Mixed method for reducing the order of a linear discrete time interval system

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Abstract— **The paper proposes a mixed method for reducing the order of discrete time interval systems. The numerator and denominator of the reduced order model is obtained by forming $\alpha - \beta$ table and combining factor division method and Cauer second form. Bilinear transformation (z = p+1) is used during the process of higher order discrete interval systems before applying reduction method. The proposed method shows that it is well suited to reduce both denominator and numerator interval discrete polynomials. The proposed method is compared with the existing methods of discrete interval system reduction. A numerical example has been discussed to illustrate the procedure. Accuracy of the system stability has been verified by using step response.**

Keywords— **Cauer second form, discrete interval systems, α table, β table, factor division, mixed method.**

I.   Introduction

In field of control engineering, model order reduction of high-order systems transfer function to low order transfer function has been an important subject for many years. Many systems such as Cold rolling mill, Continuous stirred tank reactor, Oblique wing aircraft and Robot manipulator etc. are formulated in terms of uncertain parameters and called interval systems. For the interval systems, arithmetic rules [1] were introduced by Hansen E. Kharitonov VL proposed Kharitonov's theorem [2] to study the stability of interval systems. Lot of work has been done in the field of model order reduction of continuous interval systems but in the field of discrete interval systems very little literature is available. The model order reduction of interval systems derived from the existing method for systems with fixed coefficients. Routh-Pade approximation [3] was introduced for reducing the complexity of higher order continuous interval systems. Routh approximation has been shown that a stable reduced model can be obtained by a direct truncation of the Routh table of the continuous interval system, and the numerator of the reduced order model can be obtained by matching interval time moments. γ-δ method [4] for interval systems was proposed by Bandyopadhyay.

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It has been shown that then γ-δ parameters of rth order model can be obtained by retaining first r of the higher order continuous interval system. Hwang C and Yang SF [5] commented on the above two methods. They claimed that an unstable reduced interval Routh approximation may be obtained for a stable high order interval system. Sastry propose a new method for reducing the high order interval system using Routh approximation and the method requires only γ table [6], that it is computationally superior to existing method [4] (γ-δ). Dolgin Y and Zehed E introduced a modified method [7] of direct truncation of the Routh table for interval systems. Dolgin Y and Zeheb E proved that direct truncation of the Routh approximation [3] does not guarantee a stable family of reduced order interval system if the original higher order system is stable. Yang SF [8] proved that the method proposed by Dolgin Y and Zeheb E [7] cannot guarantee the stability of the reduced order interval models and also given justification that Routh approximation method cannot preserve the stability when it is extended to the model order reduction of interval systems. Dolgin Y [9] included two conditions to modify the method of direct truncation of interval systems to obtain a stable reduced order. But this method has a disadvantage that Dolgin modified the subtraction rule for retain the stability for reduced order system. Selvaganesan proposed a mixed method based on generalized Routh approximation and Factor division method [10]. Sani DK and Prasad R applied two mixed evolutionary techniques for order reduction of linear continuous interval system based on minimization of the ISE by Genetic Algorithm and Particle Swarm Optimization [11]. Kranthi et al.   proposed differentiation method [12] to obtain stable reduced order interval system if the original system is stable.

For reduction of discrete interval systems, dominant pole [13]-[15] based methods were introduced. From the above literature, we can conclude that the Routh approximation based method cannot preserve the stability when it is extended to interval systems. In this paper new method of model order reduction based α approach, employed in the discrete time domain system is introduced. The proposed method does not require transformation of the interval system to the s-plane for order reduction. Reduction of model denominator polynomial can be obtained by means of the α table for discrete interval systems using bilinear transformation $\left(z = p + 1 \right)$ The numerator is obtained by different mixed methods, such as β table, factor division and Cauer second form. These methods ensure that the reduced order model is stable if the high- order interval system is stable. The outline of this paper is as follows: Section II contains problem statement. Section III contains proposed method. Numerical example is presented in section IV and conclusions in section V.

II. Problem Statement

Let the transfer function of a higher order discrete interval systems be:

$$G_n(z) = \frac{[c_{21}^-, c_{21}^+] + [c_{22}^-, c_{22}^+]z + \dots + [c_{2,n-1}^-, c_{2,n-1}^+]z^{n-1}}{[c_{11}^-, c_{11}^+] + [c_{12}^-, c_{12}^+]z + \dots + [c_{1,n}^-, c_{1,n}^+]z^n} = \frac{N(z)}{D(z)}$$

(1)

where $[c_{2j}^-, c_{2j}^+]; 1 \le j \le n-1$ and $[c_{1j}^-, c_{1j}^+]; 1 \le j \le n$ are known as scalar constants.

Let the reduced order model of $R_k(z)$ is expressed as

$$R_k(z) = \frac{[d_{21}^-, d_{21}^+] + [d_{22}^-, d_{22}^+]z + \dots + [d_{2,k-1}^-, d_{2,k-1}^+]z^{k-1}}{[d_{11}^-, d_{11}^+] + [d_{12}^-, d_{12}^+]z + \dots + [d_{1,k}^-, d_{1,k}^+]z^k} = \frac{N_k(z)}{D_k(z)}$$

(2)

where $[d_{2j}^-, d_{2j}^+]; 1 \le j \le k-1$ and $[d_{1j}^-, d_{1j}^+]; 1 \le j \le k$ are known as scalar constants.

The rules of the interval arithmetic have been defined in [1], and are presented below.

Let [e, f] and [g, h] be two intervals.

Addition:

$$[e,f] + [g,h] = [e+g, f+h]$$

(3)

Subtraction:

$$[e,f] - [g,h] = [e-h, f-g]$$

(4)

Multiplication:

$$[e,f] \times [g,h] = [Min(eg, eh, fg, fh), Max(eg, eh, fg, fh)]$$

(5)

Division:

$$\frac{[e,f]}{[g,h]} = [e,f] \times \left[\frac{1}{h}, \frac{1}{g} \right]$$

(6)

III. Proposed Method

The proposed method consists of the following steps for obtaining reduced order model.

Determination of the denominator polynomial of the kth order reduced model as given in Eq (2) by using α table:

Step 1: Reciprocal of higher order denominator

Step 2: Substitute $(z = p+1)$

$$D(p+1) = [c_{11}^-, c_{11}^+](p+1)^n + [c_{12}^-, c_{12}^+](p+1)^{n-1} + \dots + [c_{1,n}^-, c_{1,n}^+]$$

(7)

Step 3: Construct α table.

| TABLE I. | ALPHA TABLE |

	$a_0^0 = [c_{11}^-, c_{11}^+]$	$a_2^0 = [c_{13}^-, c_{13}^+]$	....
	$a_0^1 = [c_{12}^-, c_{12}^+]$	$a_2^1 = [c_{14}^-, c_{14}^+]$	....
$\alpha_1 = \dfrac{a_0^0}{a_0^1}$	$a_0^2 = a_2^0 - \alpha_1 a_2^1$	$a_2^2 = a_4^0 - \alpha_1 a_4^1$	....
$\alpha_2 = \dfrac{a_0^1}{a_0^2}$	$a_0^3 = a_2^1 - \alpha_2 a_2^2$	......	
$\alpha_3 = \dfrac{a_0^2}{a_0^3}$	$a_0^4 = a_2^2 - \alpha_3 a_2^3$	.....	
....	....		

Let $A_k(p+1)$ denote the denominator of kth Routh convergent respectively

$$A_1(p+1) = \alpha_1 p + 1$$

$$A_2(p+1) = \alpha_1 \alpha_2 p^2 + \alpha_2 p + 1$$

. . . .

$$A_k(p+1) = \alpha_1 A_{k-1}(p+1) + A_{k-2}(p+1)$$

where

$$A_{-1}(p+1) = 0; A_0(p+1) = 0.$$

The reduced order depends upon the order of the system reciprocal of $A_k(p+1)$.

Step 4: Reciprocal transformation of step 3.

Step 5: Substitute $(p = z-1)$

Determine the numerator coefficients of the kth order reduced model by using β table, factor division method and Cauer second form method.

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Case 1: Determination of the numerator polynomial of the kth order reduced model by using β table:

Step 1: Reciprocal of higher order numerator

Step 2: Substitute $(z = p+1)$

$$N(p+1) = [c_{21}^-, c_{21}^+](p+1)^n + [c_{22}^-, c_{22}^+](p+1)^{n-1} + ... + [c_{2,n-1}^-, c_{2,n-1}^+]$$
(8)

Step 3: Construct β table

TABLE II.   BETA TABLE

	$b_0^1 = [c_{21}^-, c_{21}^+]$	$b_2^1 = [c_{23}^-, c_{23}^+]$	....
	$b_0^2 = [c_{22}^-, c_{22}^+]$	$b_2^2 = [c_{24}^-, c_{24}^+]$	....
$\beta_1 = \dfrac{b_0^1}{a_0^1}$	$b_0^3 = b_2^1 - \beta_1 a_2^1$	$b_2^3 = b_4^1 - \beta_1 a_4^1$	....
$\beta_2 = \dfrac{b_0^2}{a_0^2}$	$b_0^4 = b_2^2 - \beta_2 a_2^2$	......	
$\beta_3 = \dfrac{b_0^3}{a_0^3}$	$b_0^5 = b_2^3 - \beta_3 a_2^3$	.....	
……….	……… ………		

Let $B_k(p+1)$ denote the numerator of kth Routh convergent respectively.

$$B_1(p+1) = \beta_1$$

$$B_2(p+1) = \alpha_2 \beta_1 p + \beta_2$$

$$. . . . .$$

$$B_k(p+1) = \alpha_k p B_{k-1}(p+1) + B_{k-2}(p+1) + \beta_k$$

where

$$B_{-1}(p+1) = 0; B_0(p+1) = 0.$$

The reduced order depends upon the order of the system reciprocal of $B_k(p+1)$.

Step 4: Reciprocal transformation of step 3.

Step 5: Substitute $(p = z-1)$

Case 2: Determination of the numerator polynomial of the kth order reduced model by using factor division method:

Any method of reduction which relies upon calculating the reduced denominator first and then the numerator, where $D_k(z)$ has already been calculated.

$$G_n(z) = \dfrac{N(z)D_k(z)\big/ D(z)}{D_k(z)}$$
(9)

$$N(z)D_k(z) = [u_{11}^-, u_{11}^+] + [u_{12}^-, u_{12}^+]z + ... + [u_{1,k-1}^-, u_{1,k-1}^+]z^{k-1}$$
(10)

$$\dfrac{N(z)D_k(z)}{D(z)} = \dfrac{\left[u_{11}^-, u_{11}^+\right] + \left[u_{12}^-, u_{12}^+\right]z + ... + \left[u_{1,k-1}^-, u_{1,k-1}^+\right]z^{k-1}}{\left[c_{11}^-, c_{11}^+\right] + \left[c_{12}^-, c_{12}^+\right]z + ... + \left[c_{1,n}^-, c_{1,n}^+\right]z^n}$$
(11)

Therefore,

$$[\alpha_{11}^-, \alpha_{11}^+] = \dfrac{[u_{11}^-, u_{11}^+]}{[c_{11}^-, c_{11}^+]} \left\{ \dfrac{[u_{11}^-, u_{11}^+][u_{12}^-, u_{12}^+].....}{[c_{11}^-, c_{11}^+][c_{12}^-, c_{12}^+]....} \right\}$$

$$[\alpha_{12}^-, \alpha_{12}^+] = \dfrac{[r_{11}^-, r_{11}^+]}{[c_{11}^-, c_{11}^+]} \left\{ \dfrac{[r_{11}^-, r_{11}^+][r_{12}^-, r_{12}^+].....}{[c_{11}^-, c_{11}^+][c_{12}^-, c_{12}^+]....} \right\}$$

$$[\alpha_{13}^-, \alpha_{13}^+] = \dfrac{[s_{11}^-, s_{11}^+]}{[c_{11}^-, c_{11}^+]} \left\{ \dfrac{[s_{11}^-, s_{11}^+][s_{12}^-, s_{12}^+].....}{[c_{11}^-, c_{11}^+][c_{12}^-, c_{12}^+]....} \right\}$$

$$. . . . . . \qquad . . . .$$

$$[\alpha_{1,k-2}^-, \alpha_{1,k-2}^+] = \dfrac{[x_{11}^-, x_{11}^+]}{[c_{11}^-, c_{11}^+]} \left\{ \dfrac{[x_{11}^-, x_{11}^+][x_{12}^-, x_{12}^+]}{[c_{11}^-, c_{11}^+][c_{12}^-, c_{12}^+]} \right\}$$

$$[\alpha_{1,k-1}^-, \alpha_{1,k-1}^+] = \dfrac{[y_{11}^-, y_{11}^+]}{[c_{11}^-, c_{11}^+]} \left\{ \dfrac{[y_{11}^-, y_{11}^+]}{[c_{11}^-, c_{11}^+]} \right\}$$

where

$$[r_{1,i}^-, r_{1,i}^+] = \left[u_{1,i+1}^-, u_{1,i+1}^+\right] - \left[\alpha_{11}^-, \alpha_{11}^+\right]\left[c_{1,i+1}^-, c_{1,i+1}^+\right]$$

here, i= 0,1, 2,. . . ,k-2.

$$[s_{1,i}^-, s_{1,i}^+] = \left[r_{1,i+1}^-, r_{1,i+1}^+\right] - \left[\alpha_{12}^-, \alpha_{12}^+\right]\left[c_{1,i+1}^-, c_{1,i+1}^+\right]$$

where, i= 0,1, 2,. . . ,k-3.

$$. . . . .$$

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$$[y_{1,i}^-, y_{1,i}^+] = \left[x_{1,i+1}^-, x_{1,i+1}^+\right] - \left[\alpha_{1,k-2}^-, \alpha_{1,k-2}^+\right]\left[c_{11}^-, c_{1,1}^+\right]$$

The reduced transfer function given by

$$R_k(z) = \frac{[\alpha_{11}^-, \alpha_{11}^+] + [\alpha_{12}^-, \alpha_{12}^+]z + \ldots + [\alpha_{1,k-1}^-, \alpha_{1,k-1}^+]z^{k-1}}{D_k(z)} \tag{12}$$

where

$$[\alpha_{11}^-, \alpha_{11}^+] = [d_{21}^-, d_{21}^+]$$

$$[\alpha_{12}^-, \alpha_{12}^+] = [d_{22}^-, d_{22}^+]$$

$$. . . . .$$

$$[\alpha_{1,k-1}^-, \alpha_{1,k-1}^+] = [d_{2k}^-, d_{2k}^+]$$

Case 3: Determination of the numerator polynomial of the kth order reduced model by using Cauer second form:

Coefficient values from Cauer second form $[h_i^-, h_i^+]$ (i =1 , 2 , 3 . . . . k) are evaluated by forming Routh array as

$$[h_1^-, h_1^+] = \frac{[c_{11}^-, c_{11}^+]}{[c_{21}^-, c_{21}^+]} \begin{Bmatrix} [c_{11}^-, c_{11}^+][c_{12}^-, c_{12}^+]\ldots \\ [c_{21}^-, c_{21}^+][c_{22}^-, c_{22}^+]\ldots \end{Bmatrix}$$

$$[h_2^-, h_2^+] = \frac{[c_{21}^-, c_{21}^+]}{[c_{31}^-, c_{31}^+]} \begin{Bmatrix} [c_{21}^-, c_{21}^+][c_{22}^-, c_{22}^+]\ldots \\ [c_{31}^-, c_{31}^+][c_{32}^-, c_{32}^+]\ldots \end{Bmatrix}$$

$$[h_3^-, h_3^+] = \frac{[c_{31}^-, c_{31}^+]}{[c_{41}^-, c_{41}^+]} \begin{Bmatrix} [c_{31}^-, c_{31}^+][c_{32}^-, c_{32}^+]\ldots \\ [c_{41}^-, c_{41}^+][c_{42}^-, c_{42}^+]\ldots \end{Bmatrix}$$

$$. . . . .$$

The first two rows are copied from the original system numerator and denominator coefficients and rest of the elements are calculated by using well known Routh algorithm.

$$[c_{1,j}^-, c_{1,j}^+] = [c_{i-2,j+1}^-, c_{i-2,j+1}^+] - [h_{i-2}^-, h_{i-2}^+][c_{i-1,j+1}^-, c_{i-1,j+1}^+] \tag{13}$$

where i= 3, 4, ... and j = 1, 2, ...

$$[h_i^-, h_i^+] = \frac{[c_{i,1}^-, c_{i,1}^+]}{[c_{i+1,1}^-, c_{i+1,1}^+]} \tag{14}$$

The coefficient values of $[d_{ij}^-, d_{ij}^+]$ (j = 1,2,.....(k+1)) and Cauer quotients $[h_i^-, h_i^+]$ (i = 1. 2, . . . k) are matched for finding the coefficients of numerator of the reduced model $R_k(z)$.

The inverse Routh array is constructed as

$$[d_{i+1,1}^-, d_{i+1,1}^+] = \frac{[d_{i,1}^-, d_{i,1}^+]}{[h_i^-, h_i^+]} \tag{15}$$

where i = 1,2, . . .,k and k ≤ n.

Also,

$$[d_{i+1,j+1}^-, d_{i+1,j+1}^+] = \frac{[d_{i,j+1}^-, d_{i,j+1}^+] - [d_{i+2,j}^-, d_{i+2,j}^+]}{[h_i^-, h_i^+]} \tag{16}$$

where i = 1. 2, ... , (k-j) and j = 1,2 ,. . . . , (k-1)

Using the above equations, the numerator coefficients of the reduced model will be obtained.

IV. **Numerical Example**

This section includes an example to illustrate the method.

Example : Consider a third order system described by the transfer function [13]

$$G_3(z) = \frac{[1,2]z^2 + [3,4]z + [8,10]}{[6,6]z^3 + [9,9.5]z^2 + [4.9,5]z + [0.8,0.85]} \tag{17}$$

Second order reduced model are shown in table 3

TABLE III.   PROPOSED METHODS FOR EXAMPLE

Methods	Second order model
$\alpha - \beta$	$\dfrac{[0.2307,0.3352]z + [0.1409,0.4577]}{z^2 + [-0.3336,-0.2403]z + [0.0615,0.2522]}$
α and factor division method	$\dfrac{[6.2565,11.5084]z + [0.579,3.1525]}{z^2 + [-0.3336,-0.2403]z + [0.0615,0.2522]}$
α and Cauer second from	$\dfrac{[-28.5712,-4.5714]z + [0.5791,3.1525]}{z^2 + [-0.3336,-0.2403]z + [0.0615,0.2522]}$
Dominant Pole and Pade approximation [13]	$\dfrac{[0.5921,0.6055]z + [0.8845,0.9]}{z^2 + [0.8041,1.2465]z + [0.1437,0.3805]}$

form. The proposed method of reduced model guarantees the stability if the original system is stable. The method is conceptually simple and it yields comparatively better results than those obtained by existing methods proposed in the literature.

Figure 1.   Comparison of step response (lower limit)

——————   Original System

----------   Proposed model (α-β)

00   Ismail [13]

Figure 2.   Comparison of step response (Upper limit)

——————   Original System

----------   Proposed model (α-β)

00   Ismail [13]

The original and reduced order model is compared by the step response as shown in Fig1 and Fig2. From the figure we can conclude that the proposed model can give accurate response compared with existing method [13]

V.   **Conclusions**

In this paper mixed methods are employed for order reduction. The denominator polynomial of reduced model is obtained by using α table. The numerator polynomial is obtained by β table, factor division method and Cauer second

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[15]   Singh VP and Chandra, Reduction of discrete interval system using clustering poles with Pade approximation: a computer aided approach. Int. J. of Engineering, Science and Technology,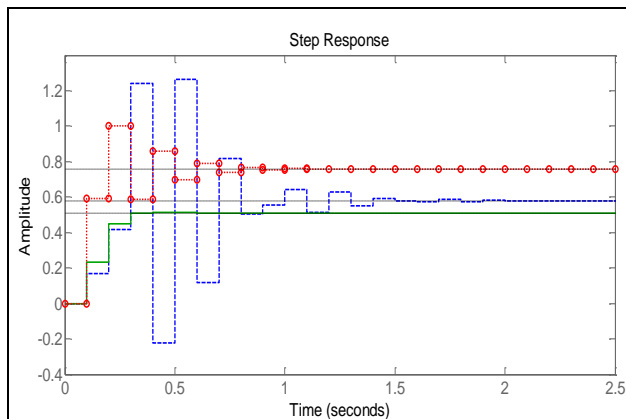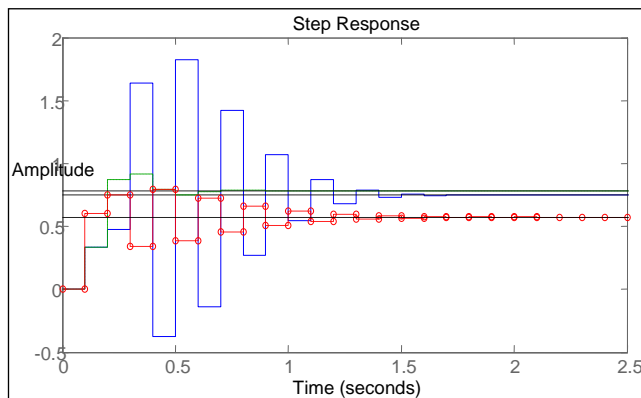 vol.4, no. 1 , pp.97-105, 2012.