.5. It is shown that the friction can evidently make the hysteretic loop plumper, thus dissipating more energy. The recentring ability is not influenced by friction. This preliminary study indicates that a friction-superelastic hybrid damper system based on SMA Belleville washers may be promising. Further studies are required on this front.

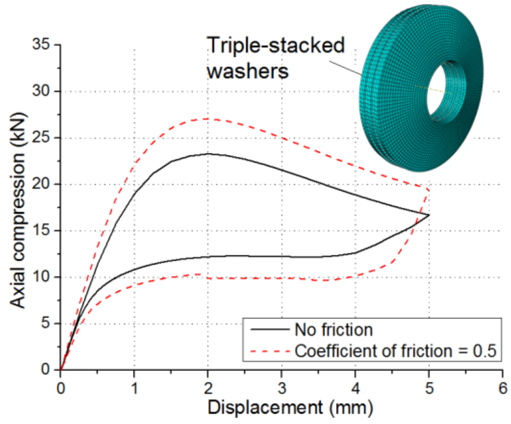


Figure 8. Influence of friction on stacked washers.

V. Conclusions

A numerical study is conducted to examine the hysteretic response of NiTi SMA Belleville washers under cyclic loading. It is found that the recentring and energy dissipation abilities of the washers are affected by the geometric configuration. A general trend is that the peak load carrying capacity and energy dissipation are increased when the cone angle and thickness (as indicated by the H/t ratio in this study) are increased. Concurrently, however, more significant decrease of the load carrying capacity after reaching the peak load is observed, which is undesirable if the washers are to act as the kernel components for dampers. From the current preliminary study, it is recommended that when a full utilisation of the distance H is expected, the upper limits for the cone angle and H/t ratio should be 15° and 1.3, respectively, such that the F_s/F_{max} ratio can be kept above 0.8. When a half utilisation of the maximum displacement (i.e. $s_{max} = 0.5H$) is expected, the limits may be relaxed to 20° and 2.5, respectively. Furthermore, the friction effect for two or more washers stacked in parallel is beneficial for improving the energy dissipation ability. It is anticipated that if appropriately designed, SMA Belleville washers may be important viable additions to the 'arsenal' against seismic hazard, especially when the benefits of various stack pattern combinations can be extensively explored.

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