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Buckling analyses of cylindrical metal silos containing bulk solids

Michal Wojcik, Mateusz Sondej, Jacek Tejchman

***Abstract*—The paper presents quasi-static 3D buckling analysis results of thin-walled cylindrical metal silos with and without bulk solids. The behaviour of the bulk solid was described with a hypoplastic constitutive model. Non-linear analyses with geometric and material non-linearity were performed with a perfect and an imperfect silo shell. Different initial geometric imperfections were considered. The influence of internally stored bulk solids on the buckling strength was compared with experimental results from the literature. The results clearly indicate a strengthening effect of the stored solid on the silo buckling strength.**

***Keywords*—silo buckling, bulk solid, hypoplasticity, FE analysis**

I. Introduction

Cylindrical silos are frequently used to store different bulk solids. In contrast to liquids and gases, solids exert both normal and shear forces on silo walls. The shear ones result from the friction between silo walls and silo fill and are mainly responsible for a buckling failure. The normal forces, however, may have a positive effect on the buckling behaviour of silos due to the fact that the normal pressure produced by the stored material decreases the amplitudes of initial wall imperfections. The solids give a lateral support for cylindrical silo walls when they move inwards during a buckling process. The positive strengthening effect in cylindrical silos is dependent upon the solid stiffness and flow pattern [1]-[4].

Michał Wójcik, adjunct
Gdańsk University of Technology
Poland
mwojcik@pg.gda.pl

Mateusz Sondej, assistant
Gdańsk University of Technology
Poland
matsonde@pg.gda.pl

Jacek Tejchman, professor
Gdańsk University of Technology
Poland
tejchmk@pg.gda.pl

The resistance of bulk solid against shell deformation is usually simulated by a Winkler foundation model which describes the bulk solid as a set of horizontal linear [1] or non-linear springs without tensile stresses [2]. However, springs are not able to realistically capture the solid shear stiffness [4].

The aim of the present research works is to perform 3D buckling quasi-static numerical analyses of thin-walled cylindrical metal silos containing cohesionless sand, and to determine in numbers the influence of the material stiffness on the buckling strength. Finite elements analyses were carried out with the commercial program Abaqus [5]. A hypoplastic constitutive model was used to describe the behaviour of sand [4], [6]. Non-linear analyses with the geometric and material non-linearity were carried out with a perfect and an imperfect silo shell. Different initial geometric imperfections were assumed. FE results were compared with corresponding experiments performed at University of Karlsruhe [2]. The experiments showed, namely, that the buckling strength of the model silo with sand was approximately by 25% higher than of the empty one. The experimental buckling coefficient $\alpha=\sigma_u/\sigma_b$ was 0.3-0.7, where σ_u – the experimental buckling strength, σ_b – the classical buckling strength of the perfect cylinder under axial loading [7]:

$$\sigma_b = \frac{E_s}{\sqrt{3(1-\nu_s)}}\frac{t}{r}, \tag{1}$$

where: E_s – modulus of elasticity of the wall material and ν_s – Poisson ratio of the wall material. The model silo exhibited a considerably post-buckling strength and was characterized by presence of many buckles in both directions [2].

II. Experimental silo

The height of the steel cylindrical model silo was 5.17 m and the diameter was 1.25 m [2]. It consisted of 4 rings, each 1 m high and a mass flow hopper. The thickness of the 3 upper rings was 2 mm. The lower fourth ring was thinner, i.e. t=0.625 mm, t=0.75 mm and t=1 mm in order to induce the buckling failure. The yield stress was 180 MPa for the rings of t=0.625 mm and t=1.0 mm and 350 MPa for the ring of t=0.75 mm. The experiments were performed with an empty silo and with a silo containing the so-called Karlsruhe sand. The measured initial imperfections of the silo shell were about 0.5–1.5 mm, which indicates the usual fabrication tolerance quality, for which the representative imperfection amplitude was [6]

$$w = \frac{t}{Q}\sqrt{\frac{r}{t}}\ , \qquad (2)$$

where: t – the wall thickness, Q=16 – the quality parameter and r – the silo radius. The representative imperfection amplitude w for t=0.625 mm, 0.75 mm and 1 mm was 1.23 mm, 1.35 mm and 1.56 mm (Eq.2), respectively.

III. Buckling FE analyses

A. *Input data*

First, the model silo was filled up with sand by means of the layer-by-layer method, i.e. by dividing the silo fill into 4 horizontal layers, to which gravity was linearly applied to each layer starting from the bottom. In the second step, the vertical displacement was applied to the top edge of the cylinder until the silo failed.

Fig. 1 presents the FE mesh of the entire silo model used in calculations. The 4-node thin shell elements were employed to represent the wall and 8-node linear brick elements to model the solid [5]. The total number of finite elements was 80’000. The mesh of the lower part of the cylinder and of the solid located close to the wall was significantly refined. The hopper part was not taken into account in the calculations. All three displacement directions at the cylinder bottom were fixed. At the cylinder top, the displacements were fixed in a radial and circumferential direction. The wall was assumed to be smooth.

A linear buckling (eigenvalue) analysis (LBA) and a geometrically and materially non-linear analysis (GMNA) of the perfect and imperfect silo (by tracing a force-deflection path) were carried out. The steel was assumed to be elastic-perfectly plastic in a non-linear analysis with the following elastic properties: E=210 GPa, v_s=0.3 and the yield stress f_y=180 MPa and 350 MPa. Two types of initial imperfections were taken into account, namely, the 1st eigenmode of the perfect shell (obtained from LBA) and the axisymmetric circumferential weld depression according to [9] (Fig.2).

B. *Bulk solid model*

The FE-analyses were carried out with a hypoplastic constitutive model for sand [6] which is able to describe the essential properties of granular bodies during shear localization in a wide range of pressures and densities. It includes barotropy (dependence on pressure level), pycnotropy (dependence on density), dilatancy, contractancy and material softening during shearing. This constitutive model describes the evolution of the effective stress tensor with the evolution of the deformation rate tensor by isotropic linear and non-linear tensorial functions. In contrast to elasto-plastic models, a decomposition of deformation components into elastic and plastic parts, the formulation of a yield surface, plastic potential, flow rule and hardening rule is not needed.

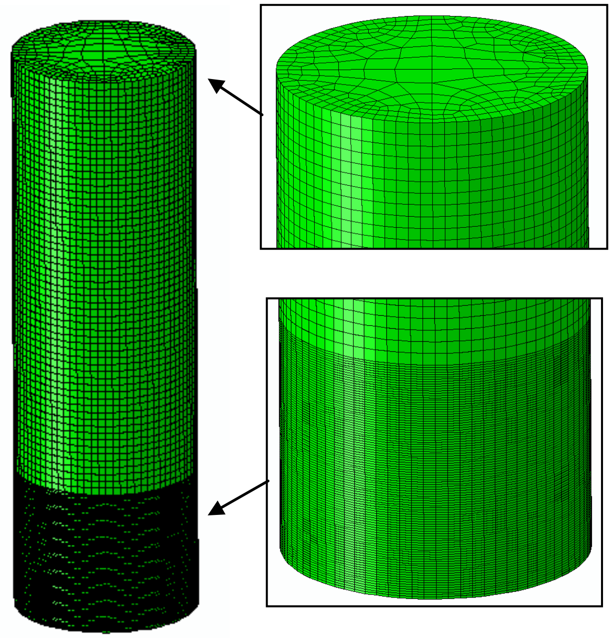
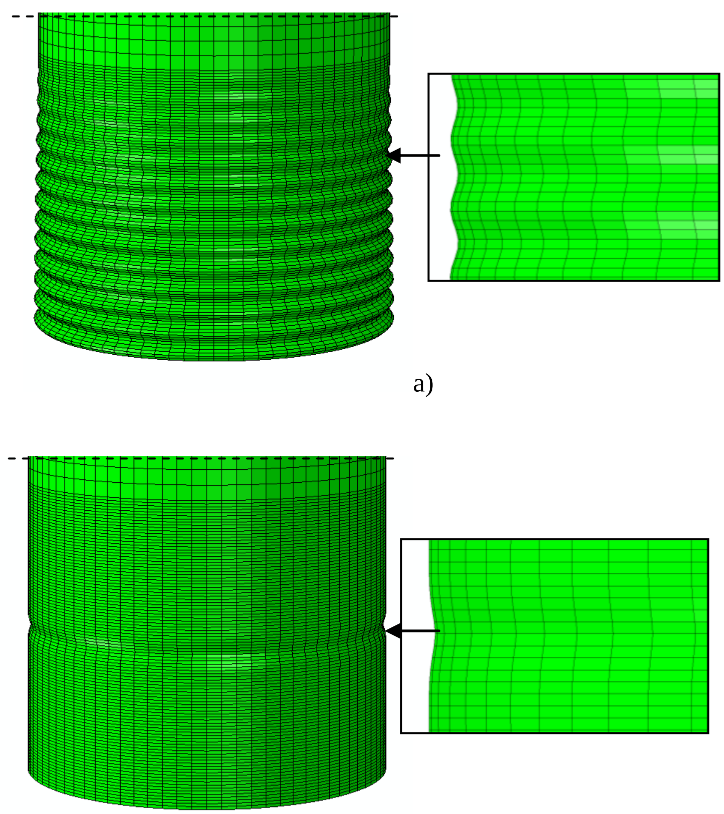


Figure 1. FE-mesh of entire model silo containing bulk solid



a)

b)

Figure 2. Lower part of imperfect bin with two different initial wall geometric imperfections: a) 1st eigenmode of perfect bin and, b) axisymmetric circumferential weld depression [9]

The local hypoplastic constitutive law can be summarized as follows [10]:

$$\mathring{\mathbf{T}}_s = f_e f_b \frac{1}{tr\hat{\mathbf{T}}_s^2}\left(F^2\mathbf{D} + a^2 tr(\hat{\mathbf{T}}_s\mathbf{D})\hat{\mathbf{T}}_s + f_d a F(\hat{\mathbf{T}}_s + \hat{\mathbf{T}}_s^*)\|\mathbf{D}\|\right), \quad (3)$$

where $\mathring{\mathrm{T}}_s$ is the objective (Jaumann) stress rate tensor, D is the stretching tensor and $\hat{\mathrm{T}}_s^*$ is the devatoric part of the stress ratio $\hat{\mathrm{T}}_s$. The constitutive relationship requires 8 material constants: e_{i0}, e_{d0}, e_{c0}, ϕ_c, h_s,, n, β and α. The calculations were carried out with the following material constants for dry cohesionless Karlsruhe sand [11]: e_{i0}=1.0, e_{d0}=0.55, e_{c0}=0.84, ϕ_c=30^o, h_s=5.8 GPa, n=0.28, β=1 and α.=0.13. The initial void ratio for initially dense sand was e_o=0.60.

C. ***Empty silo***

For a perfect bin, the buckling load factor α was equal to 1 with the ring thickness t=0.75 mm and yield stress f_y=350 MPa. In two remaining cases (t=0.75 mm and 1 mm), the buckling load factor was smaller due to the lower yield stress (f_y=180 MPa): α=0.91 and α=0.71, respectively (Fig.3). The initial geometric imperfection in the form of the 1st eigenmode of the perfect shell was significantly more detrimental as compared to the axisymmetric circumferential weld depression (Fig.3). For both imperfections, the highest buckling load factor was calculated at t=0.75 mm and the lowest one at t=1.0 mm. The first imperfection type (the eigenmode of the perfect shell) resulted in an appearance of many buckles around the cylinder perimeter (Fig.4a), where the second imperfection (circumferential weld depression) caused the axisymmetric plastic deformation in the form of an initially assumed imperfection (Fig.4b).

D. ***Silo containing sand***

Initially, the silo was filled up by means of the “layer-by-layer” method. The wall initial imperfection was taken in the form of an axisymmetric weld depression with the maximum amplitude w=0.53 mm. Fig. 5a presents the calculated normal wall pressure distribution along the height of the cylinder for different filling heights. Each normal stress distribution followed the Janssen theory, i.e. the stress gradient diminished during continuous filling. The distribution of the normal wall pressure at the end of the filling was almost identical as the characteristic one calculated by Eurocode 1 [13]. A strongly non-uniform wall pressure distribution at the height of 0.5 m from the bin bottom was caused by the presence of the initial imperfection.

Figs. 5b and 6 show that during buckling when a solid is considered in the silo, the normal pressure is dependent upon the wall movement in a radial direction due to the solid stiffness. In a silo with a weld depression imperfection, as long as the wall moved inwards, the wall pressure considerably increased. For outward wall motions at the imperfection edges, the wall pressure considerably decreased.

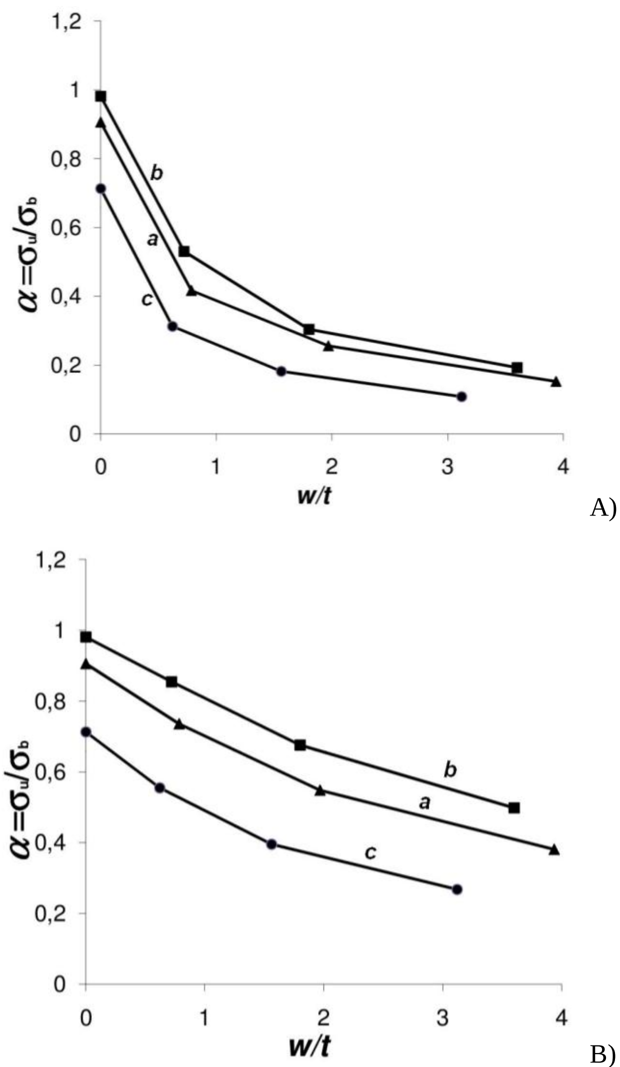
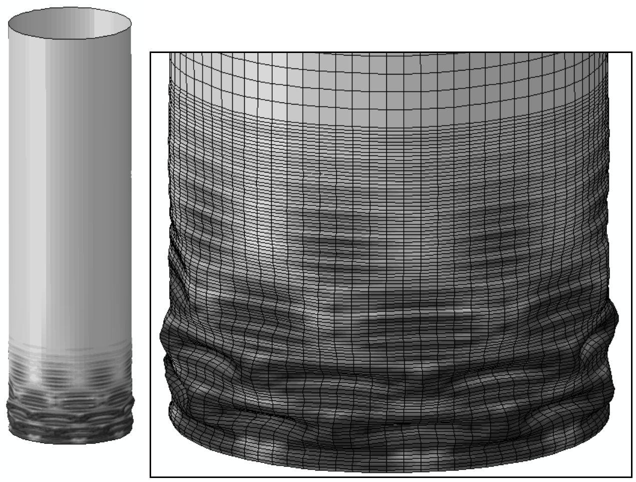
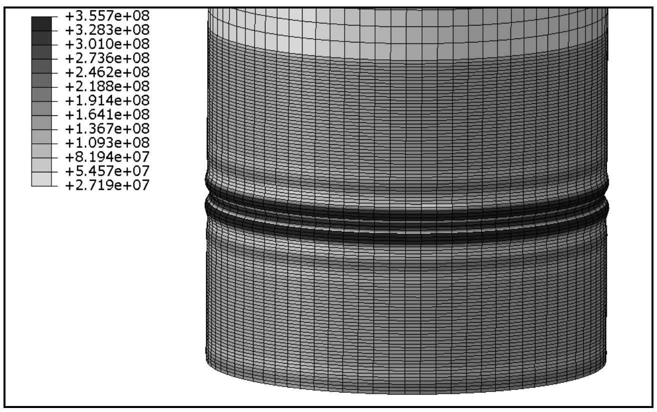


Figure 3. Change of buckling load factor α for different ring thickness of and different maximum horizontal amplitudes w of initial wall geometric imperfection: A) in the form of 1st eigenmode of perfect shell and B) in the form of axisymmetric circumferential weld depression, a) t=0.625 mm, b) t=0.75 mm, c) t=1 mm (t - bottom ring thickness).

Fig. 7 presents the buckling load factor α against ratio r/t and Fig. 8 shows the evolution of the load factor against the vertical deflection u at the cylinder top for an empty silo and a silo containing sand. The results indicate an increase of the buckling strength of a silo with sand as compared with an empty one. The increase is 22%, 13% and 9% for a silo with the wall thickness t=0.625 mm, t=0.75 mm and t=1 mm, respectively.

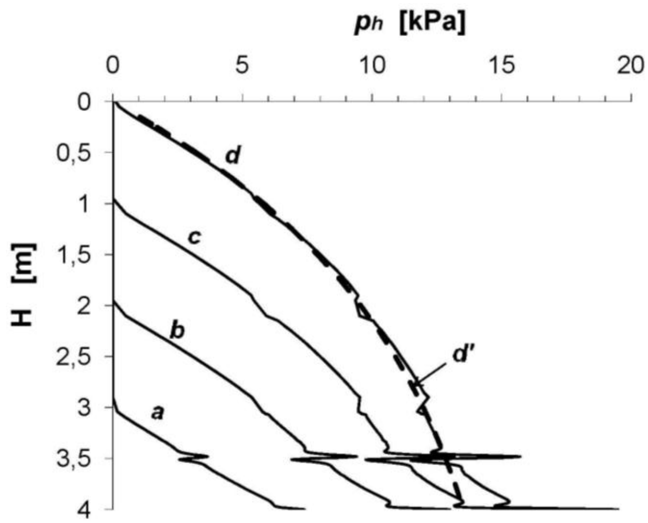


a)

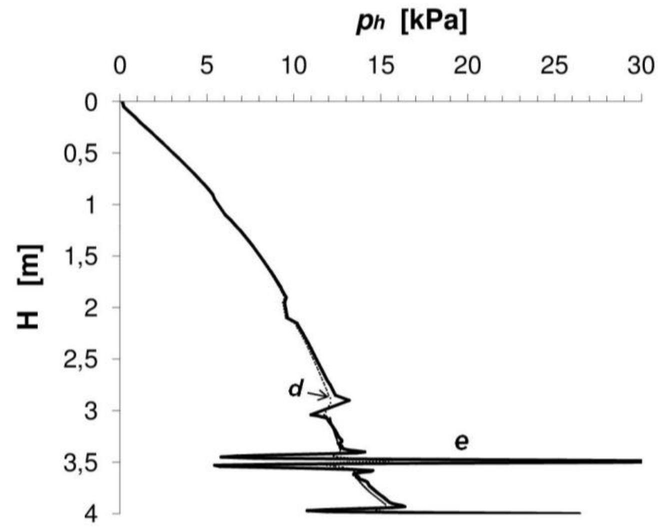


b)

Figure 4. Deformed silo walls (with stresses by von Mises in [Pa]) for two different initial geometric imperfections: a) 1st eigenmode of perfect shell and b) axisymmetric circumferential weld depression (GMNA, silo with bottom ring thickness t=0.75 mm and imperfection amplitude w=0.53 mm)



a)



b)

Figure 5. Distribution of horizontal normall stress p_h along silo height H at different filling heights: a) ¼ H, b) ½ H, c) ¾ H, d) H, d') standard curve [12], e) during buckling (GMNA, silo with sand and bottom ring thickness t=0.75 mm, initial imperfection in the form of axisymmetric weld depression with amplitude w=0.53)

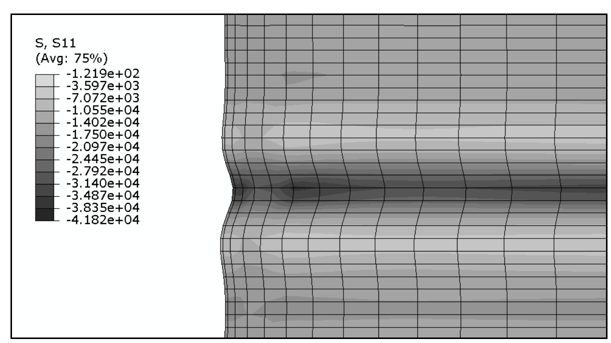


Figure 6. Deformed silo wall (with normal stresses in [Pa]) in region of initial imperfection in the form of axisymmetric weld depression with amplitude w=0.53 (GMNA, bottom ring thickness t=0.75 mm)

IV Conclusions

The obtained FE results show that the initial geometric imperfection in the form of the 1st eigenmode of the perfect shell was more detrimental as compared to the axisymmetric circumferential weld depression. It resulted in an appearance of many buckles around the cylinder perimeter similarly as in experiments. The numerical buckling results of a silo containing sand clearly show a strengthening effect of 9%-22% of the stored bulk solid on the silo buckling strength.

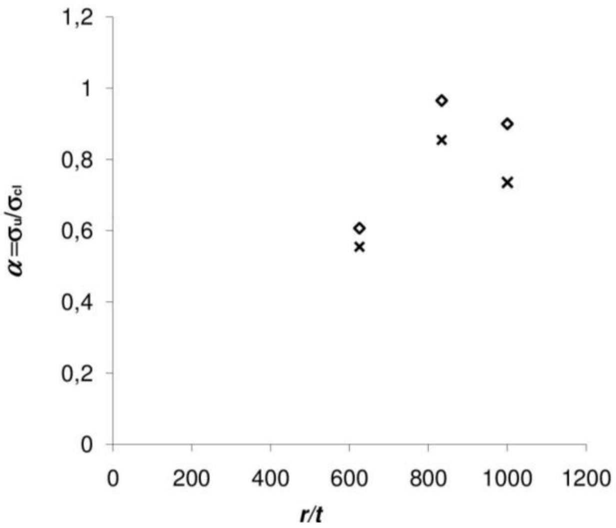


Figure 7. Load buckling factor against ratio *r*/*t* for empty silo (crosses) and silo containing sand (diamonds) from FE analyses with initial geometric imperfection in the form of axisymmetric weld depression with amplitude *w*=0.53 (*r* – silo radius, *t*- wall thickness)

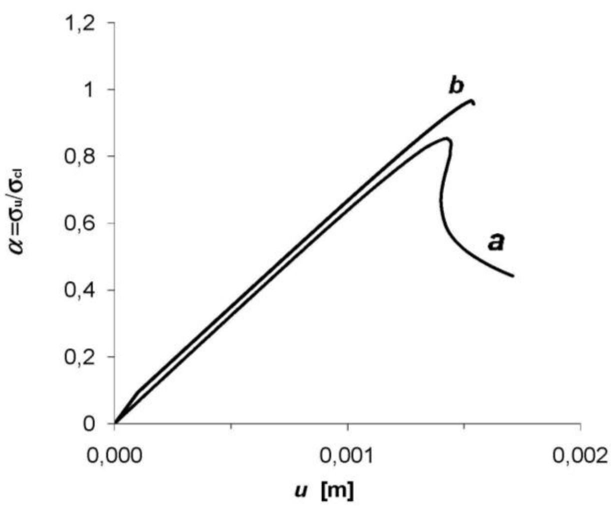


Figure 8. Load factor against vertical wall displacement *u* from FE analyses: a) empty silo, b) silo with sand (GMNA, ring thickness *t*=0.75 mm, initial geometric imperfection in the form of axisymmetric weld depression with amplitude *w*=0.53)

References

[1] JM. Rotter, Q. Zhang, "Elastic buckling of imperfect cylinders containing granular solid". J Struct. Eng. 1990;116:2253–71.

[2] P. Knoedel, "Stabilitatsuntersuchungen an kreiszylindrischen stahlernen Siloschussen". Dissertation, University of Karlsruhe, 1995.

[3] T. Ummenhofer, "Stabilitatsverhalten imperfekter zylindrischer Stahlsiloschalen - experimentelle und numerische Untersuchungen." Dissertation, University of Karlsruhe, 1996.

[4] J. Tejchman, T. Ummenhofer, "Bedding effect in bulk solids in silos: experiments and a polar hypoplastic approach". Thin-Walled Structures" 37, 2000, 333-361.

[5] Hibbit: Karlsson & Sorensen Inc. Abaqus. Theory manual, version 6.10, 2010.

[6] D. Masin, "Hypoplastic models for fine-grained soils", PhD Thesis, Charles University, Prague, 2006.

[7] Eurocode 3, BS EN 1993-4-1: Design of Steel Structures. Part 4-1: Silos, Tanks and Pipelines-Silos, 2007.

[8] SP. Timoschenko, "Einige Stabilitatsprobleme aus der Elastizitatstheorie". Z Math Phys 1910;58:337–85.

[9] PA. Berry, JM. Rotter. "Partial axisymmetric imperfections and their effect on the buckling strength of axially compressed cylinders". In: Proceedings of the Workshop on Imperfections in Metal Silos in Lyon, 1996:35–47.

[10] PA. von Wolffersdorfer. "A hypoplastic relation for granular materials with the predefined limit state surface: Mechanics of Cohesive-Frictional Materials, 1:251-271, 1996.

[11] I. Herle, G. Gudehus, "Determination of parameters of a hypoplastic constitutive model from properties of grain assemblies". Mechanics of Cohesive-Frictional Materials, 4:461-486, 1999.

[12] Eurocode 1, BS EN 1991-4: Actions on Structures. Part 4: Silos and Tanks. General Principles and Actions for the Structural Design of Tanks and Silos, 2009.

About Authors:

Dr. Eng. Michal Wójcik graduated from Civil Engineering Department at Gdańsk University of Technology, Poland, where he wrote his PhD Thesis in 2009. His research interests include experiments and numerical analyses of silo flow

Mr. Mateusz Sondej graduated from Civil Engineering Department at Gdańsk University of Technology in Poland in 2010. Currently he is a PhD student and a researcher at Chair for Structural Mechanics and Bridges of Gdańsk University of Technology. His research interests include FE simulations of shell buckling

Prof. Jacek Tejchman graduated from Civil Engineering at Gdańsk University of Technology in Poland. He received his PhD degree at Technical University of Karlsruhe in Germany in 1989. He became professor in 1998 and head of the Department of Building Structures and Material Engineering at Gdańsk University of Technology (Faculty of Civil and Environmental Engineering). His main research interests concern numerical modelling of the behaviour of bulk solids, soils, concrete and reinforced concrete