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Spectral Finite Element method for Wave Propagation problem

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***Abstract*— Wave propagation based automatic structural health monitoring techniques and the warning-alert systems are widely adopted in aerospace and nuclear industry. These techniques are however, not in a matured state of development for civil structures and are evolving. Hence research in this area is of paramount importance. This paper deals with axial wave propagation in concrete beam with structural dis-continuities. Dis-continuity (impedance mismatch) is induced in the beam by varying its cross section area along its length. Numerical simulations are performed using the spectral finite element method (SFEM) and compared with experimental studies conducted on four different concrete beams using instrumentation for Pulse echo configuration and Pitch catch configuration methods.**

***Keywords*— Wave propagation, Spectral finite element method, damage detection**

I. Introduction

In the field of the structural engineering, wave propagation based tools have found increasing application in the area of Structural Health Monitoring. Wave propagation is a transient phase of dynamic response before the on-going wave and the in-coming wave superimpose over each other giving rise to a standing wave pattern. This dynamic phenomenon happens for structures of finite dimensions and the initial transient phase of response is well handled by wave propagation and the steady state response that follows this initial response is handled by classical structural dynamics. For structures of long aspect ratios, in-coming wave shall be totally damped out with only on-going wave. The way in which such structures can be analyzed is using wave propagation mechanics. For wave propagation problems wherein the frequency content of the input is very high, many higher order vibrational modes participate in the motion.At these higher frequencies, the wavelengths are very small and hence to capture these modes effectively, FE meshes need to be very fine.

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This is due to the requirement that the element sizes should be of the same order as the wavelength of the signal. A conventional modal method, when extended to the high frequency regime, becomes computationally inefficient since many higher modes that participate in motion will not be represented. Among many frequency domain methods, the Spectral Finite Element (SFEM) method developed by Doyle [1], which has been found suitable for analysis of wave's propagation in real engineering structures as the formulation is based on dynamic equilibrium under harmonic steady state excitation. The pioneering work and the connected papers, which inspire the present work, include that of Gopalakrishnan [2], Palacz and Krawczuk, [3], Ostachowicz et al. [4], Ostachowicz [5], and Ruka [6] .The other papers in this field are described subsequently and by no-means the list is exhaustive.

In this paper, axial wave propagation in concrete beam (considered as bar element) with a discontinuity of varying cross-section along its length is analyzed. The elastic wave motion (longitudinal waves) is investigated in detail. Numerical simulations were performed using the spectral finite element method and compared with experimental studies conducted on four different concrete beams of varying cross section using instrumentation for Pulse echo configuration and Pitch catch configuration methods.

II. Spectral Finite Element Method

An elementary wave theory for a thin rod or bar element assumes the presence of one-dimensional uniform axial stress only, and neglects the lateral contraction. F(x, t) is the axial force in the bar and u(x,t) is the axial displacement of the bar. Writing the sum of forces in the x-direction yields

$$-F(x,t)+F(x,t)+\frac{\partial F(x,t)}{\partial x}-\mu A\frac{\partial^2 u(x,t)}{\partial t^2}=0 \qquad (1)$$

The governing PDE for the bar is

$$\frac{E}{\mu}\frac{\partial^2 u(x,t)}{\partial x^2}=\frac{\partial^2 u(x,t)}{\partial t^2} \qquad (2)$$

For representation of displacement field,

$$u(x,t)=X(x)T(t) \qquad (3)$$

Assume $T(t) = \bar{D}e^{i\omega t}$ and $X(x) = \bar{A}e^{ikx}$

On simplifying gives a characteristic relation between the temporal frequency and the spatial wave number

$$-\frac{E}{\mu}(k^2) + \omega^2 = 0 \quad (4)$$

By solving Wave Number (k),

$$k_1 = \sqrt{\frac{\mu\omega^2}{E}} \; k_2 = -\sqrt{\frac{\mu\omega^2}{E}} \quad (5)$$

A total solution for the displacement field,

$$u(x,t) = (\bar{A}e^{ik_1 x} + \bar{B}e^{ik_2 x})e^{i\omega t} \quad (6)$$

The dynamic stiffness matrix for harmonic forcing can now be formulated based on the representation of the displacement field. The spectral bar element uses the axial displacement at two nodes for DOF. The dynamic stiffness matrix, [K_{Dyn}], by definition relates the nodal displacements,{d}, to the nodal loads,{f}

$$\{f\} = [K_{Dyn}]\{d\} \quad (7)$$

The dynamic stiffness matrix that relates the nodal displacements to the nodal forces is then

$$[K_{Dyn}] = [F][D]^{-1} \quad (8)$$

The explicit form of the dynamic stiffness for the spectral bar element is

$$[K_{Dyn}] = EA\begin{bmatrix} \frac{ik_1 e^{ik_2 L} - ik_2 e^{ik_1 L}}{e^{ik_2 L} - e^{ik_1 L}} & \frac{ik_2 - ik_1}{e^{ik_2 L} - e^{ik_1 L}} \\ \frac{(k_1 - k_2) i e^{ik_1 L} e^{ik_2 L}}{e^{ik_2 L} - e^{ik_1 L}} & \frac{ik_2 e^{ik_2 L} - ik_1 e^{ik_1 L}}{e^{ik_2 L} - e^{ik_1 L}} \end{bmatrix} \quad (9)$$

III. Experimental Investigation With Structural Discontinuities

A concrete beam with a cross section of 200mm×200mm and length of 6000 mm is considered as a testing specimen. The beam which is considered as an axial element (bar) has free–free boundary conditions. The measurements have been made on four different specimen with discontinuities of cross-section. The descriptions of specimens are as follows (Fig.1 & 6):

- Beam 1: The Depth of the specimen-1 is 0.1 m for the center 1 m length. (Reduction in c/s area)
- Beam 2: The Depth of the specimen-2 is 0.1 m for the center 1 m length from 1m from toe. (Reduction in c/s area)
- Beam3: The Depth of the specimen-3 is 0.3 m for the center 1 m length. (Increase in c/s area)
- Beam 4: The Depth of the specimen-4 is 0.3 m for the center 1 m length from 1m from toe. (Increase in c/s area)

A. Testing procedure and Results obtained from Pulse Echo Instrumentation

The testing requires the attachment of the highly sensitive accelerometer to the beam top with viscous material. Accelerometer is connected to Data Acquisition System with special purpose signal conditioning and A/D converter. Tests are repeated at the beam head as well as toe to assess the validity of the testing. The beam head is to be struck by small hand-held hammer. After hammer impact downward compressive wave is generated which travels with wave speed "c". When this initial wave encounters a cross section change or concrete quality change at length 'L', it generates an upward traveling wave which is observed at the beam top at a time equal to twice the distance of the cross section change from the top divided by the wave speed c (2L/c). The rest of the initial wave travels down to the beam toe and reflects. Consider, Wave speed, c = 3750 m/sec. The beam 1 and beam 3 are considered to have symmetric damage (@ 2.5 m from head) and beam 2 and beam 4 are considered to have un-symmetric damage (@ 4 m from head).

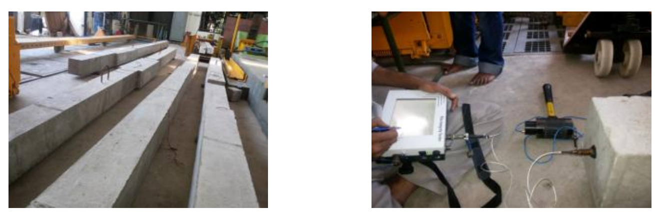
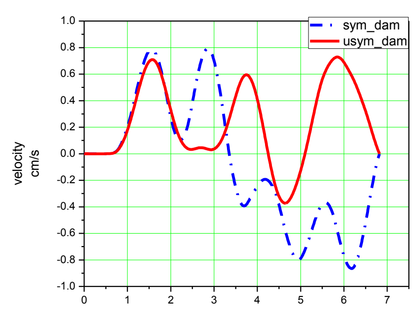
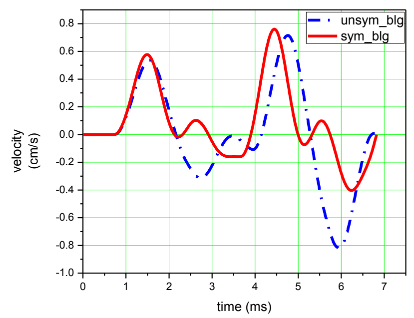


Figure1. *Experimentally tested specimen and Pulse Echo configuration Instrumentation*



(a)



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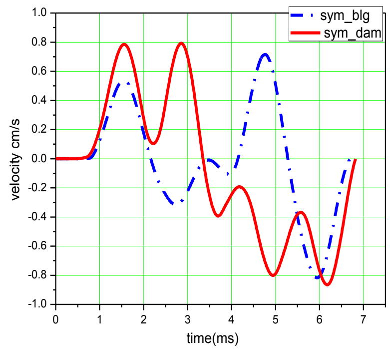

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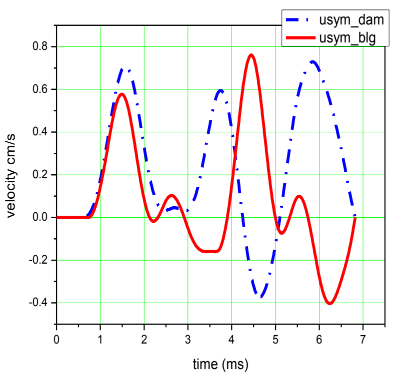


(d)

Figure2. *Experimentally measured velocity profile signals of longitudinal waves*

TABLE I. EXPERIMENTAL RESULTS OBTAINED FROM PULSE ECHO CONFIGURATION

	Incident (msec)	Damage Reflection (msec)	Location (m)	Toe reflection (msec)	Location (m)
Symmetry damage					
B1	0.6	2.2133	3.024	3.6933	5.8
B3	0.7	2.2133	2.837	3.79	5.793
Un-Symmetry damage					
B2	0.6	2.98	4.087	3.74997	5.9
B4	0.7	2.8376	4.008	3.8933	5.987

-From (Table1.), for the beam 1, damage location is 3.024m instead of 2.5m. This is because; the damage is located very near to the head. When the force is given the wave propagates such that it could not identify the damage exactly at 2.5m. After few milli-seconds it identifies the reduced area say at 3m. Same is the case for beam 3.

-For the beam 2 and beam 4, damage location is at 4.087m and 4.008m respectively, which is approximately 4m and it is quite good damage reflection. It is observed that the time gap between the peaks of reflected pulse however maintains the spatial difference of 1.5 m (4.0-2.5) and also it maintains the damage length of 1m.

-The plot clearly shows that the reduction area causes the positive velocity profile and increased area causes the negative velocity profile.

-The toe reflection is good for all the cases because it is approximately 6m (beam length).

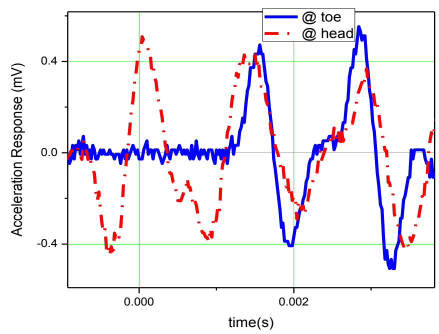
B. *Testing procedure and Results obtained from Pitch Catch Configuration*

Another set of experiment is conducted to study the axial wave propagation phenomenon in the beam as free-free rod element. This experiment is an example of Pitch – Catch Configuration method, i.e. accelerometer is kept at head as well as toe region and the response is measured through channels connected the instrument. Impulsive load is applied in the axial direction using a hammer with steel tips.

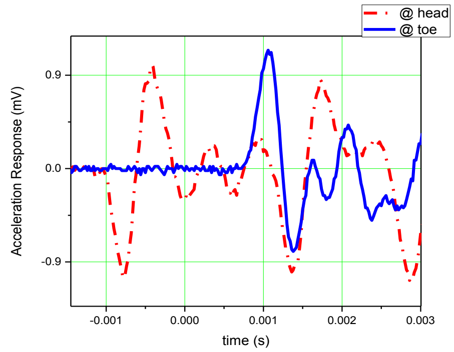
Figure3. *Instrumentation for Pitch catch configuration method*

-From (Fig. 4(a-d)) we can calculate the time taken for the wave to travel from head to toe is 1.831E-3 sec (approx. for all four cases) which gives the length of the beam approximately say 6m. Wave speed = 3500 m/sec.

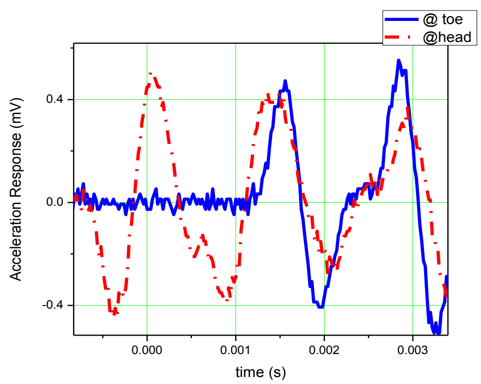
-Figure. 5 gives the comparison of the responses at head alone for the set of beams with symmetric and un-symmetric damage. It helps us to realize how the wave pattern is at the increased area and reduced area. The location of the damage is being identified (@ 2.5m and 4m) at respective places for all four cases approximately using pitch catch configuration method. On superimposing two responses, it is clear that there is variation for the increased and reduced part of the beam 1-3 and 2-4.



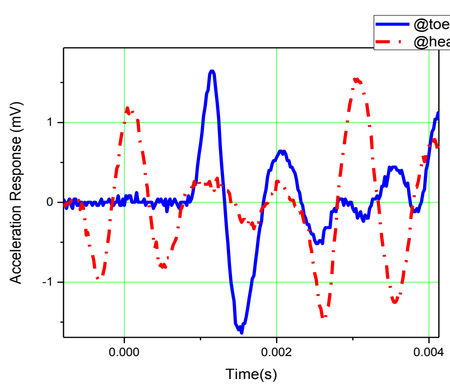
(a) B1



(b) B2

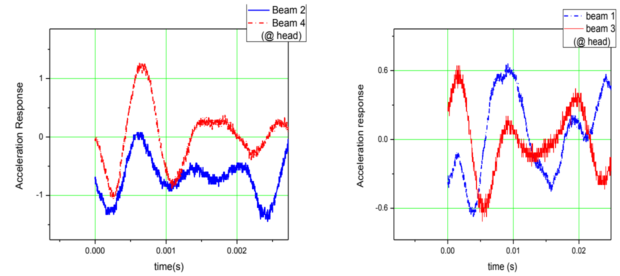


(c) B3



(d) B4

Figure4. *Experimentally obtained Response from Pitch catch configuration*

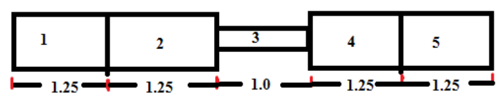


(b)

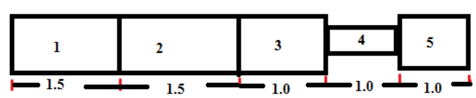
Figure5. *Comparison between responses of symmetry and un-symmetry damage obtained through Pitch catch configuration*

IV. Spectral Finite Element analysis of Wave Propagation in bar element with Discontinuities

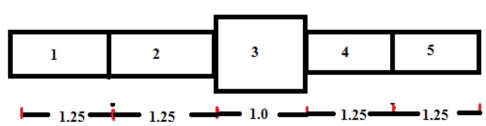
Modeling of axial wave propagation in rods was performed by the spectral finite element method. The amplitude of the external force was set as one of the response obtained from the hammer hit. The discretization of the specimen in SFEM is shown in (Fig.6). In Numerical analysis the specimen is considered as bar element with free- free boundary conditions. The response obtained from the numerical analysis in shown in (Fig. 7). The wave velocity, c is 3500 m/s.



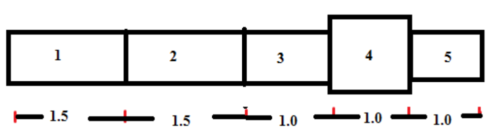
(a) Beam 1



(b) Beam 2



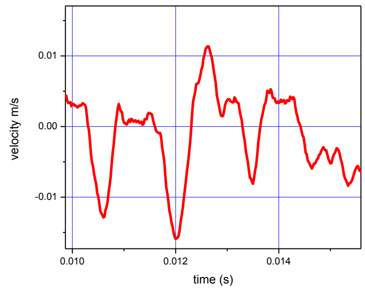
(c) Beam 3



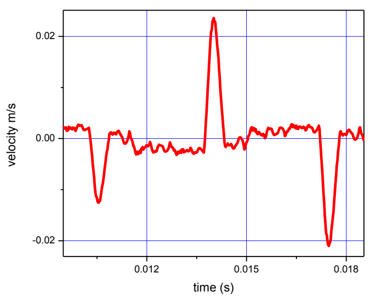
(d) Beam 4

Figure6. *Discretized specimens for SFEM*

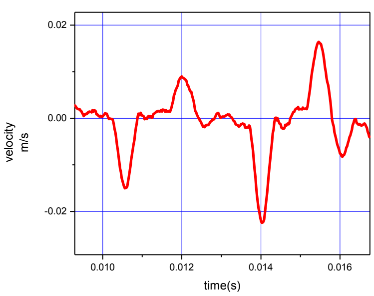
Results of numerical studies (Table. 2) lead to the following conclusions: It is observed that the time gap between the peaks of reflected pulse always maintains the spatial difference of 1.5 m (4.0-2.5) and also it maintains the damage length of 1m. The damage reflection and toe reflection matches the exact values (2.5m and 4m) with slight variations. Thus the discontinuity location is identified numerically using SFEM.



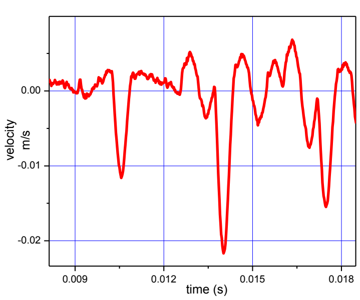
(a) B1



(b) B2



(c) B3



(d) B4

Figure7. *Numerically obtained Response from Spectral Finite Element Method*

TABLE II. NUMERICAL RESULTS OBTAINED FROM SPECTRAL FINITE ELEMENT METHOD

	Incident (sec)	Damage Reflection (sec)	Location (m)	Toe reflection (sec)	Location (m)
Symmetry damage					
B1	0.01026	0.01202	3.08	0.0135	5.8
B3	0.01026	0.012	3.045	0.01404	6.6
Un-Symmetry damage					
B2	0.01026	0.01288	4.58	0.01406	6.6
B4	0.01026	0.01288	4.5	0.014	6.54

V. Conclusion

In this paper, axial wave propagations in a bar element is investigated experimentally on four different concrete beams using Pulse echo configuration and Pitch catch configuration methods. In particular, detection of damage in form of discontinuity of cross-section is identified by analyzing wave speeds and reflections in the recorded velocity signals. Numerical results are compared with experimental data for the elementary rod theory and it is noted that numerical simulations are in good agreement with the experimental data. The proposed damage models provided velocity results compatible with experimentally measured signals.

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