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Detection of Nonlinearity in Structures Using Principal Component Analysis

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Abstract—**Presence of nonlinearity in the structure can affect the global dynamic behavior. Hence detection of nonlinearity has a greater significance in the context of structural health monitoring. In this paper we present a technique based on principal component analysis for detecting the presence of nonlinearity in the structure. The angle between response subspaces is taken as a feature to detect nonlinearity. The major advantages of the proposed technique is that it uses only acceleration time history data and can be used with ambient vibration data, which is ideally suited for civil structures. Numerical simulation studies have been carried out using a cantilever beam with nonlinear cubic stiffness attachment. Studies presented in this paper clearly indicate that the proposed technique is robust.**

Keywords—**principal components, subspace angle**

I. Introduction

Systems are often referred to as being linear or nonlinear. However, all real structures are inherently nonlinear. Large elastic deformations introduce geometric nonlinearity, deformation-dependent material properties result in material nonlinearity, and other nonlinearities can be caused by backlash, clearances between mounting brackets, geometric constraints on deformation, misalignment of substructures, dry friction, and many types of nonlinear hysteretic damping, and material damping of shape memory alloys and other materials. This structural dynamic behavior must generally be taken into account in the design of systems in order to insure their performance and reliability.

Nonlinear system identification is a very challenging inverse engineering problem. It can be viewed as succession of three steps: detection, characterization and parameter estimation. This paper focuses on the detection step that enables to know whether or not the structure has a significant level of non-linear behavior and whether or not it can be safely neglected.

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Several methods have been evolved for detecting nonlinearity and can be found in literature. The Frequency domain data methods such as the homogeneity test examines distortions in the Frequency Response Functions (FRF) for several levels of excitation (FRF is invariant for a linear system); the Hilbert transform that differs from the original FRF [1, 2]. The stepped-sine excitation gives the best result or well-defined FRF where the distortion clearly appears in the case of a nonlinear system. Fewer techniques exist that use only time data. For example, the Hilbert transform is applied to signals in the time domain in order to extract the time varying instantaneous phase and frequency [3] by which nonlinearity is detected. The Continuous wavelet transform uses the free responses of a nonlinear system to detect nonlinearities by looking at distortions in the amplitude and phase of the wavelet [4].

In this paper, we present a technique to detect nonlinearity using Principal Component Analysis. This approach uses random excited time domain response data so that no signal processing transformation of the measurement is needed and all the information is conserved. The concept of subspace angles .between two response subspaces is taken as a feature for detecting the presence of nonlinearity in the structure.

II. Principal Component Analysis

Principal component analysis (PCA) [5] is a mathematical procedure that uses an orthogonal transformation to convert a set of observations of possibly correlated variables into a set of values of linearly uncorrelated variables called principal components. The number of principal components is less than or equal to the number of original variables. PCA is closely related to Singular Value Decomposition (SVD) and Proper Orthogonal Decomposition, also known as Karhunen-Loeve decomposition. One specific application of PCA in the field of structural dynamics is to find the subspaces spanned by the principal directions that contain most of the system's energy without calculating the modes shapes.

It is always more efficient to identify directly the principal components, also called principal directions, rather than performing an exact modal identification to compute the trajectories covered by the measurements. However, under certain assumptions, principal components may represent the vibration modes of the system.

In the present work, it is assumed that the number of sensors 'n' is greater than the number of structural modes (m+1) involved in order to maintain the redundancy of the data. Let Q denote a discrete block time-history of n x b (where b >> n) sampled responses

$$Q = \begin{bmatrix} x_1(t_{j+1}) & .... & x_1(t_{j+b}) \\ . & .... & . \\ x_n(t_{j+1}) & .... & x_n(t_{j+b}) \end{bmatrix} \quad (1)$$

The singular value decomposition (SVD) of the block data Q gives:

$$Q = USV^T \quad (2)$$

where U is an orthonormal matrix (n X n) whose columns define the principal components (PCs) and form a subspace spanning the data. Each column of U is associated with the (b X b) time coefficient matrix V. The singular values, given by the (n X b) diagonal matrix S and sorted in descending order, can be related to the energy associated with the corresponding principal components of U. This means that the structure will react mainly in the directions of the principal components associated with the highest energies. We may note here that it is computationally more efficient to calculate the SVD of:

$$QQ^T = US^2V^T \quad (3)$$

Theoretically, only the first m + 1 eigen values of Q are nonzero. Nevertheless, we know that test data contains measurement noise. Since noise has much lower energy than the structural modes, the components of U associated with eigen values presenting an order of magnitude much lower than others have to be discarded from the principal component base. In the linear case, the principal directions extracted from test data, always lie in the subspace (or hyper-plane) generated by the participating modes

Mathematically speaking, this means that the so-called principal hyper-plane is invariant, even if the directions of the principal vectors are dependent on the structural excitation. Nevertheless, the principal hyper-plane is dependent on the structural characteristics. The PCA may be then considered as a powerful and straightforward approach to compute a modal metrics of test data and to detect nonlinearity by comparing reference and current structural subsets.

A. *Angle between Subspaces*

Given a set of data, the active principal components–PCs define a subspace (or hyper-plane) that characterizes the dynamic behavior of the system. A change in the system modifies consequently its dynamic state and affects the subspace spanned by the PCs. This change may be estimated using the concept of angles between two subspaces introduced by Golub and Van Loan [7]. This concept allows quantifying the spatial coherence between two time-history blocks of an oscillating system. Let $A \in \mathbb{R}^{n_s x p}$ and $B \in \mathbb{R}^{n_s x p}$ be two subsets, each with linearly independent columns. First a QR factorization allows computing the orthonormal bases of A and B:

$$\begin{aligned} A &= Q_A R_A \qquad Q_A \in \mathbb{R}^{n_s x p} \\ B &= Q_B R_B \qquad Q_B \in \mathbb{R}^{n_s x q} \end{aligned} \quad (4)$$

Thus, the singular values of $Q_A^T Q_B$ define the q cosines of the principal angles θ_i between A and B.

$$SVD(Q_A^T Q_B) \rightarrow \ Diag(\cos(\theta_i)) \ i=1, ...q \quad (5)$$

The largest angle allows quantifying how the subspaces A and B are globally different.

B. *Limit of Linearity*

The nonlinearity in the structure can be detected using these principal angles. The responses of the subspaces between reference data, i.e., pristine linear system and the current data i.e., nonlinear system, are compared by computing the principal angles. If the principal angle between these two subspaces is appreciably high, it can be concluded that the system differs due to nonlinearity. Theoretically, for a linear system in undamaged condition, the angle between the subspaces spanned by reference data and the current data should be zero. However, practically, it will not be zero due to environmental variances and also the noises present in the measurement process. In order to avoid the false positive detection, a large number of reference data sets are collected by taking measurements at different time instants and partitioned into several sets. The principal angle between the subspace spanned by each of these sets and the subspace spanned by the whole data set is computed, which gives us a collection of different subspace angle values. When dealing with the current data set, an alarm is issued when the monitored angle exceeds the upper control limit (UCL) defined as the mean angle plus three times its standard deviation. This corresponds to a 99.7% confidence interval for a normal distribution.

The subspace angle spanned by the system is computed and that if it is above the proposed limit of linearity, the system is non-linear. The main interest of this limit of linearity is that it avoids any kind of personal and subjective interpretation of the results or the conclusions on the presence or not of non-linearity in the structure.

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III. Application to a Numerical Example

The cantilever steel beam 1m*0.014m*0.014m, shown in Figure.1, with 11 node and 22 degree-of-freedom (translations along the y-axis and rotations around the z-axis) in the finite element model [5]. A non-linear cubic spring K_{NL} (K_{NL}=1e7Nm^{-3}) is added between the free end and the ground. The non-linear force, F_{KNL}, due to the nonlinear stiffness can be expressed as functions of the translational displacement x_{11} of node 11 along y direction:

$$F_{KNL} = K_{NL} x_{11}^{3} \tag{6}$$

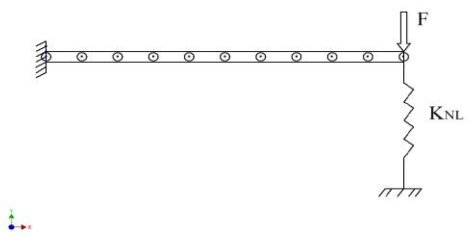

Figure1. *Cantilever Beam*

The beam is excited on node 11 with a 1.2 sec., constant amplitude random excitation. In order to compare various levels of non-linearity, different input amplitudes are considered. The first input has a very low amplitude level 0.35NRMS, which guarantees a linear behavior. These responses are designated as the reference data. The various other RMS amplitude levels 2, 4, 12, 40NRMS are also considered, where non-linear behaviors are expected.

The acceleration time history response is computed using a Newmark time integration technique combining with a Newton Raphson algorithm. The measurement data corresponding only to the translational degree of freedom of nodes 2–11 along the y axis are considered.

The data subsets are generated by partitioning the entire acceleration time history response into windows of equal or varied length. The acceleration time history data of the cantilever beam shown in Figure 1, is partitioned into 6 subsets of 750 time steps each, up to 4500th time instant, where the system is behaving linearly and the time history response after 4500th time instant is partitioned into 3 subsets of each 500 time steps.

In order to identify the presence of nonlinearity, the concept of subspace angles of different data subsets as presented earlier is employed. For this purpose six data subsets for the linear system are generated varying magnitude of random loading and with varied noise levels. Even though the angles are expected to be closer to zero, when the system is linear some marginal values of subspace angles will be obtained due to noises present in the response. Based on the subspace angles obtained the average value and the standard deviations are computed to arrive at the control limit.

UCL_1= 0.0911+3(0.0203) = 0.152 considering one principal component

UCL_2=0.0080+3(0.0048) = 0.0223 considering two principal components

UCL_3=0.0991+3(0.0150) = 0.1441 considering three principal components

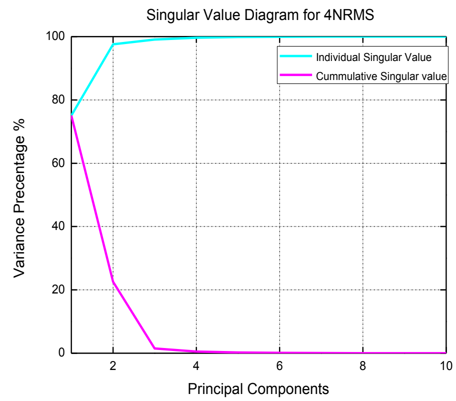

Figure2. *Singular Value Diagram*

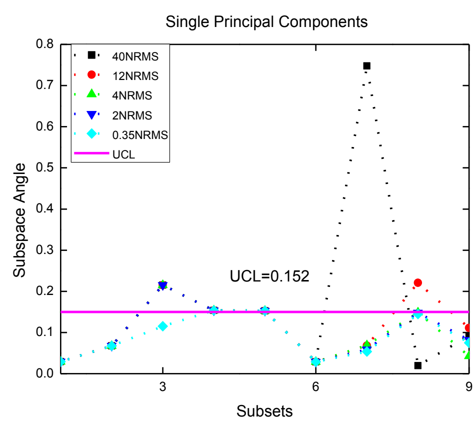

Figure3. *Detection of nonlinearity with one principal component*

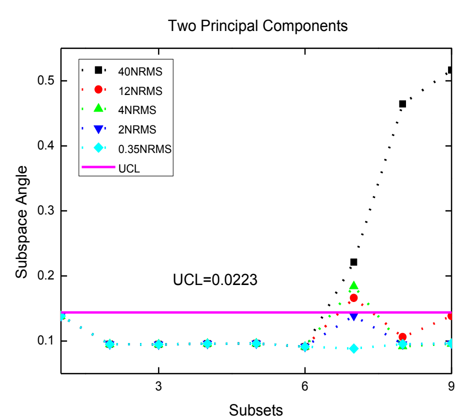

Figure4. *Detection of nonlinearity using two principal components*

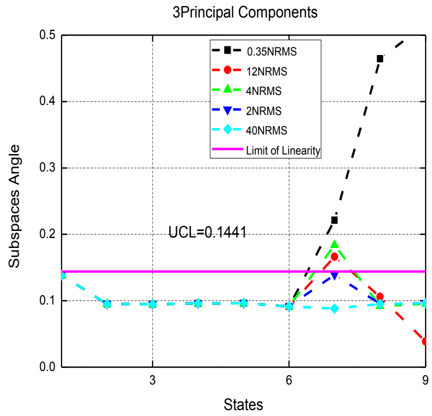

Figure5. *Detection of nonlinearity using three principal components*

The subsapce angles computed using the subsapces generated from the accleration time history response of the canliver beam, consiting of both linear and nolinear responses are presented in Figures 3 to 5 for varied number of principal components. In Figure 2 ,the energy levels and cumulative energy levels present in the singualr values associated with principal components is presented. This will help in chhosing approate number of principapl components. It is clear from figure 2, that it will be sufficient to take maximum of three active principal components whose energy works out to be 99.5% for detection of nonlinearity. The subspace angles computed with only one principal component and shown in figure 3, one can easily observe that nonlinearity can be detected (subset 7) exactly only when the excitation levels are high (12 and 40NRMS). However with two or three principal components given in figure.4 and figure.5 respectively, the nonlinearity detection is quite robust even for low levels of excitation (4NRMS).

IV. Conclusion

In this paper a technique based on principal component analysis to detect the presence of nonlinearity using ambient vibration data is presented. The angle between the two response subspaces is considered as a feature to detect nonlinearity. Numerical simulation studies have been carried out by considering a cantilever beam with nonlinear cubic stiffness attached element. The studies presented in this paper clearly indicate that the proposed technique is robust. The major advantage of the technique is that it directly uses acceleration time history data and does not require any signal processing or transformations. Further the proposed technique is well suited for civil engineering applications as it uses ambient vibration data.

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References

[1] G.Kerschen, K. Worden, A.F.Vakakis, J.C.Golinval, “Past,present and future of non-linear system identification in structural dynamics”, Mechanical Systems and Signal Processing, Vol.20, PP.505-592, 2006

[2] K.Worden, G.R.Tomlinson, “Nonlinearity in Structural Dynamics, Detection, Identification and Modeling”, Institute of Physics Publishing, 2001

[3] Micheal Feldman, “Hilbert Transform Applications in Mechanical Vibration”, John Wiley & Sons, Ltd. Publishers, First edition, 2011

[4] M.N.Ta, J.Lardies, “Identification of weak nonlinearities on damping and stiffness by the continous wavelet transform, Journal of Sound and vibration”, Vol.293, PP.16-37, 2006

[5] A.Hot, G.Kerschen, “Detection and quantification of non- linear structural behavior using principal component analysis”, Mechanical Systems and Signal Processing, Vol.26,PP.104-116, 2012

[6] P.D.Boe, J.C.Golinval, “Principal component analysis of a piezosensor array for damage localization”, Structural Health Monitoring, Vol.2, PP.137-144, 2003

[7] G.H.Golub, C.F.Van Loan, “Matrix computations”, third revised ed., Johns Hopkins University press, 1996