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Robust Optimum Design of Liquid Column Dampers in Seismic Vibration Control

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Abstract- The optimization of liquid column damper (LCD) system considering model parameter uncertainty is usually obtained by minimizing the performance measure obtained by the total probability theory without any consideration to the variation of the performance of LCD system due to parameter uncertainty. However, such a design method does not necessarily correspond to an optimum design in terms of maximum response reduction as well as its minimum dispersion. The present study is focused on robust design optimization (RDO) of liquid column vibration absorber (LCVA) system in seismic vibration control of structure considering uncertain but bounded (UBB) type system parameters. This involves optimization of LCVA parameters allowing uncertainty in the properties of the primary structure as well as ground motion parameters. The RDO is performed by minimizing the weighted sum of the nominal value of the root mean square displacement (rmsd) of the primary structure and its dispersion is minimized. The conventional interval analysis based bounded optimum solution is also obtained to demonstrate the effectiveness of the RDO approach. A numerical study elucidates the effect of parameter uncertainty on the RDO of LCVA parameters by comparing the RDO results with the optimum solution obtained by solving usual interval optimization procedure.

Index terms- Seismic vibration control, liquid column vibration absorber, bounded uncertain parameters, robust optimization.

I. INTRODUCTION

The application of LCD to control the effect of wind and seismic induced vibration effect is quite known [1-3]. In fact, the optimal design of passive control devices like Tuned Mass Dampers (TMD) and LCD is well established [4, 5]. The most commonly used approach of damper parameter optimization is to consider the load like earthquake or wind actions as the only source of randomness. The loads are suitably modelled as a stochastic process in the standard random vibration theory and the stochastic structural optimization (SSO) is performed by considering the structural displacement covariance of the protected system as the performance measure. The approach assumes that all parameters except the load are unaffected by any source of uncertainty. A major limitation of such deterministic assumption is that the uncertainties in the performance-related decision variables cannot be included in the optimization process. But, the complete information about a dynamical system and its environment are never available, the system and excitation is not modeled exactly. Therefore, the design of LCD based on a single nominal model of the system may fail to create a control system that provides satisfactory performance. The efficiency of damper may reduce if the parameters are not properly tuned to the vibrating mode it is designed to suppress due to unavoidable presence of uncertainty in the system parameters. Hence, for an efficient design of damper system, uncertainty associated with excitation as well as modeling of structure should be explicitly taken into account. In recent years, the vibration control problem considering uncertain system parameters has attracted a great deal of interest. The optimal TMD design under uncertain parameters was introduced by Jensen et al. [6]. The RBDO for passive control applications was originally proposed by Papadimitriou et al.[7] in which the unconditional probability of failure of the primary structure is minimized for systems with probabilistic parameters uncertainties and stochastic excitation. May and Beck [8] introduce the concept of robust reliability against failure to serve as an important metric by which the quality of controlled systems can be judged. The unconditional failure probability obtained by the total probability theorem is defined as the robust failure probability. Taflanidis et al. [9] studied robust RBDO of liquid dampers based on the total probability theory concept considering random system parameters under earthquake excitation. Taflanidis et al. [10] presented a theoretical analysis of RBDO for passive or active structural control applications that optimizes a control system explicitly to minimize the probability of failure of structure.

The studies on optimization of damper parameters considering model parameter uncertainty primarily apply the total probability theory concept to obtain the unconditional response or the failure probability of the system which is subsequently used as the performance measure. But, such design approach does not consider the possible dispersion of such performance. Hence, it is important to achieve a balance where an optimum design will also assure less sensitivity with respect to the variations of parameters due to uncertainty, thereby producing robustness in the design. The robustness is generally expressed in terms of the dispersion of performance function from its nominal value. The dispersion is usually measured in terms of the variance and percentile difference [11]. Furthermore, it is of worth mentioning here that though, the probabilistic methods are powerful, the approach cannot be applied in many real situations when the required detailed information on the statistical variations of parameters is unavailable. The maximum possible ranges of variations expressed in terms of percentage of the corresponding nominal values of the parameters are only known and can be only modelled as uncertain but bounded (UBB) type parameters. In such situations, the interval analysis method in the framework of set theoretical description is used [12, 13]. However, the bounded solutions obtained by such approach are the worst case measures and has little importance for practical design. The robust design optimization (RDO) in which the bounds on the magnitude of uncertain parameters are only required will be a viable alternative. The concepts of RDO have been developed independently in different scientific disciplines and the developments in recent past are noteworthy [14-16]. However, there have been a few applications of RDO with respect to reduction of vibration levels of structures. Hwang et al. [17] have minimized the mean and variance of displacement at the first resonance frequency of an automobile mirror system with both stiffness and mass variation. Son and Savage [18] proposed a probabilistic approach of designing vibration absorber parameters to reduce both the mean and variance of the dynamic performance measure over the excitation frequency range. Marano et al. [19, 20] studied the RDO criterion in probabilistic framework for use of TMD in seismic vibration control.

The studies on the RBDO and RDO of TMD parameters optimization considering random system parameters are noteworthy. However, the same is not the case for liquid dampers, except a study by Taflanidis et al. [9] where the model parameter uncertainty is considered in the framework of the total probability theory concept. However, the optimization was performed without any consideration to the variation of response reduction capability of the liquid damper due to uncertainty. It may be realized that such a design method does not necessarily correspond to an optimum design in terms of maximum response reduction and its minimum dispersion. In order to obtain a more realistic optimum design of LCVA parameters to mitigate the vibration effect, a RDO is more desirable which can optimizes a performance index expressed in terms of mean value (the performance index obtained by the so called RBDO) as well as the variability of the performance function due to the presence of system parameter uncertainty. Thereby, a LCVA configuration is achieved so that the final response reduction capability of the system will be less sensitive to the variation of system parameters due to uncertainties. Moreover, in many real life problems, the most and the least conservative estimates (mini-max criteria) may only provide a range of variations. These estimates, though unsuitable for RBDO, can be integrated into a RDO process.

The primary objective of the present study is to propose an RDO procedure to obtain the optimum LCVA parameters to mitigate seismic vibration effect of structures characterized by *UBB* type uncertain parameters. The maximum root mean square displacement (rmsd) of the primary structures is considered as the performance index. The RDO is obtained by using a two-criterion equivalent deterministic optimization problem, where the weighted sum of the nominal value of the performance function and its dispersion, is optimized. The conventional interval analysis based bounded design optimization (BDO) is also performed to demonstrate the effectiveness of the proposed RDO approach. A numerical study is performed to elucidate the effect of parameter uncertainty on RDO of LCVA parameters by comparing the present RDO results with those obtained from the conventional BDO procedures.

II. STOCHASTIC DYNAMIC RESPONSE OF LCVA-STRUCTURE SYSTEM

A. Description of the system

A LCVA is a U-shaped liquid column tube attached to the primary structure. The basic structure of a LCVA is generally modeled as a SDOF system with properties in accordance with the specified mode of vibration required to be controlled. The horizontal and vertical cross sectional area, length of the horizontal portion and density of liquid mass of LCVA are denoted by A_h, A_v, B_h and ρ, respectively. A simplified model of LCVA structure system is shown in figure 1. The system is subjected to random base acceleration $\ddot{z}_b(t)$ due to seismic motion. Ignoring the mass of the container of LCVA (which can be included in the mass of the primary structure), the total mass of the damper system can be expressed as: $m_l = (\rho A_h B_h + 2\rho h A_v)$. The ratio of the total damper mass to that of the structure i.e. mass ratio is denoted by, $\mu = (\rho A_h B_h + 2\rho h A_v)/m_0$. Further, notations introduced are: area ratio, $r = A_V/A_h$, length ratio, $p = B_h / L_e$ where, the total length of liquid column, $L_e = (2h + B_h)$, the liquid frequency, $\omega_l = \sqrt{2g/L_{ee}}$ where, $L_{ee} = B_h r + (L_e - B_h) = L_e[1 + p(r-1)]$ and tuning ratio, $\gamma = \omega_l / \omega_0$.

The structure-damper system is subjected to base acceleration, $\ddot{z}_b(t)$ due to earthquake motion. If $x(t)$ and $y(t)$ represents the horizontal displacement of the SDOF system relative to the ground and the displacement of liquid surface, the equation of motion of the liquid column can be approximated as:

$$\rho A_h L_{ee}\ddot{y}(t) + \frac{1}{2}\rho A_h \xi |\dot{y}(t)| \dot{y}(t) + 2\rho g A_h y(t) = -\rho A_h B_h \{\ddot{x}(t) + \ddot{z}_b(t)\} \quad (1)$$

The constant ξ is the coefficient of head loss controlled by the opening ratio of the orifice typically placed at centre of the horizontal portion of the damper. It can be considered as the overall head loss induced by flow motion in the liquid column, although it is mainly induced by flow passing through the orifice.

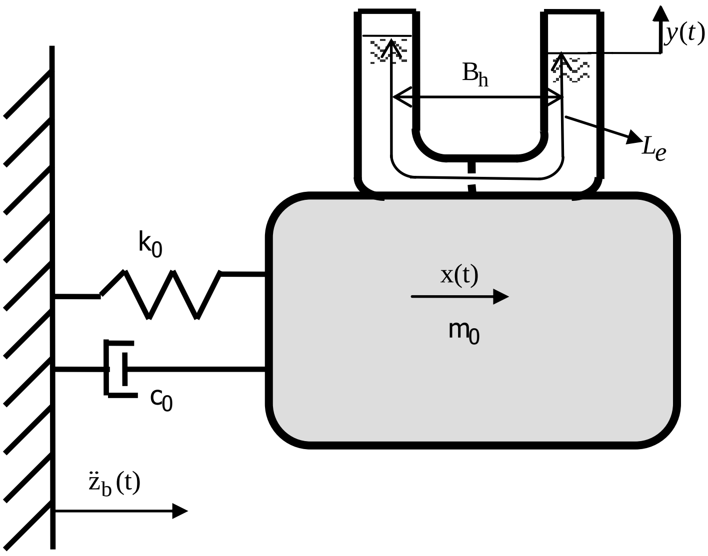

Fig.1: The LCVA-SDOF system

The above equation is non-linear due to the drag-type forces induced by the orifice as indicated by the second term of the left hand side of Eq. 1. Using equivalent linearization techniques above equation can be approximated as [21]:

$$\rho A_h L_{ee}\ddot{y}(t)+2\rho A_h c_p \dot{y}(t)+2\rho A_h g y(t)=-\rho A_h B_h\left\{\ddot{x}(t)+\ddot{z}_b(t)\right\} \qquad (2)$$

Here, c_p represents the equivalent linearization damping co-efficient and can be expressed as [1],

$$c_p=\frac{\dot{\sigma}_y \xi r^2}{\sqrt{2\pi}} \qquad (3)$$

Where, $\dot{\sigma}_y$ is the standard deviation of the liquid velocity. From Eq. 3, it can be noted that c_p depends on the response of the liquid, $\dot{\sigma}_y$ which is not known a priori. Thus, an iterative solution procedure is required. Normalizing Eq. 2 with respect to mass of the liquid in the container ($\rho A_h L_{ee}$) gives:

$$\ddot{y}(t)+\frac{2c_p}{L_{ee}}\dot{y}(t)+\frac{2g}{L_{ee}}y(t)+p\frac{L_e}{L_{ee}}\ddot{x}(t)=-p\frac{L_e}{L_{ee}}\ddot{z}_b(t) \qquad (4)$$

2.2 Structure motion equation

The vibration of SDOF system (as shown in Fig. 1) having mass of m_0, stiffness k_0 and structural damping c_0 (damping ratio of ξ_0) is to be reduced using LCVA. The equation of motion of the primary structure with LCVA can be written as,

$$\{m_0+m_l\}\ddot{x}(t)+c_0\dot{x}(t)+k_0x(t)=-\{m_0+m_l\}\ddot{z}_b(t)-\rho B_h r A_h \ddot{y}(t) \qquad (5)$$

Normalization of Eq. 5 with respect to mass, m_0 leads to

$$\{1+\mu\}\ddot{x}(t)+2\xi_0\omega_0\dot{x}(t)+\omega_0^2 x+\mu\frac{pL_e}{L_{em}}\ddot{y}(t)=-\{1+\mu\}\ddot{z}_b(t) \qquad (6)$$

Where, $L_{em}=(B_h/r+2h)$. Now, rewriting Eqs. 4 and 6 in matrix form yields:

$$\mathbf{M}\ddot{\mathbf{Y}}(t)+\mathbf{C}\dot{\mathbf{Y}}(t)+\mathbf{K}\mathbf{Y}(t)=-\mathbf{M}\bar{\mathbf{r}}\ddot{z}_b \qquad (7)$$

In which **M**, **C**, **K** represents the mass, damping and stiffness matrix of combined system defined as,

$$\mathbf{M}=\begin{bmatrix}1 & pL_e/L_{ee}\\ \mu pL_e/L_{em} & (1+\mu)\end{bmatrix},\mathbf{K}=\begin{bmatrix}2c_p/L_{ee} & 0\\ 0 & 2\xi_0\omega_0\end{bmatrix}\text{ and }\mathbf{C}=\begin{bmatrix}2g/L_{ee} & 0\\ 0 & \omega_0^2\end{bmatrix} \qquad (8)$$

And $\mathbf{Y}=[y,x]^T$ is relative displacement vector and $\bar{\mathbf{r}}=[0\ 1]^T$. Introducing the state space vector, $\mathbf{Y}_s=(y,x,\dot{y},\dot{x})^T$, Eq. 7 can be written in state space form as [23]:

$$\dot{\mathbf{Y}}_s=\mathbf{A}_s\mathbf{Y}_s+\mathbf{r}\ddot{z}_b(t) \qquad (9)$$

Where, $\mathbf{A}_s=\begin{bmatrix}0 & \mathrm{I}\\ \mathbf{H}_k & \mathbf{H}_c\end{bmatrix}$ is the structural system matrix, $\mathbf{r}=[0,0,1,1]^T$, I and 0 is the 2x2 unit and null matrices, respectively and $\mathbf{H}_k=\mathbf{M}^{-1}\mathbf{K}$ and $\mathbf{H}_c=\mathbf{M}^{-1}\mathbf{C}$

B. Response Covariance Analysis

The load represents the random seismic acceleration $\ddot{z}_b(t)$ that excites the system at base of the primary structure. A widely adopted model in stationary case for $\ddot{z}_b(t)$ is obtained by filtering a white noise process, acting at the bed rock, through a linear filter which represents the surface ground. This is the well known Kanai-Tajimi stochastic process [22] which is able to characterize the input frequency content for a wide range of practical situations. The process of excitation at the base can be described as:

$$\begin{aligned}\ddot{x}_f(t)+2\xi_f\omega_f\dot{x}_f+\omega_f^2x_f&=-W(t)\\ \ddot{z}(t)=\ddot{x}_f(t)+W(t)&=2\xi_f\omega_f\dot{x}_f+\omega_f^2x_f\end{aligned} \qquad (10)$$

Where, $W(t)$ is a stationary Gaussian zero mean white noise process, representing the excitation at the bed rock, ω_f is the base filter frequency and ξ_f is the filter or ground damping.

The global state space vector is defined as:

$$\mathbf{Z}=[y,x,x_f,\dot{y},\dot{x},\dot{x}_f]^T \qquad (11)$$

Using above notations, Eqs. 9 and 10 leads to an algebraic matrix equation of order six i.e. the so called Lyapunov equation [23]:

$$\mathbf{AR}+\mathbf{RA}^T+\mathbf{B}=0 \qquad (12)$$

Where, the state space matrix **A** and **B** are as following:

$$\mathbf{A}=\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -m_{11}\frac{2g}{L_{ee}} & m_{12}\omega_0^2 & 0 & -m_{12}\frac{2c_p}{L_{ee}} & m_{12}2\xi_0\omega_0 & 0 \\ m_{21}\frac{2g}{L_{ee}} & -m_{22}\omega_0^2 & +\omega_f^2 & m_{21}\frac{2c_p}{L_{ee}} & -m_{22}2\xi_0\omega_0 & +2\xi_0\omega_0 \\ 0 & 0 & -\omega_f^2 & 0 & 0 & -2\xi_f\omega_f \end{bmatrix} \quad (13)$$

$$\text{where, } m_{11}=\frac{1+\mu}{d_e},\ m_{21}=\frac{\mu p\frac{L_e}{L_{em}}}{d_e},\ m_{12}=\frac{p\frac{L_e}{L_{ee}}}{d_e}, m_{22}=\frac{1}{d_e}, d_e=(1+\mu)-\mu p^2\frac{L_e}{L_{ee}}\frac{L_e}{L_{em}}$$

and

$$\mathbf{B}=\begin{bmatrix} 0 & \cdots & \cdots & \cdots & \cdots & 0 \\ \vdots & \cdots & \cdots & \cdots & \cdots & \vdots \\ \vdots & \cdots & \cdots & \cdots & \cdots & \vdots \\ \vdots & \cdots & \cdots & \cdots & \cdots & \vdots \\ \vdots & \cdots & \cdots & \cdots & \cdots & \vdots \\ 0 & \cdots & \cdots & \cdots & \cdots & 2\pi S_0 \end{bmatrix} \quad (14)$$

The state space covariance matrix $\mathbf{R}$ is obtained as the solution of the Lyapunov equation. The state space covariance matrix of size 4x4 is represented as:

$$\mathbf{R}=\begin{bmatrix} \mathbf{R}_{zz} & \mathbf{R}_{z\dot{z}} \\ \mathbf{R}_{\dot{z}z} & \mathbf{R}_{\dot{z}\dot{z}} \end{bmatrix} \quad (15)$$

So that $R_{zz}, R_{z\dot{z}}, R_{\dot{z}z}$ and $R_{\dot{z}\dot{z}}$ are the sub-matrices of $\mathbf{R}$. The root mean square of displacement (rmsd) of liquid and the primary structure can be obtained as:

$$\sigma_y=\sqrt{R_{ZZ}(1,1)} \quad \text{and} \quad \sigma_x=\sqrt{R_{ZZ}(2,2)} \quad (16)$$

III. OPTIMIZATION OF LCVA PARAMETERS

The optimum LCVA parameters are obtained by minimizing the vibration effect of a structure under dynamic load. The problem of optimization of the LCVA system of protection requires to determine the tuning ratio (γ) and coefficient of head loss (ξ) of the damper system. The design vector (DV) can be thus defined as: $\bar{b}=(\gamma\ \ \xi)^T$. The SSO problem under random earthquake load can be formulated as the search of a suitable set of DVs, over a possible admissible domain $\mathbf{\Omega}$ to minimize desired objective. For stochastically excited structures, a tractable measure of performance can be given in terms of mean square responses (displacement, acceleration, stress etc.). The failure probability of the structure or the total life-cycle cost of the structure can be also used as the performance index. In the present study, the rmsd of the primary structure is considered as the objective function. The SSO problem so defined leads to a standard nonlinear programming problem [24]:

$$\text{Find } \bar{b}\in\mathbf{\Omega} \text{ to minimze, } f=\sigma_x \quad (17)$$

IV. PARAMETER UNCERTAINTY AND OPTIMUM LCVA SYSTEM

It can be noted that the matrix **A** and **B** as described by Eqs. 13 and 14 are functions of various system parameters characterizing the primary structures and the stochastic load. This includes the properties of primary structure and ground motion model parameters. The response statistic evaluated under stochastic earthquake load to solve the SSO problem as described by Eq. 17 intuitively assumes that these parameters are completely known. However, the uncertainties in these system parameters may lead to unexpected excursion of responses affecting the desired safety of structure [25-26]. Thus, in the design of optimum LCVA parameters, apart from the stochastic nature of earthquake**,** the uncertainty with regard to these parameters, expected to have influences on the optimization results should be taken into account. This will involve sensitivity analysis of stochastic dynamic system. In the present section related formulations are briefly presented.

A. Uncertain Parameter Model and Response Sensitivity

In many cases, even though some experimental data are available about the system parameters, it is not enough to construct the probability density function reliably. The available data can be used, particularly in combination with engineering experience, to set some tolerances or bounds on uncertainties. If $\bar{x}_i$ is the nominal value of the ith UBB parameter viewed as the mean value and $\pm\Delta x_i$ represents the maximum deviation from the nominal value, then the UBB parameter value deviates from the nominal value can be expressed as,

$$x_i^I=[x_i^l,x_i^u]=[\bar{x}_i-\Delta x_i,\bar{x}_i+\Delta x_i]=\bar{x}_i+\Delta x_i[-1,1]=\bar{x}_i+\Delta x_i e_\Delta$$
$$\text{where, } \bar{x}_i=\frac{x_i^l+x_i^u}{2}, e_\Delta=[-1,\ 1] \quad (18)$$

Thus, the ith interval variable can be written as: $x_i=\bar{x}_i+\delta x_i$, where, $|\delta x_i|\le\Delta x_i, i=1,2,..,m$.

The system matrix **A** and **B** as well as the response covariance matrix **R** can be expanded with respect to `m` numbers of such UBB parameters in Taylor series about the nominal values in first order terms of δx_i as,

$$\mathbf{A}=\bar{\mathbf{A}}+\sum_{i=1}^{m}\frac{\partial\mathbf{A}}{\partial x_i}\delta x_i+.., \mathbf{B}=\bar{\mathbf{B}}+\sum_{i=1}^{m}\frac{\partial\mathbf{B}}{\partial x_i}\delta x_i+..$$
$$\mathbf{R}=\bar{\mathbf{R}}+\sum_{i=1}^{m}\frac{\partial\mathbf{R}}{\partial x_i}\delta x_i+.., \delta x_i\in[-\Delta x_i,\Delta x_i], i=1,2,..,m \quad (19)$$

In the above, the over bar represents the matrices correspond to the nominal values of the UBB parameters. The derivatives are evaluated at the nominal value of the model parameters i.e. at $x_i=\bar{x}_i$. To avoid the complicacy in presentation, the notations of various derivatives are not explicitly mention. However, all the first order derivatives used in the text means that those are evaluated at the mean point of the associated uncertain variable.

Substituting Eq. 19 in Eq. 12 for ith UBB parameters and equating the equal order term after neglecting the higher order term the following can be readily obtained as:

$$\bar{\mathbf{A}}\bar{\mathbf{R}} + \bar{\mathbf{R}}\bar{\mathbf{A}}^T + \bar{\mathbf{B}} = \mathbf{0} \qquad (20)$$

$$\bar{\mathbf{A}}\frac{\partial \mathbf{R}}{\partial x_i} + \frac{\partial \mathbf{R}}{\partial x_i}\bar{\mathbf{A}}^T + \mathbf{B}^I = \mathbf{0} \qquad (21)$$

where, $\mathbf{B}^I = \frac{\partial \mathbf{A}}{\partial x}\bar{\mathbf{R}} + \bar{\mathbf{R}}\frac{\partial \mathbf{A}^T}{\partial x}$

The mean covariance matrix $\bar{\mathbf{R}}$ is readily obtained by solving Eq. 20 considering the mean value of system parameter matrix $\bar{\mathbf{A}}$ and $\bar{\mathbf{B}}$ obtained by substituting the mean value of the model parameters. The first order sensitivities of the covariance matrix $\frac{\partial \mathbf{R}}{\partial x_i}$ can be obtained by solving Eq. 21. It may be noted that the equation needs to be solved for each uncertain parameter involved in the problem.

B. Bounded Optimization of LCVA Parameters

The performance function i.e. the rmsd as defined by Eq. 17 is also a function of the uncertain parameters and can be expanded in first order Taylor series as the mean and fluctuating part as following:

$$\sigma_x = \bar{\sigma}_x + \sum_{i=1}^{m}\frac{\partial \sigma}{\partial x_i}\delta x_i + ... \qquad (22)$$

In above, $\bar{\sigma}_x$ is obtained by using the solution of Eq. 17 in Eq. 16. The sensitivity of rmsd can be obtained by differentiating the appropriate expression of Eq. 14 with respect to the ith UBB parameters as following:

$$\frac{\partial \sigma_x}{\partial x_i} = \frac{1}{2}\left[\frac{\partial \mathbf{R}(2,2)}{\partial x} / \sqrt{\mathbf{R}(2,2)}\right] \qquad (23)$$

In which, $\partial \mathbf{R}/\partial x_i$ is obtained by solving Eq. 21. Now, by making use of interval extension in interval mathematics assuming monotonic responses, the interval extension of the above expression can be obtained as,

$$\sigma_x^I(\mathbf{X}) = \bar{\sigma}_x(\mathbf{X}) + \sum_{i=1}^{m}\frac{\partial \sigma_x}{\partial x_i}\Delta x_i e_\Delta + .. \qquad (24)$$

The interval region of the function involving the UBB variables can be then separated out to the upper and lower bound as below:

$$\sigma_x^u = \bar{\sigma}_x + \sum_{i=1}^{m}\frac{\partial \sigma_x}{\partial x_i}\Delta x_i + .. \text{ and}$$
$$\sigma_{xl} = \bar{\sigma}_x - \sum_{i=1}^{m}\frac{\partial \sigma_x}{\partial x_i}\Delta x_i + .. \qquad (25)$$

The optimization problem now involves two separate objective function yielding the upper and lower bound solutions. It can be noted that the rational approximation of the function is a linear function of the bounded uncertainties and each such variable appear once in the expression and the interval solution obtained is unique.

C. Robust Optimization of LCVA Parameters

The RDO can be considered as a design procedure that is insensitive (or less sensitive) to the changes in the input variables within the prescribed ranges of interest. It also ensure specified safety if sufficient information about the uncertain inputs are available or satisfy certain indirect measures of safety in the absence of sufficient statistical information.

The robustness of performance is generally expressed in terms of the dispersion of performance function, Δf from its nominal value, $\bar{f}$. The performance of the design is characterized by a function $f(\mathbf{x})$, $\mathbf{x}$ composed of the design parameters. Using first-order Taylor series expansion about $\bar{\mathbf{x}}$; $\bar{f}$ and Δf can be approximated as [27]:

$$\bar{f} = f(\bar{\mathbf{u}}) \qquad \Delta f = \sum_{i=1}^{N}\left|\frac{\partial f}{\partial u_i}\right|\Delta u_i \qquad (26)$$

The dispersion Δf can be visualized as a gradient index, which is a function of the gradients of the performance function with respect to uncertain variables. The objective of an ideal design is to achieve the optimal performance as well as less sensitivity of the performance with respect to the variation of system parameters. Thus, one needs to minimize the performance as well as its dispersion. The two criteria often conflict with each other. The problem is dealt as a multi-objective optimization, where the conventional objective function and its dispersion are two objectives that need to be optimized. Thus, the RDO problem is stated as the minimization of the mean and variance of the objective function as well as its variance, leading to a two criteria RDO problem, expressed as: find $\mathbf{x}$, to min. $\{\bar{f}, \Delta f\}$. The two-criterion optimization problem is transformed to minimization of an equivalent single objective as:

$$f = \alpha * \sigma_{mean} + (1-\alpha) * \Delta\sigma \qquad (27)$$

Where, α is a weighting factor in the bi-objective optimization problem. The maximum robustness will be achieved for α =1.0, and α = 0 indicates optimization without any consideration for robustness. In Eq. 27, $\bar{f}^*$ and Δf^* are the optimal solutions at two ideal situations obtained for, $\alpha = 1.0$ and 0.0, respectively.

V. NUMERICAL STUDY

A SDOF primary system with an attached LCVA as shown in Fig.1 is undertaken to elucidate the proposed RDO of LCVA system in seismic vibration control of structure characterized by UBB type system parameters. The uncertainties are considered in. $\omega_0, \xi_0, \omega_f, \xi_f$ and S_0. Uncertainty of such ith parameter, x_i is described by Δx_i, representing the maximum possible dispersion expressed in terms of the percentage of corresponding nominal value $(\bar{x}_i)$. The PSD of white noise process, S_0 is related to the standard deviation $\ddot{\sigma}_z$ of ground acceleration by [28]: $S_0 = \frac{2\xi_f \sigma_{\ddot{z}_b}^2}{\pi\left(1+4\xi_f^2\right)\omega_f}$. Unless mentioned otherwise, the following nominal values are assumed in present numerical study: $T = 2\sec$, $\xi_0 = 1\%$, μ=3%, p=0.7, r=1.5, PGA=0.2g, Δx_i =10% and it is assumed that $PGA = 3\ddot{\sigma}_z$. Based on this, the rmsd of the unprotected system i.e. without LCVA is computed to be 11.08 cm.

The rmsd of the primary structures is optimized by the RDO procedure as proposed by Eq. 27. The optimum mean value of the performance function i.e. the rmsd of the structure versus mass ratio is plotted in Fig. 2 for different settings of weight factor α. The rmsd results are normalized with respect to the rmsd of unprotected structure for convenience to study the nature of variation of performance of LCVA for varying degree of robustness imposed on the design. The associated dispersion of the rmsd of the primary structures is shown in Fig. 3. The corresponding optimum tuning ratio and head loss coefficient are shown in Figs. 4 and 5, respectively. The BDO results are shown in the same plot for ease in comparison with the present RDO results. The lower bound solution though efficient in terms of response reduction; the associated dispersion of the design is more. The efficiency of the RDO solution is less compare to that of the lower bound solution and lies in between these two bounded solution. However, the dispersion of the design is much lower than the dispersion of the lower bound case solution. The RDO solution show the tendency to give greater damping values in comparison to those required in the conventional BDO approach. Moreover, one can notice that the damping ratio varies in more ample ranges. The tendency is more marked when α decreases.

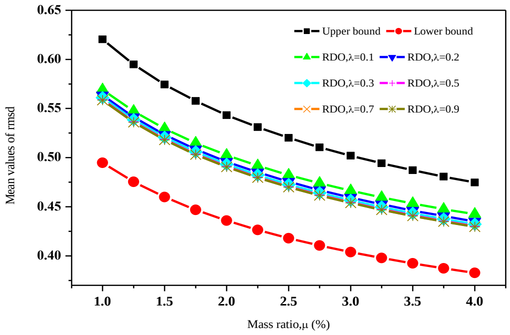

Fig.2: The variation of the mean value of the rmsd of the primary structure with varying mass ratio.

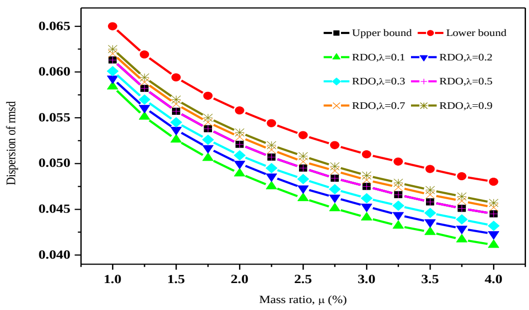

Fig.3: The variation of the dispersion of rmsd of the primary structure with varying mass ratio.

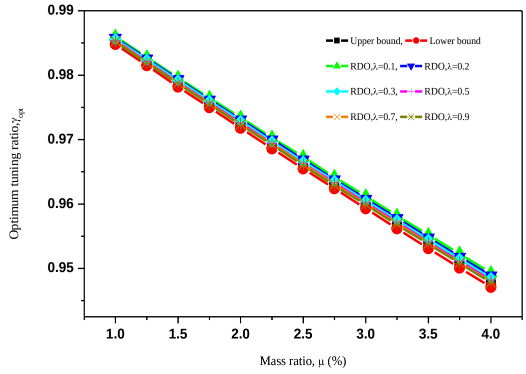

Fig.4: The optimum tuning ratio with increasing mass ratio

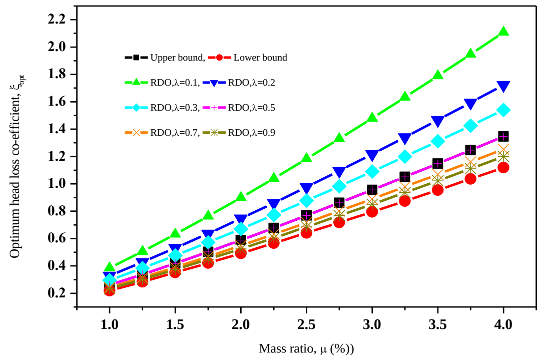

Fig.5: The optimum head loss coefficient with increasing mass ratio

The normalized mean value of the rmsd of the primary structure versus uncertainty range is plotted in Fig. 6 for different value of weight factor α. The associated normalized dispersion of the rmsd of the primary structures is shown in Fig. 7.

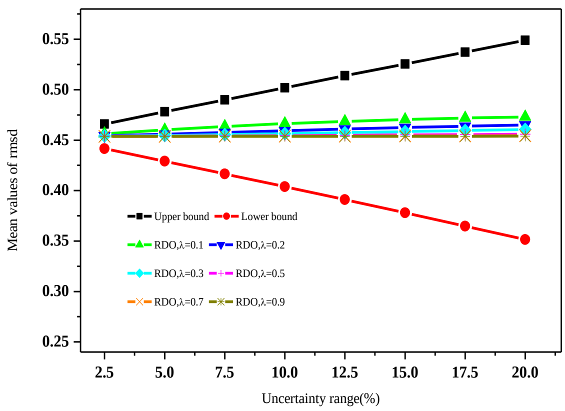

Fig. 6: The variation of mean value of the rmsd of the primary structure with varying uncertainty range.

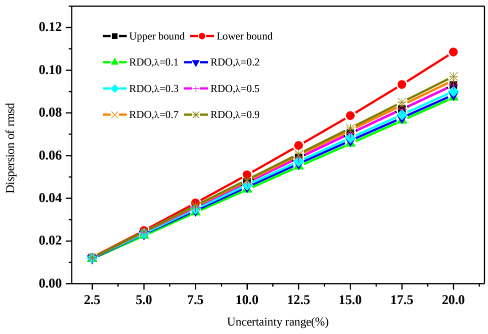

Fig. 7: The variation of dispersion of the rmsd of the primary structure with varying uncertainty range.

The corresponding optimum tuning ratio and head loss coefficient are shown in Figs. 8 and 9, respectively. The width of the bounded solution increases sharply with increasing level of uncertainty. However the change of the optimum rmsd is nominal by the proposed RDO case. As expected, the dispersion of the rmsd value increases with increasing level of uncertainty for all robust design cases i.e. for all settings of α. However, the dispersion of design is much smaller than the dispersion of lower bound case irrespective of level of uncertainty. The change in the head loss coefficient as shown in Fig. 9 with increasing level of uncertainty is e notable. To study the sensitivity of various parameters involved in the RDO procedure, further results are developed.

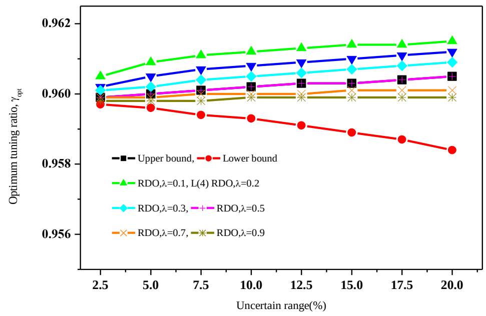

Fig. 8: The optimum tuning ratio with increasing level of uncertainty

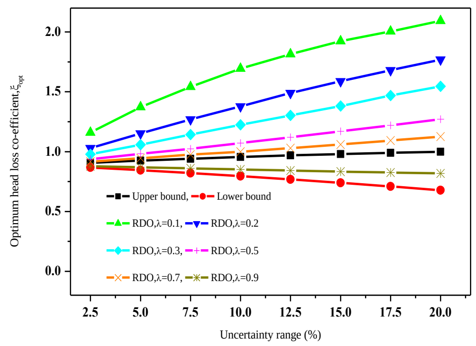

Fig. 9: The optimum head loss coefficient with increasing level of uncertainty.

The normalized rmsd of the primary structure and its dispersion with varying length ratio are shown in Figs. 10 and 11 and in Figs. 12 and 13 for varying time period. Further similar results are shown in Figs. 14 and 15 for varying area ratio. It may be observed from these plots that without affecting the efficiency of the LCVA system of protection much, the dispersion of the performance can be reasonably reduced. Thereby, improved robustness in achieved in the design by the RDO approach. The results show that the trends of RDO results remain same over wide range of length ratio and area ratio.

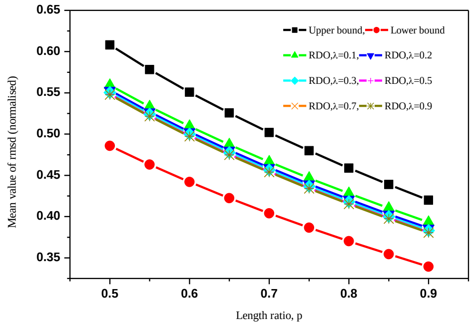

Fig. 10: The variation of the mean value of the rmsd of primary structure with varying length ratio

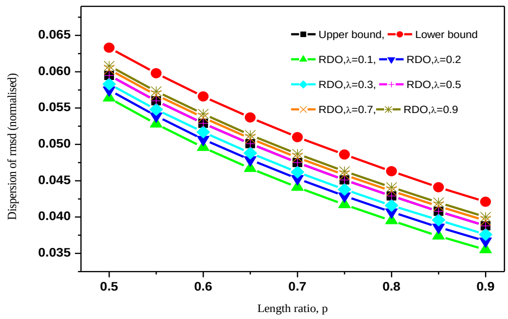

Fig. 11: The variation of the dispersion of the maximum rmsd of the primary structure with varying length ratio

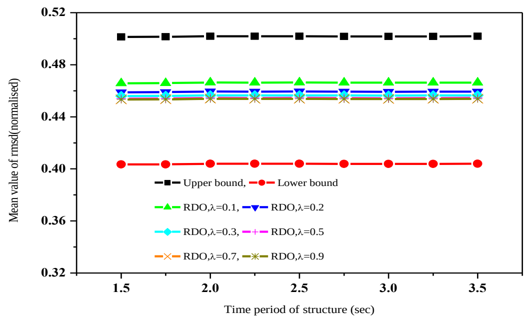

Fig. 12: The variation of the mean value of rmsd of the primary structure with varying time period

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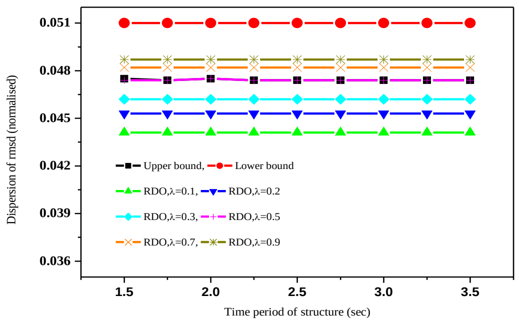

Fig. 13: The variation of the dispersion of the maximum rmsd of the primary structure with varying time period

The normalized rmsd of the primary structures and its dispersion with area ratios are plotted in figures 14 and 15, respectively for different values of the weight factor λ. The results of BDO procedure are also shown in the same plot.

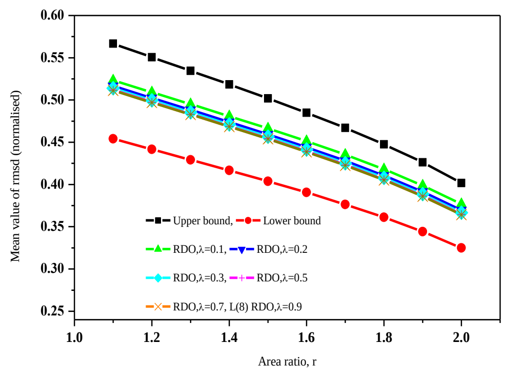

Fig. 14: The variation of the mean value of rmsd of the primary structure with increasing area ratio

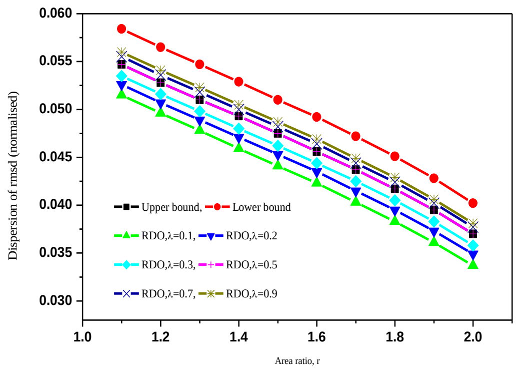

Fig. 15: The variation of the dispersion of the maximum rmsd of the primary structure with increasing area ratio

To study the trade-off between the objective values of a design and its robustness, the Pareto front is generated by solving the RDO by varying the weight factor α and the results are plotted in Fig. 16 for different mass ratio. The uncertainty range for all variables and parameters are taken as 10% of the nominal values. It can be observed from the plot that more robustness is achieved at the cost of sacrificing the optimum weight. The corresponding optimum parameters are shown in Figs. 17 and 18, respectively.

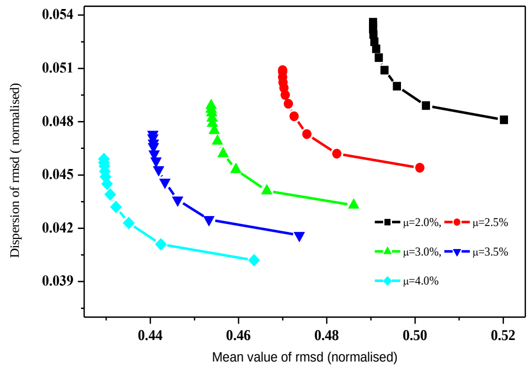

Fig. 16: The Pareto front of the RDO of the LCVA system

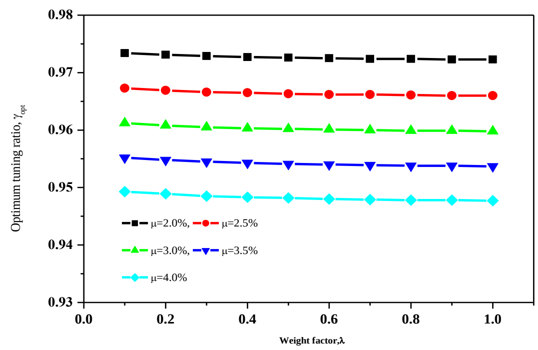

Fig. 17: The variation of optimum tuning ratio with varying weight factor λ

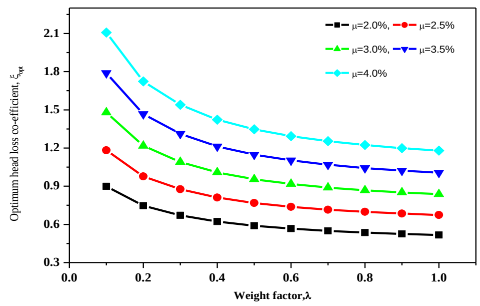

Fig. 18: The variation of optimum head loss coefficient with varying weight factor λ

6. CONCLUSIONS

The RDO of LCVA system to mitigate the seismic vibration effect of structures characterized by UBB type system parameters is studied in the present work. The usual BDO solutions are too far apart and conservative upper bound solution usually suggested is of little use for practical design application. Moreover, such approach fails to provide information about the possible dispersion of the design performance. But, the RDO approach provides the necessary flexibility to the designer to achieve the desired level of performance efficiency (i. e. reduction of vibration level) and its dispersion under uncertain environment through suitable choice of weight factor. It is generally observed that more robustness is achieved at the cost of sacrificing the optimum weight, an obvious characteristic of results obtained from any multi-objective optimization problem. Though, the efficiency of RDO solution is comparatively less compare to that of the lower bound solution, the dispersion of the design is much lower than the dispersion of the lower bound case solution. It may be noted that the approach being generic in nature, can be applied for robust optimum design of LCVA for vibration control of more complex MDOF system, which needs further study.

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