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Adding an Extra Link Reduce Travel Time in Road Networks – Myth or Fact?

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Abstract—**Any real life situation where there are nodes and edges, can be modeled as graphs. Road networks are one such example where roads can be modeled as edges and nodes can be modeled as cities. Such a model can be used to analyze and optimize various processes that take place in a road network. These models are commonly used to optimize the distance or time of travel between cities. In this study a model is developed where the speed of a vehicle is a linear function of the traffic volume. This model can be used identify locations of possible traffic congestion. When traffic congestion is observed in a road network traffic planners tend to add extra links to divert some of the traffic. This act which is done with the good intention sometimes worsens the situation. Such a situation can occur in a road network where there are two parallel roads between two cities and the two roads are connected in between by one or more links. This paper explores the criteria under which this problem can arise and suggest recommendations. A simulation run done with hypothetical data in a road network between Piliyandala and Horana in Sri Lanka is used in the study to illustrate the problem.**

***Keywords*—optimization, road network, Braess paradox, traffic engineering, transport algorithm.**

I. Introduction

Graph theory is generally used in modeling road networks. Cities are modeled as nodes and the roads are modeled as edges. Nodes generally have labels and edges are model with labels and weights. Names of the cities are modeled as labels of nodes and road classification and speed are modeled as labels of edges. Time of travel or distance between cities is modeled as weights of edges.

The objective of the road network planner is to minimize the distance between the two cities. However the average road user is more interested in the time of travel than distance between two cities.

There are several algorithms available to calculate the shortest distance between two cities in a road network. One such is Dijkstras algorithm. This algorithm can also be applied to find the shortest time if the speed of the vehicle is uniform.

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Several researchers have done work in finding the shortest time between cities assuming that the speed is independent of the number of vehicles travelling in a given stretch of a road [1]. This is true in cases where the road lengths are long and capacious. However, there are situations where the time taken to travel between two cities depends on the number of vehicles on the road [2]. Some of the conditions that might result in this situation are

- A narrow bridge on the road
- Presence of a city,
- Sudden drop in speed limits
- Poorly maintained road

In such situations the shortest distance algorithm cannot be modified to find the shortest time. The reason is that shortest distance algorithm assumes a constant travel time between two cities; in cooperate of the traffic load.

II. Purpose of this Research

Purpose of this research is to develop a methodology to find the shortest time between two nodes in a road network where the time taken of travel between depends on the number of vehicles on the road. This methodology is then used to evaluate the viability of adding extra links.

A. *Scope of this Research*

Due to complexity of the problem, the time taken to travel between any two cities is assumed to be a linear function of the number of vehicles travelling along a given edge.

Another assumption made in this study is that the drivers are selfish. As such they are keen on optimizing the travel time of the individual and not that of the group.

B. *Braess Paradox*

The general consensus is that when there is congestion, adding an extra link will ease the congestion. However, under certain conditions adding an extra link may worsen the situation. This is known as Braess paradox named after the German engineer who observed this phenomenon. Later in this paper, two conditions under which Braess paradox can occur are discussed in detail [3].

A real life Braess paradox occurred in NewYork in 1990 when the 42nd street was closed on Earth day. Mayor of New York warned public to expect extraordinary traffic jams. “But to everyone's surprise, Earth Day generated no historic traffic jam. Traffic flow actually improved when 42d Street was closed’

III. Methodology

Initially a hypothetical road network of 4 nodes and 5 edges was modeled with simulated data to study the impact of traffic load on travel times.

A. *Road Networks Selected For The Study*

For the purpose of this research two different road networks where Braess Paradox can occur were studied [4]. Both networks have with four nodes and four edges.

In the first scenario, there are two roads (North and South) to go from A to B. On North road there is a bridge *P*, on the South road is a bridge *Q* (Fig 1). Drivers seek to minimize the time to get from A to B. However, they are unable to act independently. The traffic load and the actions of other drivers influence the individual action.

The road sections are so capacious that the travel time on them is independent of the number of cars. Let us assume this to be 15 minutes. The bridges, however, are bottlenecks and the time taken to cross each bridge varies and is proportional to the rate at which cars are crossing it. If *X* is the flow rate (number of cars per hour) across a bridge, then the time to cross the bridge is assumed to be *X*/6000. The problem one needs to solve is, how long it takes to get from A to B, if a steady stream of 1000 cars per hour enters the network at A.

When the conditions are steady, both routes should take the same amount of time. If not, drivers will switch to the route which takes lesser time. It is assumed that the drivers are familiar with traffic conditions and decide on the route to travel accordingly.

Let *X* be the number of cars on the North road, and *Y* be the number of cars on the South road.
Then the time taken to travel on the North road is,

$$X/6000 + 15/60$$

And, time taken to travel on South road is

$$Y/6000 + 15/60$$

At steady state these two times are equal. Hence we get

$$X = Y$$

But

$$X + Y = 1000$$

Therefore

$$X = Y = 500$$

Hence, the time taken is

$$15/60 + (500 * 60/6000) = 20\ mins$$

Now let us assume that a centre road is constructed connecting the two routes as shown in Figure 2. Let us assume that Travel time on this road is 7.5 mins, and that under steady conditions z cars go on this road.
With the addition of this link three routes are available to go from A to B. (Fig 2) North road, South road, the one through bridge P expressway and bridge Q
.
Time taken to travel on the northern road is

$$(X + Z)/6000 + 15/60$$

Time taken on southern road is

$$(Z + Y)/6000 + 15/60$$

Time taken through the expressway

$$(X + Z)/6000 + 7.5/60 + (Z + Y)/6000$$

At steady state these three times are equal,

Also

$$X + Y + Z = 1000$$

Solving these equations gives

$$Z = 500, \quad X = 250 \text{ and } Y = 250$$

Hence time taken is 22.5 min. Which is more than 20, the time without link. So the travel time increased for everybody by 2.5 min.
It should be noted that this situation is not very common. In this problem the decisive factor is the time of travel in the connecting road. The above calculation shows that when the time of travel in the centre road is 7.5 min total travel time for everybody increases by 2.5 min. Now let us analyze the conditions under which this situation may occur.

B. *Mathematical Analysis*

Let us keep flow rate constant at 1000 and vary the time taken on the center road.

Let the number of cars on the North road is X, South road is Y , centre road is Z and the time of travel on the centre road is t. Calculating the total time taken to travel from A to B for values of t varying from 5 to 10 following table is obtained.

TABLE I. TRAVEL TIMES FOR 1000 CARS WITH LINK

x	y	z	t	Total Time
0	0	1000	5	25
100	100	800	6	24
200	200	600	7	23
250	250	500	7.5	22.5
300	300	400	8	22
400	400	200	9	21
500	500	0	10	20

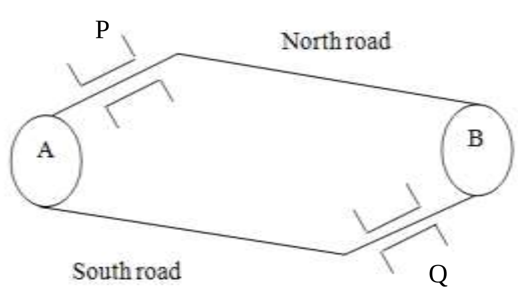

Figure 1. Road Network without link

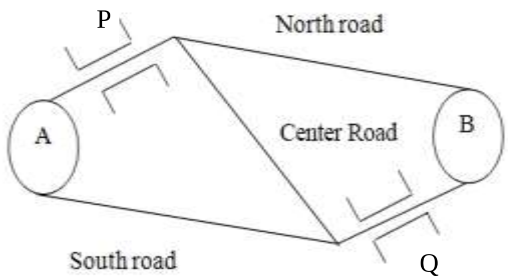

Figure 2. Road Network with link

It can be seen that time taken to travel from A to B is more with link than without link, when the time of travel in the centre road is between 5 and 10 minutes. For all other values time of travel from A to B decreases as a result of adding the extra link.

IV. Second Scenario

A. *Network configuration*

The second scenario consists of four nodes and five edges as shown in Fig 3. Vehicles enter through node A and exit through node D.

Under this set up there are only three paths ABD, ACD and ABCD available to go from A to D. The problem that we are going to address is that whether the link BC helps to reduce the travel time from A to D.Let

X	- Number of cars on an edge
Ax+1	- Time of travel from A to B and C to D
x+B	- Time of travel from A to C and B to D
C	- Time of travel from B to C

Where A, B and C are positive integers. The values of A, B and C depend on the conditions of the road and the number of lanes. These can be calculated from a traffic survey data, whose calculation procedure is outside scope of this paper.

B. *Sample Calculation*

For the purpose of sample calculation the following values were assumed

A=2, B=20, C=10

Travel time between B and C is independent of the number of cars on the link. In real life situation this represents high-speed link.

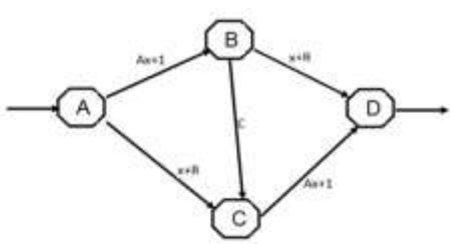

Figure 3. Road Network with five edges and four nodes

Let us consider the situation where there are no cars in the system. When the first driver arrives at point A he has three options ABD, ACD and ABCD to go to point D. When x=0 the time taken to travel in these three paths are 21, 21 and 7 respectively. The driver being selfish will select the path that gives the shortest time. In this case the driver will select ABCD. Similar computation shows that when the second driver arrives at A the time taken for him travel paths ABD, ACD and ABCD are 23, 23 and 11 respectively. As in the previous case second driver select path ABCD. (Refer table II.). When there are seven cars in the system the time taken for the driver to travel paths ABD, ACD and ABCD are 37, 37 and 39 respectively. Now the driver being selfish will select one of the two paths ABD or ACD [5]. Let us assume that the driver select ABD. When the next driver comes time taken on the three paths are 40, 37 and 41 respectively. This driver will select ACD. Table II shows the time taken on three paths for values of x varying from 0 to 11. According to this table it can be seen that when the number of cars in the system is more than 8 the driver will never select the path ABCD. After x goes beyond 8 the least time path sways between ABD and ACD.

Now let us consider the situation where the link BC is not constructed. In this situation the minimum time taken sways between ABD and ACD. Table III shows the times taken on ABD and ACD for different values of x. Comparing table II and table III shows that when the number of cars in the system is 5 the least time taken without link is 27 as well as with link. On the other hand when the number of cars on the system is 6 the minimum time taken is 30 without link and 31 with link. For all values of x>6 the time taken go from A to D is more with link than without link. This means that the general consensus of traffic planners that adding an extra link reduce the travel time between nodes is not true.

C. *Mathematical Analysis*

Let us now establish the condition under which this situation can arise. Assume that there are no cars on the system. When the first driver arrives at the node he will decide one of the paths ABD, ACD or ABCD which is the least time path. If this is not ABCD the driver will never select ABCD. Refer table III. When there are no cars on the system time taken on path ABD is B+1 and ACD is B+1 and ABCD is C+2.

Therefore the driver will never choose ABCD if B-1>C. If this condition is satisfied the driver will never use the link.

Table II Show the optimum path taken by the drivers for different values of x.

TABLE II. OPTIMUM PATHS TAKEN BY THE DRIVERS (WITH LINK)

X	ABD	ACD	ABCD	Min Path
0	21	21	7	ABCD
1	23	23	11	ABCD
2	25	25	15	ABCD
3	27	27	19	ABCD
4	29	29	23	ABCD
5	31	31	27	ABCD
6	33	33	31	ABCD
7	35	35	35	ABCD
8	37	37	39	ABD
9	40	37	41	ACD
10	40	40	43	ABD
11	43	40	45	ACD

D. *Establishing the optimal path*

The step by step procedure used in establishing the minimum time of travel between A and B is given below.

Let us assume that x is number of cars entering the node A. When the conditions are stable the number of cars leaving B is also be x.
Step 1: Assume that $x=0$. ie. There are no cars on the road.
Step 2: Calculate the time taken for the first car to travel from A to B in paths ABD, ACD and ABCD.
Step 3: Assume that the first car take the path where the time is minimum.
Step 4: Assume car number 1 decide to take the path optimum path chosen in step 3.Now go back to step2 and repeat the procedure for car number 2. Repeat this procedure for several cars up to a maximum of about 50 cars.

TABLE III. OPTIMUM PATHS TAKEN BY THE DRIVERS (WITHOUT LINK)

X	ABD	ACD	Min Path
0	21	21	ABD
1	24	21	ACD
2	24	24	ABD
3	27	24	ACD
4	27	27	ABD
5	30	27	ACD
	30	30	ABD
7	33	30	ACD
8	33	33	ABD
9	36	33	ACD
10	36	36	ABD
11	39	36	ACD

E. *Summary of results*

This procedure was repeated for different values of C, keeping the values of A and B constant at 2 and 20 respectively, whose results are presented in Fig. 4.

F. *Discussion*

A general feeling among average road user is that when there are two parallel roads between two given cities addition of a link connecting the two edges reduces the travel time. This is true in many cases. However there are situations where the addition of a link can worsen the situation. In this research an attempt was made to establish the conditions under which this paradox can occur.
When A=2 and $B=20$ and $C=5$ the threshold occurs at $x=6$. Which means if the number of cars on the road is more than 6 travels time is less if the link is not there. Threshold number of cars for selected values of A, B and C are given in table IV.

It should be noted that the threshold number of cars depend on the values of A, B and C. In this research these values were arbitrarily chosen. However, in real life situation these values need to be calculated from the traffic flow data collected on the network.

Travel times with and without link for scenario 2

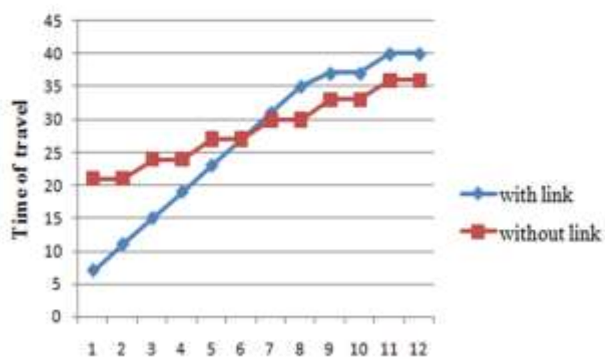

Figure 4. Travel times with and without link when A=2,B=20 and C=5

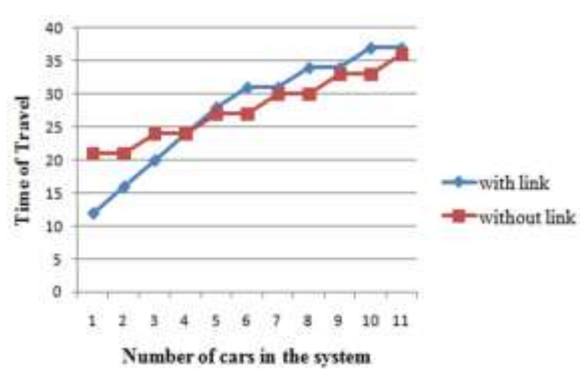

Figure 5. Travel times with and without link when A=2,B=20 and C=10

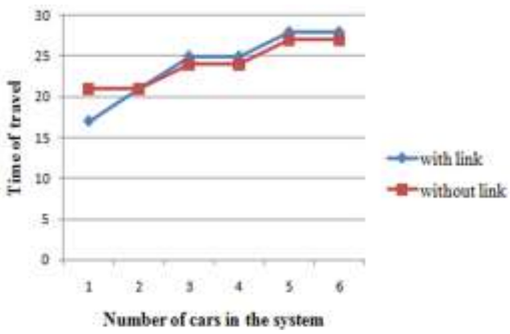

Figure 6. Travel time with and without link when A=2,B=20 and C=15

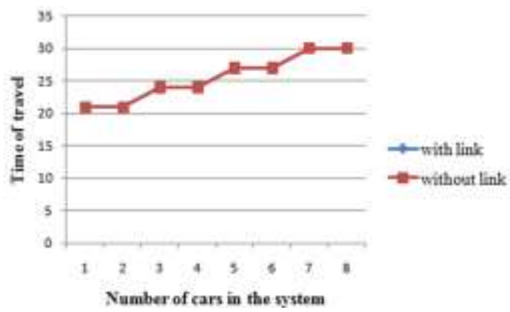

Figure 7. Travel time with and without link whenA=2,B=20, C=40

A real life situation of this condition exists on the Colombo Horana road between Piliyandala and Horana, in Sri Lanka as shown in Fig 8.

Under certain conditions of traffic load and travel time, closing the road from Kesbewa to Bandaragama might decrease the travel time from Piliyandala to Horana.

G. *Conclusion*

In road network design the tendency is to add extra links to ease traffic congestion. However, in situations where the time of travel between two nodes is a function of the number of vehicles on the road adding an extra link can worsen situation.

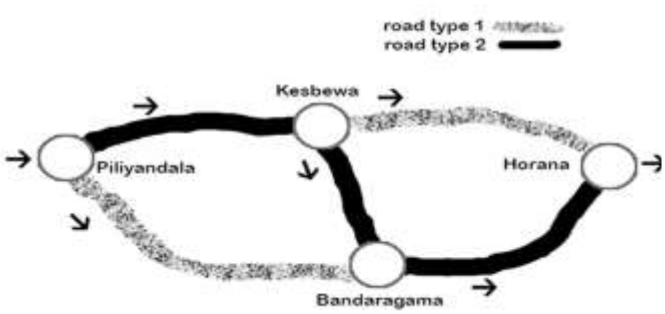

Figure 8. Potential Braess Paradox location

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References

[1] Sara Hartse Sean Colin-Ellerin Comparison of Optimization Techniques for Road Networks 'Supercomputing challenge, final report, Desert Academy, Santa Fe, New Mexico, Apr 2013

[2] Jürg Andreas Stückelberger, A Weighted-Graph Optimization Approach for Automatic Location of Forest Road Networks, A dissertation submitted to the ETH Zurich for the degree of Doctor of Sciences, ISBN 978-3-7281-3217-8 , 2008

[3] Anna Nagurney, David Boyce Preface to "On a Paradox of Traffic Planning"TRANSPORTATION SCIENCE, Vol. 39, No. 4, November 2005, pp. 443–445, issn 0041-1655

[4] J. N. Hagstrom, R. A. Abrams, Characterizing Braess's Paradox for Traffic Networks, Information and Decision Sciences, University of Illinois at Chicago

[5] Victor L. Knoop, Serge P. Hoogendoorn, Bart van Arem A paradox in dynamic traffic assignment Dynamic extension of the Braess paradox Sixth Triennial Symposium on Transportation Analysis, Bangkok, Thailand, June 2007

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