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Health Monitoring and Damage Assessment by Parameter Estimation of Framed Structures From Static Responses

Jaydeep Chowdhury[1] and Priyabrata Guha[2]

***Abstract*—**Structural damage assessment of framed structure utilizing a sensitivity based parameter identification method is presented in this paper. A subset of applied static forces and subset of measured strains are used to identify the elemental stiffness parameters of all or a portion of a finite element model of the structure. Here we developed a method for parameter estimation of linear elastic structures using static strain measurements, preserving structural connectivity and determines the changes in cross-sectional properties, including large changes or elemental failure for stable structures. To linearize the associated non linear problem a first order Taylor series expansion is used as an iterative scheme. The algorithm automatically adjusts the structural element stiffness parameters in order to improve the comparison between a measured and theoretical response in an optimal way. In this study we have artificially generated the required measured input. The identified cross-sectional element properties can be used for damage assessment of any structure. This procedure also identifies the selection of limited number of degrees of freedom required to perform successful parameter identification, as well as reduces the impact of measurement errors of the identified parameters. Two numerical examples, including two-dimensional (2D) and three-dimensional (3D) frame structures are presented and the element stiffness are successfully and accurately evaluated, and to analyze any frame structure with this numeric process easily, a program is implemented in MATLAB programming according to the explained process, so that we can easily change the input data for analyzing different types of frame structures.

Keywords—*Finite element model; Static strain Measurement; Damage detection;Error elimination;*

Introduction

In recent years data-driven structural health monitoring (SHM) has been actively investigated by the civil engineering community as an important tool for future infrastructure maintenance. To properly manage civil infrastructure, its condition, or serviceability must be assessed. Measurements and proper data processing are expected to give a reasonable assessment of serviceability that can be improved based on the assessment. A static condensation method is proposed to adjust the parameters of the finite element model (FEM), by minimizing the difference between the analytical data and the nondestructive test (NDT) data. The parameter adjustmentsbased on trial and error is not appropriate for structures with large number of parameter values. To overcome this type of difficulties parameter estimation process is used, in which the major differences between estimated and expected parameters are classified as damage. In all cases, damage can severely affect the safety and serviceability of the structure. Hence an early detection of any damage is necessary. Owing to its practical importance, damage identification in structures, especially the monitoring of structural health has been the subject of extensive investigations over the last two decades. As a result, a great deal of research work on using either the static or the dynamic response of structures for damage detection has been carried out.

Displacement measurements from applied loads are used for static finite element method. It can be a difficult task to make displacement measurement on full-scale structure; for this a frame of reference must be established. There is having a number of non-destructive tests to assess the material strengths of the structure without damaging the structure. However, all these techniques normally assess the damage qualitatively. System identification techniques, based on dynamic data, have been developed extensively, compared to static data. But, the static responses are more locally sensitive than the frequency in structural damage detection.

The model-based approach is implemented by using computer model of the structure of interest, such as Finite-Element Method (FEM), to identify structural parameters based on the measured test data. Though non-contact displacement processes are available, but, they are costly and difficult. Use of strain gauges to measure the static strain can overcome those limitations, but some small errors can occur, which we have to deal with or control. In this paper for structural health monitoring and damage assessment of structure, static strain measurement process is used. Static strain measurements are definitely more useful than displacement measurements, as it reduces the use of costly equipments and also avoids the difficulties of displacement measurements on full scale structures.

Implementing a damage identification strategy for aerospace, civil, and mechanical engineering infrastructures is referred to as Structural Health Monitoring (SHM). There are two main approaches to SHM: (a) non-model-based approach and (b) model-based approach. Both approaches have been successfully used for damage detection in structures. The non-model-based approach relies on the signal processing of experimental data, while the model-based approach relies on mathematical descriptions of structural systems. The alternatives to the non-model-based method include: modal analysis, dynamic flexibility measurements, matrix update methods and wavelet transform technique, which are used to determine changes in structures to identify damage. In model-

1. *Jaydeep Chowdhury.*
Narula Institute of Technology (West Bengal University of Technology). India.

2. *Priyabrata Guha.*
Narula Institute of Technology (West Bengal University of Technology). India.

base approach uses computer model, such as Finite-Element Method (FEM), to identify structural parameter based on measured test data.

The focus of the present work is the sensitivity based damage detection method. In this paper a method of estimating the parameter of linear-elastic structures using static strain measurements, preserving structural connectivity is presented. Sensitivity based update methods are based on the first order Taylor series, which minimizes the error of non-linear problem occurrence. This process allows single or several static forces to be applied at a subset of degree of freedoms (DOF), and the strains to be measured at a subset of structural components. It is also possible to identify all or a portion of structural cross-sectional properties, including element failure. The proposed method has numerically demonstrated in this paper for two-dimensional (2D) and three-dimensional (3D) frame structures, and the cross-sectional parameters of those structures are successfully identified.

Estimation of Parameter:

A sensitivity based parameter estimation process is presented in this paper to modify the parameters of a finite element model with simulated static strain measurements. Forces are induced at a subset of degrees of freedoms and with respect to that strains are measured on a limited number of structural elements.

Damage Assessment Using Finite Element Model:

The static finite element equation for frame structure can be expressed as

$$[K]\{U\} = \{F\} \qquad (1)$$

Where, [K] is the global stiffness matrix and {F} and {U} are force and displacement vector, respectively. Finite element model is based on the stiffness relationship between forces and displacements. The relationship will be created in the form of an element mapping vector $\{M_n\}$ in global coordinates.

First step is to create $\{M_n\}$ in the local coordinates such that the following relation is satisfied for an elemental strain (η_n) and displacement $\{\bar{U}_n\}$. The relation is,

$$\eta_n = \{\bar{M}_n\}\{\bar{U}_n\} \qquad (2)$$

then $\{U_n\}$ is transformed from the local coordinates to the global coordinates

$$\{\bar{U}_n\} = [T_n]\,\{U_n\} \qquad (3)$$

Where, $[T_n]$ = mapping matrix; and $\{U_n\}$ = element nodal displacement in the global coordinates.

and,

$$\{M_n\} = \{\bar{M}_n\}\,[T_n] \qquad (4)$$

By substituting the value of $\{\bar{U}_n\}$ and $\{\bar{M}_n\}$ from (3) and (4) in (2), we can get the global mapping relation for one structural element. That is

$$\eta_n = \{M_n\}\{U_n\} \qquad (5)$$

From equation (5), we got the strain-displacement relation for system of *n* elements. Now by vertically augmenting the elemental strains and aligning the system degree of freedoms horizontally with the system [M] matrix, we get,

$$\{\eta\} = [M]\,\{U\} \qquad (6)$$

Now, by assembling $\{\varepsilon\}$ = element strain vector of size NEL×1; {U}= global displacement vector; we got the [M] matrix of size NEL×NDOF.

Creation of mapping vector :

For linear elastic behavior of frame element, the actions of axial deformation and bending both causes axial strains on the element surface at a known distance from the neutral axis are superimposed. These strains are measured length wise, which are in $\bar{x}$ direction.

$$\eta_n = -\bar{y}\frac{d^2\bar{v}}{d\bar{x}^2} \qquad (7)$$

where, $\bar{y}$= distance from the neutral axis to strain measurement surface and $\bar{v}$ = translation in $\bar{y}$ direction. A natural coordinate system ξ is induced for finite element analysis, ξ can be defined as

$$\xi = \frac{2\bar{x}}{L} - 1 \qquad (8)$$

ξ ranges from -1 to +1, for the substituted *x* value, which ranges from 0 to *L*.

We can express mapping vector $\{M_n\}$ as

$$\{M_n\} = \{\bar{M}_n\}\,[T_n] \qquad (9)$$

Where, $\{\bar{M}_n\}$ = mapping vector of an element *n*; The size of $\{\bar{M}_n\}$ will be 1×6 for 2D frame and 1×12 for 3D frame, and they are represented as

$\{\bar{M}_n\}$ for 2D frame =

$$\left\{ \frac{-1}{L}\ \frac{-6\bar{y}\xi}{L^2}\ \frac{\bar{y}}{2L}(6\xi - 2)\ \frac{1}{L}\ \frac{6\bar{y}\xi}{L^2}\ \frac{-\bar{y}}{2L}(6\xi + 2) \right\}_n;$$

$\{\bar{M}_n\}$ for 3D frame =

$$\left\{ \frac{-1}{L}\ \frac{-6\bar{y}\xi}{L^2}\ \frac{-6\bar{z}\xi}{L^2}\ 0\ \frac{\bar{y}}{2L}(6\xi - 2)\ \frac{\bar{z}}{2L}(6\xi - 2)\ \frac{1}{L}\ \frac{6\bar{y}\xi}{L^2}\ \frac{6\bar{z}\xi}{L^2}\ 0\ \frac{-\bar{y}}{2L}(6\xi + 2)\ \frac{-\bar{z}}{2L}(6\xi + 2) \right\}_n;$$

and, $[T_n]$ = transformation matrix of size 6×6 for 2D frame and 12×12 for 3D frame, and represented as

$$[T_n] \text{ for 2d frame} = \begin{bmatrix} l_{\bar{x}x} & l_{\bar{x}y} & 0 & 0 & 0 & 0 \\ l_{\bar{x}x} & l_{\bar{x}y} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & l_{\bar{x}x} & l_{\bar{x}y} & 0 \\ 0 & 0 & 0 & l_{\bar{x}x} & l_{\bar{x}y} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}_n$$

and, $[T_n]$ for 3D frame =

$$\left[\begin{array}{cccccccccccc}
l_{\bar{x}x} & l_{\bar{x}y} & l_{\bar{x}z} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
l_{\bar{y}x} & l_{\bar{y}y} & l_{\bar{y}z} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
l_{\bar{z}x} & l_{\bar{z}y} & l_{\bar{z}z} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & l_{\bar{x}x} & l_{\bar{x}y} & l_{\bar{x}z} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & l_{\bar{y}x} & l_{\bar{y}y} & l_{\bar{y}z} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & l_{\bar{z}x} & l_{\bar{z}y} & l_{\bar{z}z} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & l_{\bar{x}x} & l_{\bar{x}y} & l_{\bar{x}z} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & l_{\bar{y}x} & l_{\bar{y}y} & l_{\bar{y}z} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & l_{\bar{z}x} & l_{\bar{z}y} & l_{\bar{z}z} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & l_{\bar{x}x} & l_{\bar{x}y} & l_{\bar{x}z} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & l_{\bar{y}x} & l_{\bar{y}y} & l_{\bar{y}z} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & l_{\bar{z}x} & l_{\bar{z}y} & l_{\bar{z}z}
\end{array}\right]_n$$

where,
$l_{\bar{x}x} = cos(\alpha_x)$, $l_{\bar{x}y} = cos(\alpha_y)$, $l_{\bar{x}z} = cos(\alpha_z)$;
$l_{\bar{y}x} = cos(\alpha_z)$, $l_{\bar{y}y} = cos(\alpha_x)$, $l_{\bar{y}z} = cos(\alpha_y)$;
$l_{\bar{z}x} = cos(\alpha_y)$, $l_{\bar{z}y} = cos(\alpha_z)$, $l_{\bar{z}z} = cos(\alpha_x)$;

By substituting (1) in (6) it becomes

$$\{e\} = [M]\,[K(p)]^{-1}\{F\} \qquad (9)$$

Where, $\{\eta\}$= NEL ×1; [M]= NEL ×NDOF; [K(p)]= NDOF × NDOF; and {F}=NDOF ×1.

The element stiffness parameters of the stiffness matrix may be a cross-sectional area *A*, or a moment of inertia I_{xx}, I_{yy}, $I_{zz}$and stored in the vector of parameters {p}. Number of set of force(NSF) load cases are used in parameter identification method for solving the unknown parameters. By arranging the test data horizontally in {e} and {F}, the equation becomes

$$[\eta] = [M]\,[K(p)]^{-1}[F] \qquad (10)$$

where, [e] = NEL×NSF; and [F] = NDOF×NSF.
To eliminate the less important unmeasured strain, [e] is partitioned as $[\frac{\eta_a}{\eta_b}]$, correspond to this [M] is also partitioned as $[\frac{B_a}{B_b}]$, where $[\eta_b]$ is the unmeasured strain. After elimination of the unmeasured strain the equation becomes

$$[\eta_a] = [M_a]\,[K(p)]^{-1}[F] \qquad (11)$$

where, $[\eta_a]$ = NMS × NSF; $[M_a]$ = NMS × NDOF; and NMS = number of measured strain.

Estimation and Minimization of Error Function:

The error function is defined as the difference between the analytical strain value $[e_a]$ and the measured strain value $[\eta_a]^m$. It can be expressed as

$$[\eta(p)] = [\eta_a] - [\eta_a]^m \qquad (12)$$

The error function will be of size NMS × NSF, and in equation (11) due to the inversion of [K(p)], a nonlinear function arrives. To linearize [e(p)], the error function matrix is vectorized by concatenating all the columns vertically. This produces an {e(p)} of size NM × 1, where, the number of measurements (NM) will be the product value of NMS × NSF. To linearize this {e(p)}, a first-order Taylor series expansion is used as

$$\{e(p + \Delta p)\} = \{e(p)\} + [C]\,\{\Delta p\} \qquad (13)$$

By differentiating [e(p)] one time with respect to *p* we got the sensitivity matrix [C] of size NM × NUP and Δp is the change in parameter of size NUP×1.
A scalar performance error function *R(p)* is came form the reduction of error function {e(p)}, and it can be expressed as

$$R(p) = \{e(p + \Delta p)\}^T\,\{e(p + \Delta p)\} \qquad (14)$$

To minimize *R(p)* with respect to {*p*}, subjected to $p_i \geq 0$ for i = 1 to NUP, the gradient of *R(p)* with respect to $\{\Delta p\}^T$ is set to zero, which results a linearized system of equation as,

$$[C]^T\,[C]\,\{\Delta p\} = -[C]^{-1}\,\{e(p)\} \qquad (15)$$

If [C] is a square matrix then direct inversion may be used and {Δ*p*} will be

$$\{\Delta p\} = -[C]^T\,\{e(p)\} \qquad (16)$$

but, when [C] will not be square (NM > NUP), the least-square method results {Δ*p*} as

$$\{\Delta p\} = -([C]^T\,[C])^T\,[C]^T\,\{e(p)\} \qquad (17)$$

Examples for Parameter Estimation of Frame Structures:

2D *Frame Example:*

The first example consist of a 2D frame as shown in"Fig. 1". As it is a frame structure, it is capable of bending and axial deformation, so, it will have two parameters as: cross-sectional area (*A*) and moment of inertia (I_z). This frame is having 18 elements or 18 members, 15 number of nodes. As shown in "Fig. 1", node 11 to 15 is fixed.so, there can be a maximum of 36 unknown parameters, total number of DOF will be 30. Modulus of elasticity (*E)* for all elements of the frame is taken as 280.6 GPa. and cross-sectional properties are taken as specified in"Table-1". There are 9 cases for the parameter identification of the 2D frame example. In case 1,3,4,5,6,7,8,9 applied forces are 500 N or 200 N.m. and for cases 11,12,13,15 applied forces are 600 N or N.m,800 N or N.m,700 N or N.m,900 N or N.m respectively, for case-14 forces are 800 N or 400 N.m.

Table-1. Cross-sectional Properties of 2D Frame

Table -2. Different Applied Load Cases for Analysis:

Case	FDOF	η	Load (N or N.m)	Iterations
1	16 to 30	1 to 18	500 N or 200 N.m	3
2	2,5,8,11,14,17, 20,23,26,29	4,7,9,12, 15,18	500 N or 200 N.m	4
3	4,5,8,9, 15,21,25	1,6,12,17	500 N or 200 N.m	3
4	1,2,12,15,16 17,27,30	6,8,11,16	500 N or 200 N.m	2
5	6,9,12,16,21,27	10,11,16, 17	600 N or 600 N.m	6
6	19,20,27	2,9,10,18	800 N or 800 N.m	3
7	6,12,21,30	6,13	700 N or 700 N.m	Singular
8	2,6,18,30	5,9,15	800 N or 400 N.m	5
9	11,12,19,21	3,6,8,11	900 N or 900 N.m	3

Parameters		*Initial*	*True*
Area	A_1 to A_4 and A_{10} to A_{13}	900×10^2 mm^2	855×10^2 mm^2
	A_5 to A_9 and A_{14} to A_{18}	1600×10^2 mm^2	1760×10^2 mm^2
Moment of Inertia	I_{z1} to I_{z4} and I_{z10} to I_{z13}	67500×10^4 mm^4	81000×10^4 mm^4
	I_{z5} to I_{z9} and I_{z14} to I_{z18}	213333×10^4 mm^4	170666.4×10^4 mm^4

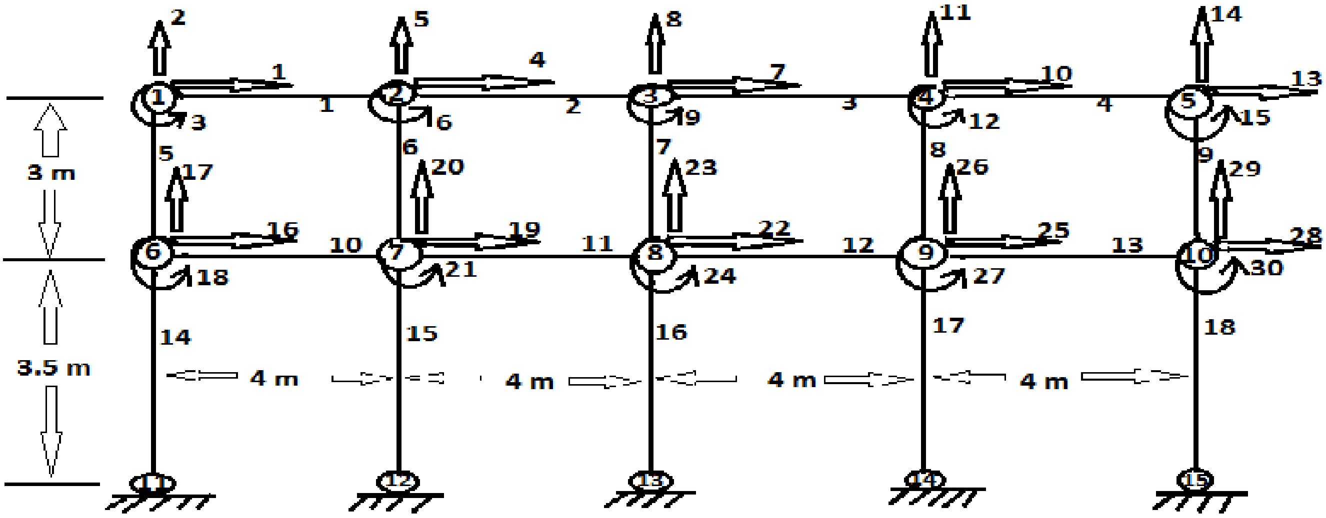
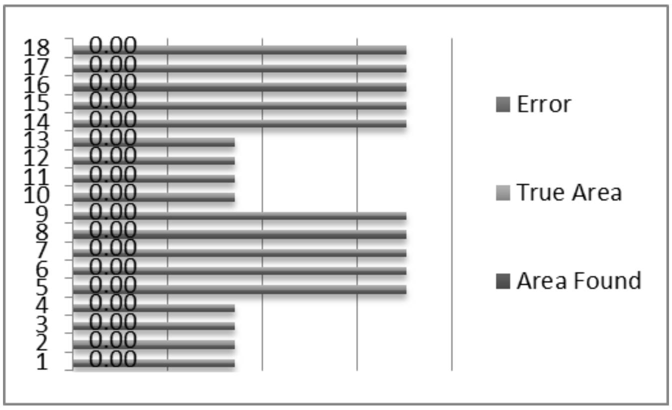
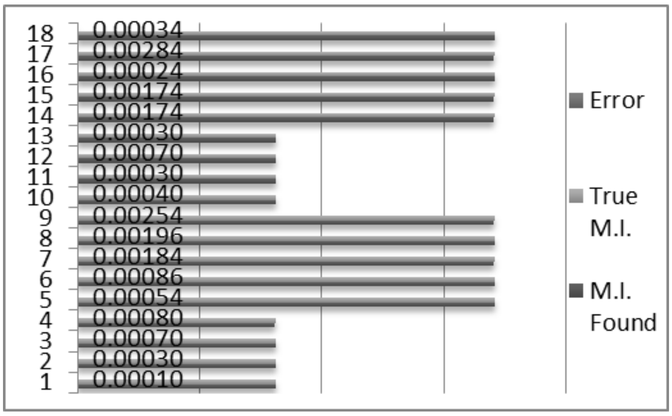


Fig. 1 – 4 Bay 2 Storied 2D Frame

Bar chart-1: Error in Cross-sectional Area (2D Frame):





Bar chart-2: Error in Moment of Inertia (2D Frame):

Load case analysis of case-9 of "Table-2" is done with the help of MATLAB and in this case we have detected some errors in moment of inertia which is very less and there is having no errors in cross-sectional area of all members they as shown in the "Bar chart-1" and "Bar chart-2".

3D Frame Example:

"Fig. 2" shows the example of a 3D frame structure. As it is also a frame structure, it will also be capable of bending and axial deformation and parameters will also be similar

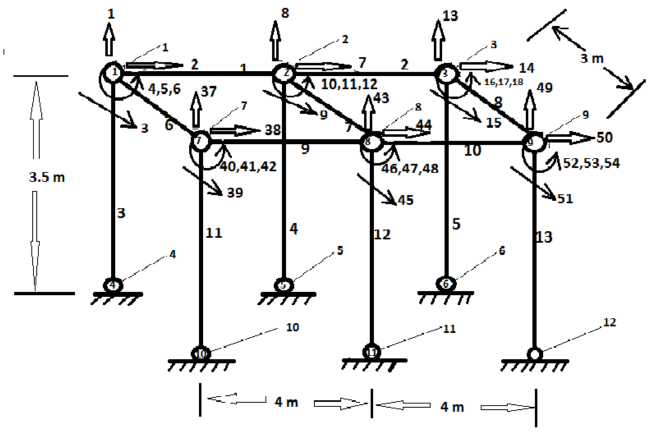


Fig. 2 – 2 Bay 1 Storied 3D Frame

to the previous frame example.This frame is having 13 elements or 13 members, 12 number of nodes. As shown in "Fig. 2", node 4 to 6 and 10 to 12 is fixed.so, there can be a maximum of 13 unknown parameters, total number of NDOF will be 36, but in "Fig. 2" the DOf s are represented as their global number. Modulus of elasticity *(E)* for all elements of the frame is taken as 280.6 GPa. torsional constant (*J*) is taken as 27795 GPa and cross-sectional properties are taken as specified in "Table-3(a)" and "Table-3(b)". There are 2 cases for the parameter identification of the 3D frame example. The load cases are specified in "Table-4".

Table-3 (a). Cross-sectional Properties of 3D Frame

Parameters		*Initial*	*True*
Area	A_1 to A_2 and A_6 to A_{10}	900×10^2 mm^2	855×10^2 mm^2
	A_3 to A_5 and A_{11} to A_{13}	1600×10^2 mm^2	1760×10^2 mm^2

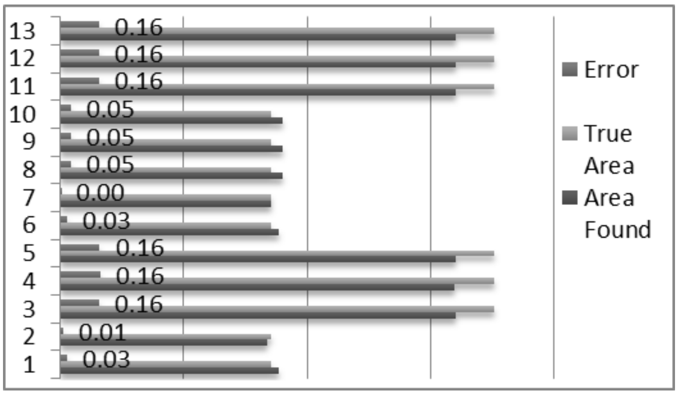
Table-3 (b). Cross-sectional Properties of 3D Frame

Parameters		*Initial*	*True*
Moment of Inertia	I_{z1} to I_{z4} and I_{z10} to I_{z13}	67500×10^4 mm^4	81000×10^4 mm^4
	I_{z5} to I_{z9} and I_{z14} to I_{z18}	213333×10^4 mm^4	170666.4×10^4 mm^4

Table -4. Different Applied Load Cases for Analysis:

Case	FDOF	η	Load (N or N.m)	Iterations
1	4 to 12 49 to 54	2,4,8,9, 11,13	200 N or 100 N.m	4
2	8,9,44,45, 50,51	3,5,7, 11,13	200 N	Singular

Bar chart-3: Error in Cross-sectional Area (3D Frame):



Load case analysis of case-1 of "Table-4" is done with the help of MATLAB and in this case we are not be able to identify the moment of inertia properly, there are having so many errors which consist of large magnitude, but in case of identification of cross-sectional area, this process has successfully identified a part of the members, more specifically the beam elements have been identified very closely, as they are having very less magnitude of error. As shown in "Bar chart-3" and "Fig. 2", member 1, 2, 6, 7, 8, 9, 10 are identified with a very less error.

Conclusions

This method has successfully identified the parameters at the element level by applying forces at some limited numbers of degrees of freedoms and also by taking strain measurements at some selective locations for linear elastic structures. The difference between analytical strain and measured strain forms the performance error function. The cross-sectional element properties for frame such as, areas and/or moments of inertias are identified by minimizing that performance error function.

Two examples are presented here and assessment has been done successfully to some extent. It is also identified that if NM ≥ NUP, it is possible to identify the parameters. So, this method can successfully identify the parameters of 2D structures. In case of 3D structures the capability of simultaneous identification parameters are reduced. Either subset of measurements has to change to avoid large changes in identified parameters or only a partial set of parameters can be identified.

Future work can include some improvement in this method to overcome the limitations of identifying all the parameters properly and the damage assessment of frame structure for dynamic responses can also be done. So that, we will be able to identify the parameters of any type of frame structures for both static and dynamic responses.

References

[1] William McGuire, Richard H. Gallagher, Ronald D Zeimian. "Matrix Structural Analysis" (Second Edition). Chapter- 4 and 5.

[2] P.Seshu. "Textbook of Finite Element Analysis" , Chapter-5.

[3] Masoud Sanayei, Michael J. Saletnik.(1996) "Parameter Eastimation of Structures From Static Strain Measurements. I:Formulation" .

[4] G. Aditya and S. Chakraborty.(2008) "Sensitivity Based Health Monitoring of Structures With Static Response".

[5] Masoud Sanayei, Oladipo Onipede.(1991) "Damage Assessment of Structures Using Static Test Data".