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The New Approach of Design Robust Stability for Linear Control System

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***Abstract* - We propose an approach to design of robust stability of dynamic control systems with one input and one output signals. Design of robust stability of control system is based upon construction A.M. Lyapunov's function. The system states the method of construction of A.M. Lyapunov's function on the basis of geometric interpretation of Lyapunov theorems concerning asymptotic stability and concepts of dynamic systems stability. It is supposed that undisturbed motion of system corresponds to origin of coordinates, and equations of state are comprised as regard to perturbances, i.e. in deviations of perturbational motion. Hence, equations of state evaluate the rate of change of perturbance (disturbance) vector and in stable system is directed to origin of coordinates. Gradient vector from Lyapunov required function is directed to the side opposite.**

Construction of Lyapunov vector function and development of robust stability of dynamic control system with undetermined parameters is based on conception of A.M. Lyapunov's direct method. Stability region comes as simplest in equations by certain parameters of controlled object and selected controller's parameters.

***Keywords* - Control systems, robust stability, superstability, Lyapunov's direct method, modelling, simulation.**

Introduction

Control system design is one of the main tasks in automation of all branches of industry, including machine manufacturing, energy sector, electronics, chemical and biological, metallurgical, textile, transportation, robotics, aviation, space systems, high-precision military systems, etc.

Robust stability can be viewed as one of the outstanding issues in control theory, which is also of a great practical interest. Assuming that the linear system is controllable, a sufficient condition is proposed to preserve the properties of object (parameters of control systems) when system uncertainties are introduced. The most important idea in the study of robust stability is to specify constraints for changes in control system parameters that preserve stability. For the purpose of studying the system dynamics and their control, we considered models of observing input and output signals of the object and the representing its behavior in the state space as most suitable.

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This paper presents the approach of the construction of Lyapunov functions based on the geometric interpretation of the Lyapunov's direct method (also called the second method of Lyapunov) and on gradient of dynamical systems in the state space of systems.

The content of this paper is organized in next way: In section 2, we introduce the basic equations of the model and their expanded form and received the Lyapunov function, geometric interpretation, gradient vector components and super stability condition of system. In section 3, we have considered the existence and robust stability, super stability of nominal system and define condition of robust stability. In Section 4 considered a case study of conditions with the simulation practical example.

II. Mathematical model formulation

Lyapunov conception of direct method is universal for development of dynamic systems stability [1,2]. Widespread application of concepts of this method is refrained by lack of general method to selection or construction of Lyapunov functions and difficulties of their algorithmic representation. In many cases real objects function in conditions of various degree of uncertainty. Upon that uncertainty can be defined by lack of knowledge of true values of control objects parameters and their unpredictable temporal variations.

That is why robust stability plays a significant part in dynamic objects control theory. In general definition development of robust stability consists in determination of restrictions to variation of control system uncertain parameters upon which stability is preserved. These restrictions are defined by stability region in uncertain and selected parameters.

Considerable number of works is devoted to development of control system robust stability. These works [3,4] mainly study robust stability of polynomial and matrix within the frames of linear development concept of stability of continuous and sampled-data control systems.

The present work offers method to Lyapunov function construction by antigradient of desired vector function [5], and all candidates of antigradient vector are given by vector of state. Stability region is shown as the simplest in equations by certain parameters of controlled objects and selected regulator's parameters. Development of system robust stability is based upon concept of A.M. Lyapunov's direct approach [2].

Let completed time-invariant control system be described by equation of state

$$\frac{dx}{dt} = Ax + bu, x \in R^n, u \in R^1 \tag{1}$$

where

$$A = \begin{Vmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -a_n & -a_{n-1} & -a_{n-2} & \cdots & a_1 \end{Vmatrix}, \quad b = \begin{Vmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{Vmatrix}, \quad x = \begin{Vmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{Vmatrix}.$$

Control law is given by scalar function:

$$u(t) = -k^T x(t) \tag{2}$$

where $k^T = \|k_1 \quad k_2, \quad \cdots, \quad k_n\|$ – matrix of dimensionality control ratio *1xn*. Then the system (1) in explicit form can be presented as:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \vdots \\ \dot{x}_{n-1} = x_n \\ \dot{x}_n = -(a_n + k_1)x_1 - (a_{n-1} + k_2)x_2 - \dots - (a_1 + k_n)x_n \end{cases} \tag{3}$$

As an instrument for development of system stability (3) we apply basic provisions of Lyapunov's direct method [2], for asymptotic stability of system balance state it is necessary and enough that there is positive Lyapunov's function *V(x)* in such form that total derivative with time along the solution of differential equation of state (3) is negative definite function, i.e.

$$\dot{V}(x) = \frac{\partial V(x)}{\partial x} \frac{dx}{dt} < 0 \tag{4}$$

Total derivative with time from Lyapunov's function (4) with regard to equation of state (3) is determined as scalar product of gradient vector $\frac{\partial V(x)}{\partial x}$ from Lyapunov's function by velocity vector $\frac{dx}{dt}$. Gradient vector from Lyapunov's function always points to maximum growth of functions, i.e. from the origin of coordinates to the maximum growth of Lyapunov's function.

When developing system stability [1,2], given motion vectors or system is equilibrium corresponds to the origin of coordinates. Equations of system state (1) or (3) are always constructed in deviations Δx from steady state $X_S (x = \Delta x = X - X_S)$.

That is why equations of state (1) or (3) evaluate rate of change of deviation vector x and we can suppose that deviation rate vector in stable system points to the origin of coordinates. Therefore, if Lyapunov's function $V(x)$ is given by vector-function $V(V_1(x), V_2(x), \dots, V_n(x))$, and from geometric interpretation of A.M. Lyapunov's theorem [2] we pick antigradients from Lyapunov function candidates equal to velocity vector candidates, i.e.

$$-\frac{dx_i}{dt} = \frac{\partial V_i(x)}{\partial x_1} + \frac{\partial V_i(x)}{\partial x_2} + \dots + \frac{\partial V_i(x)}{\partial x_n}, (i = 1, \dots, n)$$

Therefore we can write

$$\begin{cases} -\frac{dx_1}{dt} = \frac{\partial V_1(x)}{\partial x_2} = x_2 \\ -\frac{dx_2}{dt} = \frac{\partial V_2(x)}{\partial x_3} = x_3 \\ \dots \\ -\frac{dx_{n-1}}{dt} = \frac{\partial V_{n-1}(x)}{\partial x_n} = x_n \\ -\frac{dx_n}{dt} = \frac{\partial V_n(x)}{\partial x_1} + \frac{\partial V_n(x)}{\partial x_2} + \frac{\partial V_n(x)}{\partial x_3} + \dots + \frac{\partial V_n(x)}{\partial x_n} = \\ = -[(a_n + k_1)x_1 + (a_{n-1} + k_2)x_2 + \dots + (a_1 + k_n)x_n]^2 \end{cases} \tag{5}$$

Then for complete time derivative from candidates of required Lyapunov vector function we obtain.

$$\begin{cases} \frac{dV_1(x)}{dt} = -x_2^2 \\ \frac{dV_2(x)}{dt} = -x_3^2 \\ \dots \\ \frac{dV_2(x)}{dt} = -x_n^2 \\ \frac{dV_n(x)}{dt} = -[(a_n + k_1)x_1 + (a_{n-1} + k_2)x_2 + \dots + (a_1 + k_n)x_n]^2 \end{cases}$$

Of this formula it follows that complete time derivative from candidates of Lyapunov vector function will always be negative function.

Also for complete time derivative from Lyapunov function $V(x) = V_1(x) + V_2(x) + \dots + V_n(x)$ in scalar form we shall obtain

$$\frac{dV(x)}{dt} = -x_2^2 - x_3^2 - \dots - [(a_n + k_1)x_1 + (a_{n-1} + k_2)x_2 + \dots + (a_1 + k_n)x_n]^2 \tag{6}$$

Lyapunov function from (5) we can obtain in form of vector function [7] with candidates:

$$V_1(x) = (0, -\frac{1}{2}x_2^2, 0, \dots, 0)$$

$$V_2(x) = (0,0,-\frac{1}{2}x_3^2,...,0)$$

...

$$V_{n-1}(x) = (0,0,0,...,-\frac{1}{2}x_n^2)$$

$$V_n(x) = (\frac{1}{2}(a_n + k_1)x_1^2, \frac{1}{2}(a_{n-1} + k_2)x_2^2,..., \frac{1}{2}(a_1 + k_n)x_n^2)$$

Here candidates of Lyapunov vector function $V_i(i = 1,..., n)$ are constructed by gradient vector candidates:

$$\frac{\partial V_1}{\partial x_1} = 0, \frac{\partial V_1}{\partial x_2} = -x_2, \frac{\partial V_1}{\partial x_3} = 0,..., \frac{\partial V_1}{\partial x_n} = 0$$

$$\frac{\partial V_2}{\partial x_1} = 0, \frac{\partial V_2}{\partial x_2} = 0, \frac{\partial V_2}{\partial x_3} = -x_3,..., \frac{\partial V_1}{\partial x_n} = 0$$

... ...

$$\frac{\partial V_{n-1}}{\partial x_1} = 0, \frac{\partial V_{n-1}}{\partial x_2} = 0, \frac{\partial V_{n-1}}{\partial x_3} = 0,..., \frac{\partial V_{n-1}}{\partial x_n} = -x_n$$

$$\frac{\partial V_n}{\partial x_1} = (a_n + k_1)x_1, \frac{\partial V_n}{\partial x_2} = (a_{n-1} + k_2)x_2, \frac{\partial V_n}{\partial x_3} =$$
$$= (a_{n-2} + k_3)x_n,..., \frac{\partial V_n}{\partial x_n} = (a_1 + k_n)x_n$$

Lyapunov function in scalar form can be presented in form

$$V(x) = \frac{1}{2}(a_n + k_1)x_1^2 + \frac{1}{2}(a_{n-1} + k_2 - 1)x_2^2 + + \frac{1}{2}(a_{n-2} + k_3 - 1)x_3^2 + ... + \frac{1}{2}(a_1 + k_n - 1)x_n^2 \quad (7)$$

Assumption of function positive definiteness (7) with regard to negative definiteness of quadratic form (5), i.e. system stability (3) we shall obtain in form

$$\begin{cases} a_n + k_1 > 0 \\ a_{n-1} + k_2 - 1 > 0 \\ a_{n-2} + k_3 - 1 > 0 \\ ... \\ a_1 + k_n - 1 > 0 \end{cases} \quad (8)$$

III. Stability Conditions of the Steady States of the System

Usually in control systems a proximate mathematical formulation is often inaccessible. Real problems inevitably contain uncertainty and control system shall be efficient when performing constraints (8) and in case of uncertainties within parameters.

$$G = ((g_{ij})), \ g_{ij} = g_{ij}^0 + \Delta_{ij}, \ \left|\Delta_{ij}\right| < \gamma m_{ij}, \qquad i = 1,..., n$$

where nominal matrix $G_0 = g_{ij}^0$ is superstable $g_{ij} = a_{ij} - b_j k_j -$ candidates of closed system matrix $G_0 = ((g_{ij}^0))$ – face value of candidates of nominal system matrix (1), $\Delta = ((\Delta_{ij})), |\Delta| < m_{ij} -$ uncertainty, matrix $m = ((m_{ij}))$ scales changes of candidates g_{ij} of G matrix, and $\gamma > 0$ – uncertainty range.

Let's define the system by antigradient of any potential function $\dot{x} = \Delta_x V$, obtained earlier in form of Lyapunov function, i.e.

$$\begin{cases} \dot{x}_1 = -(a_n + k_1)x_1 \\ \dot{x}_2 = -(a_{n-1} + k_2 - 1)x_2 \\ \dot{x}_3 = -(a_{n-2} + k_3 - 1)x_3 \\ ... \\ \dot{x}_n = -(a_1 + k_n - 1)x_n \end{cases} \quad (9)$$

Superstability of nominal system (9) is defined by formula (4).

$$\delta(G_0) = \min(-g_{ii}^0 - \sum_{j \neq 1} g_{ij}^0) = \min((a_n + k_1), \min(a_{n-i} + k_i - 1)) \geq 0, \quad (10)$$
$$i = (2,..., n)$$

Let's claim that the condition of superstability is preserved for all matrices of the family:

$$-\left(g_{ii}^0 + \Delta_{ii}\right) - \sum_{j \neq i} \left|g_{ij}^0 + \Delta_{ij}\right| \geq 0, i = 1,..., n$$

It is clear that this in equation will be performed for all admissible Δ_{ii} then and only then, when

$$a_n^0 + k_1^0 - \gamma m_{11} > 0$$

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$$a_{n-1}^0 + k_2^0 - 1 - \gamma m_{22} > 0$$

$$\dots$$

$$a_1^0 + k_n^0 - 1 - \gamma m_{nn} > 0$$

i.e. where

$$\gamma < \gamma^* = \min(\frac{a_n^0 + k_1^0}{m_{11}}, \min\frac{a_{n-1}^0 + k_i^0 - 1}{m_{ii}}), i = 1,.., n-1$$

Thus we clearly determine radius of robust stability of interval family.

Let's demonstrate how suggested approach works by the example based on reduction of matrix A to block-diagonal form

$$\tilde{A} = P^{-1}AP = diag\{\Lambda, J_1,..., J_m, J_1',..., J_k'\}, \quad (11)$$

With diagonal quadratic blocks in form of

$$\Lambda = diag\{s_1,..., s_l\}; \quad (12)$$

$$J_i = \begin{Vmatrix} s_i & 1 & \dots & 0 & 0 \\ 0 & s_i & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & s_i & 1 \\ 0 & 0 & \dots & 0 & s_i \end{Vmatrix} \qquad N_i \times N_i, i = 1,..., m \quad (13)$$

$$J_j' = \begin{Vmatrix} \alpha_j & -\beta_j \\ \beta_j & \alpha_j \end{Vmatrix}, \quad j = 1,..., k \quad (14)$$

where $s_1,..., s_l$ - real-valued simple, s_i -real-valued, N_i - multiple, $s_j = \alpha_j \pm \beta_j$ - complex conjugate eigenvalues of A matrix, where surely $l + N_1 + ... + N_m + 2k = n$. Columns of nonsingular P matrix in canonical transformation (11) are determined by eigenvectors of A matrix, rules and calculation algorithms of which are stated, for example, in [9-11].

Let's demonstrate that stated structure (11) allows verifying the validity of suggested approach to the construction of Lyapunov function and dividing the system (1) depending on proper values of any diagonal block of $\tilde{A}$ matrix. For this purpose let's write

$$\dot{\tilde{x}} = \tilde{A}\tilde{x} + \tilde{B}u = \begin{Vmatrix} \Lambda & & 0 \\ & J & \\ 0 & & J' \end{Vmatrix} \tilde{x} + \begin{Vmatrix} \tilde{b}_1 \\ \tilde{b}_2 \\ \tilde{b}_3 \end{Vmatrix} u; \quad (15)$$

$$\tilde{u} = -\tilde{k}^T \tilde{x} = -\begin{Vmatrix} \tilde{k}_1^T \tilde{k}_2^T \tilde{k}_3^T \end{Vmatrix} \tilde{x} \quad (16)$$

$$\tilde{x} = P^{-1}x, \ \tilde{A} = P^{-1}AP, \ \tilde{b} = P^{-1}b, \ \tilde{k}^T = k^T P \quad (17)$$

and here dimensions of column matrices and row matrices $\tilde{k}_1^T$, $\tilde{k}_2^T$, $\tilde{k}_3^T$ match up with dimensions of square matrices Λ, J, J'. On the basis of (15), (16) it is easy to obtain characteristic determinant of closed system

$$\left|\lambda I - (\tilde{A} - \tilde{b}\tilde{k}^T)\right| = \left|sI_1 - (\Lambda - \tilde{b}_1\tilde{k}_1^T)\right|\left|sI_2 - (J - \tilde{b}_2\tilde{k}_2^T)\right|\left|sI_3 - (J' - \tilde{b}_3\tilde{k}_3^T)\right|,$$

which clearly shows that further problem amounts to sequential development in accordance with proposed method of accepted objects.

$$\dot{\tilde{x}} = \Lambda\tilde{x} + \tilde{b}_1 u \quad (18)$$

$$\dot{\tilde{x}} = J\tilde{x} + \tilde{b}_2 u \quad (19)$$

$$\dot{\tilde{x}} = J\tilde{x} + \tilde{b}_3 u \quad (20)$$

With matrices in form of (12) – (14).

1. Set of equations (18) is written in expanded form

$$\begin{cases} \dot{\tilde{x}}_1 = (s_1 - \tilde{b}_1\tilde{k}_1)\tilde{x}_1 \\ \dot{\tilde{x}}_2 = (s_2 - \tilde{b}_2\tilde{k}_2)\tilde{x}_2 \\ \dots \\ \dot{\tilde{x}}_l = (s_l - \tilde{b}_l\tilde{k}_l)\tilde{x}_l \end{cases}$$

For candidates of gradient vector from Lyapunov function $V(x_1,..., x_l)$ we shall obtain

$$\frac{\partial V(\tilde{x})}{\partial \tilde{x}_1} = -(s_1 - \tilde{b}_1\tilde{k}_1)\tilde{x}_1, \qquad \frac{\partial V(\tilde{x})}{\partial \tilde{x}_2} = -(s_2 - \tilde{b}_2\tilde{k}_2)\tilde{x}_2$$

$$,..., \frac{\partial V(\tilde{x})}{\partial \tilde{x}_l} = -(s_l - \tilde{b}_l\tilde{k}_l)\tilde{x}_l$$

Total derivative with time from Lyapunov function

$$\frac{dV(\tilde{x})}{dt} = \sum_{i=1}^{l} \frac{\partial V(\tilde{x})}{\partial \tilde{x}_i} \frac{d\tilde{x}_i}{dt} = \sum_{i=1}^{l} (s_i - \tilde{b}_i k_i)^2 \tilde{x}_i^2$$

will be negative function. Lyapunov function we shall obtain in form

$$V(\tilde{x}) = -(s_1 - \tilde{b}_1\tilde{k}_1)\tilde{x}_1^2 - (s_2 - \tilde{b}_2\tilde{k}_2)\tilde{x}_2^2 -,...,-(s_l - \tilde{b}_l\tilde{k}_l)\tilde{x}_l^2$$

Positive definiteness of Lyapunov function is given by inequations

$$s_1 - \tilde{b}_1\tilde{k}_1 < 0, \quad s_2 - \tilde{b}_2\tilde{k}_2 < 0 ,..., s_l - \tilde{b}_l\tilde{k}_l < 0$$

Here $s_i - \tilde{b}_i\tilde{k}_i = \mu_i, i = 1,...,l$ are eigenvalues of matrix in closed system and we shall obtain acquainted result of linear principle of stability $\mu_i = s_i - \tilde{b}_i\tilde{k}_i < 0, i = 1,...,l$.

2. Set of equations (19) we assume in expanded form for one Jordan block:

$$\begin{cases} \dot{\tilde{x}}_i == s_i\tilde{x}_i + \tilde{x}_{i+1} - \tilde{b}_i\tilde{k}_i\tilde{x}_i \\ \dot{\tilde{x}}_{i+1} == s_i\tilde{x}_{i+1} + \tilde{x}_{i+2} - \tilde{b}_{i+1}\tilde{k}_{i+1}\tilde{x}_{i+1} \qquad i = 1,..., m; \\ ... \\ \dot{\tilde{x}}_{i+N_i} == s_i\tilde{x}_{i+N_i} - \tilde{b}_{i+N_i}\tilde{k}_{i+N_i}\tilde{x}_{i+N_i} \end{cases}$$

Gradient vector candidates of Lyapunov vector function in accordance with suggested approach will be equal to:

$$\frac{\partial V_i(\tilde{x})}{\partial \tilde{x}_{i+1}} = -(s_i - \tilde{b}_i\tilde{k}_i)\tilde{x}_i; \qquad \frac{\partial V_i(\tilde{x})}{\partial \tilde{x}_{i+1}} = -\tilde{x}_{i+1}$$

$$\frac{\partial V_{i+1}(\tilde{x})}{\partial \tilde{x}_{i+1}} = -(s_i - \tilde{b}_{i+1}\tilde{k}_{i+1})\tilde{x}_{i+1}; \qquad \frac{\partial V_{i+1}(\tilde{x})}{\partial \tilde{x}_{i+2}} = -\tilde{x}_{i+2}$$

...

$$\frac{\partial V_{i+N_i}(\tilde{x})}{\partial \tilde{x}_{i+N_i}} = -(s_i - \tilde{b}_{i+N_i}\tilde{k}_{i+N_i})\tilde{x}_{i+N_i};$$

Complete derivatives with time from Lyapunov vector functions have form:

$$\frac{dV_i(\tilde{x})}{dt} = -(s_i\tilde{x}_i + \tilde{x}_{i+1} - \tilde{b}_i\tilde{k}_i\tilde{x}_i)^2$$

$$\frac{dV_{i+1}(\tilde{x})}{dt} = -(s_i\tilde{x}_{i+1} + \tilde{x}_{i+2} - \tilde{b}_{i+1}\tilde{k}_{i+1}\tilde{x}_{i+1})^2$$

...

$$\frac{dV_{i+N_i}(\tilde{x})}{dt} = -(s_i\tilde{x}_{i+N_i} - \tilde{b}_{i+N_i}\tilde{k}_{i+N_i}\tilde{x}_{i+N_i})^2.$$

Complete derivatives with time are negative functions and meet the condition of asymptotic stability.

Candidates of Lyapunov vector function will be equal to:

$$V_i(\tilde{x}) = -(s_i - \tilde{b}_i\tilde{k}_i)\tilde{x}_i^2 - \tilde{x}_{i+1}^2,$$
$$V_{i+1}(\tilde{x}) = -(s_i - \tilde{b}_{i+1}\tilde{k}_{i+1})\tilde{x}_{i+1}^{\ 2} - \tilde{x}_{i+2}^2,...,$$
$$V_{i+N_i-1}(\tilde{x}) = -(s_i - \tilde{b}_{i+N_i-1}\tilde{k}_{i+N_i-1})\tilde{x}_{i+N_i-1}^{\ 2} - \tilde{x}_{i+N_i}^2,$$
$$V_{i+N_i}(\tilde{x}) = -(s_i - \tilde{b}_{i+N_i}\tilde{k}_{i+N_i})\tilde{x}_{i+N_i}^{\ 2}.$$

Condition of positive definiteness of Lyapunov function for system (19) we shall obtain in form

$$s_i - \tilde{b}_i\tilde{k}_i < 0, \ s_i + 1 - \tilde{b}_{i+1}\tilde{k}_{i+1} < 0,..., \ s_i + 1 - \tilde{b}_{i+N_i}\tilde{k}_{i+N_i} < 0$$
$$i = 1,..., m. \qquad (21)$$

Set of inequations (21) also evaluates the condition of negativeness of real-valued roots of secular equation in closed system.

1. Let's observe the system (20) in expanded form for one block:

$$\begin{cases} \dot{\tilde{x}}_i = \alpha_i\tilde{x}_i + \beta_i\tilde{x}_{i+1} - \tilde{b}_i\tilde{k}_i\tilde{x}_i \\ \dot{\tilde{x}}_{i+1} = -\beta_i\tilde{x}_i + \alpha_i\tilde{x}_{i+1} - \tilde{b}_{i+1}\tilde{k}_{i+1}\tilde{x}_{i+1} \end{cases} \qquad i = 1,.., k$$

If we construct Lyapunov functions in form of vector functions with candidates $V_i(\tilde{x})$ and $V_{i+1}(\tilde{x})$, we shall obtain gradient vector candidates of Lyapunov function as follows

$$\frac{\partial V_i(\tilde{x})}{\partial \tilde{x}_i} = -(\alpha_i - \tilde{b}_i\tilde{k}_i)\tilde{x}_i, \qquad \frac{\partial V_i(\tilde{x})}{\partial \tilde{x}_{i+1}} = -\beta_i\tilde{x}_{i+1}, \qquad \frac{\partial V_{i+1}(\tilde{x})}{\partial \tilde{x}_i} = \beta_i\tilde{x}_i,$$
$$\frac{\partial V_{i+1}(\tilde{x})}{\partial \tilde{x}_{i+1}} = -(\alpha_i - \tilde{b}_{i+1}\tilde{k}_{i+1})\tilde{x}_{i+1}$$

Complete derivatives with time from Lyapunov vector function candidates

$$\frac{dV_i(\tilde{x})}{dt} = -(\alpha_i\tilde{x}_i + \beta_i\tilde{x}_{i+1} - \tilde{b}_i\tilde{k}_i\tilde{x}_i)^2,$$

$$\frac{dV_{i+1}(\tilde{x})}{dt} = -(-\beta_i\tilde{x}_i + \alpha_i\tilde{x}_{i+1} - \tilde{b}_{i+1}\tilde{k}_{i+1}\tilde{x}_{i+1})^2$$

are negative function and meets the conditions of asymptotic stability.

Lyapunov function in scalar form is given in form

$$V_i(\tilde{x}) = -2(\alpha_i - \tilde{b}_i\tilde{k}_i)\tilde{x}_i^2, \quad i = 1,..,k$$

Conditions of Lyapunov function positive definiteness is written

$$\alpha_i - \tilde{b}_i\tilde{k}_i < 0, \; i = 1,..,k \tag{22}$$

IV. Case Study

Condition (22) evaluates negativeness of real part of performance equation roots in closed system. Thus, the correctness of suggested approach is supported by results of linear conception of stability, q.e.d.

We find conditions for the stability of the system and the transition process of the system.

Then, as an example, let's analyze a third-order system the flow diagram of which is shown in fig. 1.

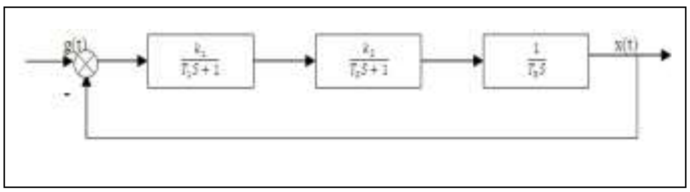

Figure1.System flow diagram

Transfer function for the open-loop system has a form

$$W(s) = \frac{k_1 k_2}{(T_1 s + 1)(T_2 s + 1)T_3 s}$$

Where T_1, T_2, T_3 - time-constants correspondingly, k_1 and k_2 - amplification factors.

Meaning $k = k_1 k_2$, we shall obtain transfer function for the closed-loop system

$$H(s) = \frac{k}{(T_1 s + 1)(T_2 s + 1)T_3 s + k}$$

Secular equation of closed system has a form

$$G(s) = (T_1 s + 1)(T_2 s + 1)T_3 s + k = 0$$

after transformation we shall obtain that

$$G(s) = b_0 s^3 + b_1 s^2 + b_2 s + b_3 = 0$$

where $b_0 = T_1, T_2, T_3, b_1 = (T_1 + T_2)T_3, b_2 = T_3, b_3 = k$

Dividing all candidates of secular equation by b_0 we shall obtain

$$G(s) = s^3 + a_1 s^2 + a_2 s + a_3 = 0$$

where

$$G = \begin{Vmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_3 & -a_2 & -a_1 \end{Vmatrix}$$

$$a_1 = \frac{b_1}{b_0} = \frac{(T_1 + T_2)T_3}{T_1 T_2 T_3}; \quad a_2 = \frac{b_2}{b_0} = \frac{T_3}{T_1 T_2 T_3}; \quad a_3 = \frac{b_3}{b_0} = \frac{k}{T_1 T_2 T_3};$$

Equations of state in closed-loop control system is written as:

$$\begin{cases} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ \vdots \\ \dot{x}_3 &= -\dfrac{k}{T_1 T_2 T_3} x_1 - \dfrac{T_3}{T_1 T_2 T_3} x_2 - \dfrac{(T_1 + T_2)T_3}{T_1 T_2 T_3} x_3 \end{cases}$$

According to developed method let's construct Lyapunov function the complete derivative in time of which will be equal to:

$$\frac{dV(x)}{dt} = -x_2^2 - x_3^2 - \left(\frac{k}{T_1 T_2 T_3} x_1 + \frac{T_3}{T_1 T_2 T_3} x_2 + \frac{(T_1 + T_2)T_3}{T_1 T_2 T_3} x_3\right)^2$$

and Lyapunov function shall be obtained in form

$$V(x) = \frac{1}{2}\frac{k}{T_1 T_2 T_3} x_1^2 + \frac{1}{2}\frac{T_3(1 - T_1 T_2)}{T_1 T_2 T_3} x_2^2 + \frac{1}{2}\frac{(T_1 + T_2)T_3 - T_1 T_2 T_3}{T_1 T_2 T_3} x_3^2$$

Conditions of system stability are reduced to the form

$$\left(\frac{1}{T_1} + \frac{1}{T_2}\right) - 1 > 0; \quad \frac{1}{T_1 T_2} - 1 > 0; \quad \frac{k}{T_1 T_2 T_3} > 0$$

We can define stability limits

1. Aperiodic stability limit (zero root s=0)

$$\frac{k}{T_1 T_2 T_3} = 0, k = 0;$$

When the initial settings are follow:

$$k = 0; \; T_1 = 0; T_2 = 5; T_3 = 0$$

The overall the transition process of the system shows on the Figure 2.

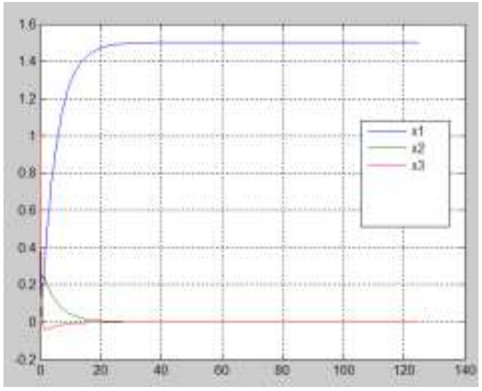

Fig. 2.The transition process, exp.1.

2. Vibrational stability limit come about from

$$(\frac{1}{T_1}+\frac{1}{T_2})=1 \text{ and } \frac{1}{T_1T_2}=1$$

The second case, when the initial settings are follow $k=5;\ T_1=2;T_2=2;T_3=3$

The overall the transition process of the system shows on the Figure 3.

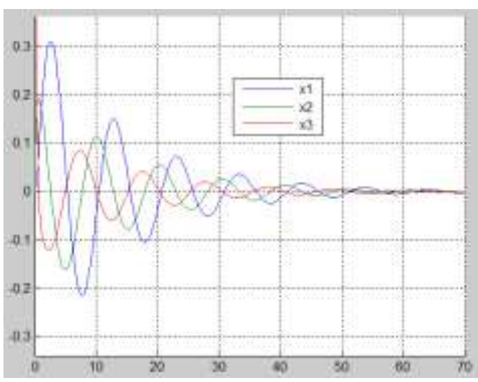

Fig. 3.The transition process, exp.2.

3. Stability limit corresponding to infinite root $(s=\infty)$ equal to $T_1T_2T_3=0$

Conclusion

In this paper robust stability perform an important function in the theory of control of dynamic objects and described in [5,13-16].

The main point of robust stability study is to specify constraints on the change control system parameters that preserve stability. These limits are determined by the region of stability in an uncertain and are selected, i.e. changing parameters.

In this paper we propose an approach of the construction of a Lyapunov function in the form of a vector function in way that it is equal to the gradient of the components, of the velocity vector (right side of the equation of state), but with the negative sign. Study of the robust stability of the system is based on the idea of a direct method A.M. Lyapunov.

The region of stability is obtained in the form of simple inequalities for uncertain parameters control object and selected regulator properties. A new theoretical method of robust stability is proposed for linear systems with uncertain valued parameters [14,15].

This method is an extension of the notion of stability where the Lyapunov function is replaced by a geometric interpretation of the Lyapunov function with dependence on the uncertain parameters [16-18].

The radius of stability coefficients interval family of positive definite functions is equal to the smallest value of the coefficients of the vector Lyapunov functions. Theoretical results obtained in this paper are an important contribution to the theory of stability, to the theory of robust stability of linear control systems.

Thus, for a wide class of systems, we believe the theory is sufficiently well developed that work can begin on developing efficient approach to aid control engineers in incorporating the parametric approach into their analysis and design toolboxes.

The practical importance of these results should motivate new theoretical studies on typical application techniques, clarification area of the robust control and design complex automated system [18-20].

Finally, this is the main results that theoretical approaches represent the most promising direction. These studies are especially important for the designing more effective automation control systems.

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