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Seismic Response Evaluation of Structures subjected to Multi-Support Excitation using Ritz vectors

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***Abstract*— The response of a multi supported structure subjected to spatially varying ground motion at its different supports is of great importance for structures like bridges, pipe lines, viaducts, tunnels etc,. Various reasons like local site conditions, wave passage effect, incoherence effect and attenuation effect tend to induce a phase difference and time lag in the ground motion which results in differential ground motions at each supports. Assuming that the differential input is available at the various supports the response of the structure can be obtained by the Pseudo-static methods. In these methods the static component of the response is expressed as a function of influence matrix and the input motion which remains independent of the dynamic response. The dynamic component is then computed by using various methods which are then combined with the static components to get the total response. In literature many authors have tried using the modal superposition method for finding the dynamic response of the structure. In this paper the seismic response of the structure is evaluated by using Ritz modes which was then compared with the modal superposition method. The methods were explained by using an example of a bridge model.**

Keywords: Multi-support excitation, spatially varying ground motions, bridge, modal vectors, Ritz vectors

I. INTRODUCTION

Dynamic response induced by non- uniformly distributed ground motions, termed as multi support excitation is important for large-span structures such as bridges, buildings of large aspect ratio, pipe lines, dams etc. Spectacular failures of bridges due to un-seating of decks (dislodging of bridge decks from the piers) reveals that there are relative displacements between the points even a few tenths of meters apart. The Loma Prieta earthquake that struck the San Francisco Bay area of California on October 17, 1989, caused major damages to the bridges due to unseating of the decks as the piers were subjected to differential movements. It was then the studies on response of structures to multi support excitation gained importance. Generally while analyzing a structure for seismic forces it was assumed that the ground motions between various supports are fully correlated in space and are represented by response spectrum, time histories, power spectral density functions etc. But this assumption will not be valid for spatially varying structures since the correlation between the grounds motion at various supports cannot be ignored.

II. LITERATURE REVIEW

In early stage, most of the studies investigated only the wave passage effect on the response of structural systems simulated using relatively simple finite element models. (*E.g .Bogdanoff et al., 1965[1]; Masri, 1976[2]*).

Leimbach and Sterkel [3] implemented the direct time integration and modal analysis method for multi support excited problem and also studied the importance of considering the multi support excitation. *Albdel-Ghaffar and Rubin (1982 and 1983) [4]* used the random vibration approach to examine the seismic response of suspension bridges under multisupport excitations. *Harichandran and Wang (1988) [5]* investigated the effect of the spatial variation of ground motions on the response of a one-span simple beam using a random vibration approach. *Zerva (1992) [6]*compared the seismic response of pipelines under spatially varying ground motions generated using two different spatial variability models proposed *by Harichandran and Vanmarcke (1986) [7]* and by *Luco and Wong (1986)[8]*. *Zerva (1994) [9]* examined the effect of the spatial incoherence and apparent propagation of the seismic ground motions on the response of lifelines subjected to ground motions generated from widely used spatial variability models. *Abdel-Ghaffer and Nazmy, 1991[10]* studied the seismic response of different models of modern cable-stayed bridges to both spatially varying and uniform excitations with input motions established using existing strong motion records. *Harichandran et al. (1996) [11]* compared the seismic response of long-span bridges to both spatially variable ground motions and uniform ones. They found that uniform excitations are generally unacceptable for long-span bridges, and that the incoherence effect should be considered for the seismic response evaluation of these structures. *Monti et al. (1996) [12]* analyzed the nonlinear seismic response of multiple-span bridges subjected to spatially variable excitations in a Monte Carlo simulation framework. *Der Kiureghian and Keshishian (1997) [13]* investigated the effect of spatially variable ground motions on the seismic response of bridges. Seismic ground motions were generated using the coherency model developed by *Der Kiureghian (1996) [14]* that considers wave-passage, incoherency, and local soil effects. They concluded that: the spatially ground motions can either increase or decrease the bridge response *Saxena et al. (2000), [15]* conducted nonlinear dynamic response evaluations of two multi-span bridges subjected to spatially variable ground motions. Two types of differential ground motions were used, i.e., all structural supports on same local soil conditions and different structural supports on different local soil conditions. The ground motions are generated using a variation of the spectral representation method *(Deodatis 1996[16])*. It was concluded that the assumption of identical ground motions yields generally conservative results. *Lin et al. (2004) [17]* examined the effect of spatially variable ground motions on the seismic response of long-span bridges using the random vibration approach. Their results showed that the wave passage effect cannot be neglected when evaluating the seismic safety of these bridges. In the present paper the response of the structure to multi-support excitation was found by direct time integration method, pseudo-static method, modal analysis and by using Ritz vectors assuming that the input motion at the supports are available.

III. FORMULATION FOR MULTI SUPPORT EXCITATION:

The equations of motion for the MDOF system with multi-support excitation can be written as[18]:

$$\begin{bmatrix} M_{ss} & M_{sg} \\ M_{gs} & M_{gg} \end{bmatrix} \begin{Bmatrix} \ddot{X}^t \\ \ddot{X}_g \end{Bmatrix} + \begin{bmatrix} C_{ss} & C_{sg} \\ C_{gs} & C_{gg} \end{bmatrix} \begin{Bmatrix} \dot{X}^t \\ \dot{X}_g \end{Bmatrix} + \begin{bmatrix} K_{ss} & K_{sg} \\ K_{gs} & K_{gg} \end{bmatrix} \begin{Bmatrix} X^t \\ X_g \end{Bmatrix} = \begin{Bmatrix} 0 \\ F_g \end{Bmatrix} \quad (1)$$

where

Mss, Kss and Css are the mass, stiffness, damping matrices corresponding to non-support degrees of freedom.

Mgg ,Kgg and Cgg are the mass, stiffness, damping matrices corresponding to the support degrees of freedom.

Msg Mgs Ksg Kgs Csg Cgs are the coupling mass, stiffness, damping matrices that express the inertia force in the non-support degrees of freedom of the structure due to the motions of the supports (inertia coupling).

X^t is the vector of total displacements corresponding to non-support degrees of freedom.

X_g is the vector of input ground displacements at the supports; a dot denotes the time derivatives.

F_g denotes forces generated at the support degrees of freedom.

For an MDOF system with single-support excitation, the total displacement of the non-support degrees of freedom is obtained by simply adding the input support motion to the relative displacements of the structures with respect to the support. For multi support excitation, the support motions at any instant of time are different for the various supports and, therefore, the total displacements of the non-support degrees of freedom (NSDF) are equal to the sum of the relative displacements of the structure with respect to the supports and the displacements produced at support degrees of freedom (SDF) due to pseudo-static motions of the supports. The latter are obtained by a pseudo-static analysis of the structure for the support motions.

Therefore the displacement X^t is given as

$$X^t = X + rX_g \qquad \text{where} \quad X_s = rX_g \quad (2)$$

Where

X_s=static displacement

X=dynamic displacement of structure

r- is an influence coefficient matrix of size n x m

n-is the number of non-support degrees of freedom

As the responses of NSDF are of interest, the first set of equations obtained from *equation (1)* are considered for the analysis, that is,

$$M_{SS}\ddot{X}^t + M_{sg}\ddot{X}_g + C_{ss}\dot{X}^t + C_{sg}\dot{X}_g + K_{ss}X^t + K_{sg}X_g = 0 \quad (3)$$

Or
$$M_{SS}\ddot{X}^t + C_{ss}\dot{X}^t + K_{ss}X^t = -M_{sg}\ddot{X}_g - C_{sg}\dot{X}_g - K_{sg}X_g \quad (4)$$

Equation (4) is in terms of total displacements of NSDF with inputs as the support displacement velocities, and accelerations. If the effects of mass and damping couplings are ignored, then *equation (4)* takes the form

$$M_{SS}\ddot{X}^t + C_{ss}\dot{X}^t + K_{ss}X^t = -K_{sg}X_g \quad (5)$$

As K_{sg} can be determined, the right hand side of the equation is known and *equation (5)* can be solved to obtain the total displacements.

An equation of motion can also be written in terms of relative displacements of the structure by substituting *equation (2) into equation (4)* leading to:

$$M_{ss}\ddot{X} + C_{ss}\dot{X} + K_{ss}X = -(M_{sg} + rM_{ss})\ddot{X}_g - (C_{sg} + rC_{ss})\dot{X}_g - (K_{sg} + rK_{ss})X_g \quad (6)$$

To find the pseudo-static displacement X_s produced due to the support displacement X_g, the pseudo static equation of equilibrium can be written as:

$$K_{ss}X_s + K_{sg}X_g = 0 \quad (7)$$

The solution for X_s gives

$$X_s = -K_{ss}^{-1}K_{sg}X_g = rX_g \quad (8)$$

Substituting *equation (8)* into *equation (7)*, it is seen that

$$rK_{ss} + K_{sg} = 0 \quad (9)$$

From *equation (8)*, it is seen that the r matrix can be obtained by knowing K_{ss} and $K_{sg.}$ It is evident from *equation (9)* that the last term *equation (6)* is zero. Furthermore, M_{sg} denoting the inertia coupling is generally neglected for most structures when lumped mass system is used. The contribution of the damping is ignored. With these two assumptions, *equation (6)* takes the form

$$M_{ss}\ddot{X} + C_{ss}\dot{X} + K_{ss}X = -rM_{ss}\ddot{X}_g \quad (10)$$

To solve the *equation (10)*, the time histories of the ground accelerations are to be applied at the supports. The solution of the equation of motion provides the responses of the non-support degrees of freedom relative to the support. In order to obtain the total (absolute) responses at the non-support degrees of freedom, *equation (2)* is used. The inertial forces in the member are obtained using the absolute responses (not the relative responses).

IV. TIME HISTORY METHOD:

The response of the structure for the multi support excitation can be obtained by using the time history analysis. The Newmark's family of algorithm [19] can be employed to the *equation (5)* to obtain the total response of the structure which is composed of static response and dynamic response. The Pseudo static methods in which the static components and dynamic components are separated *(equation 10)* can also be solved by using the time history methods. In order to illustrate the use of this family of numerical integration methods consider the solution of the linear dynamic equilibrium equations written in the following form:

$$M\ddot{X}_t + C\dot{X}_t + KX_t = F_t \quad (11)$$

The direct use of Taylor's series provided a rigorous approach to obtain the following two additional equations:

$$X_t = X_{t-\Delta t} + \Delta t\dot{X}_{t-\Delta t} + \frac{\Delta t^2}{2}\ddot{X}_{t-\Delta t} + \frac{\Delta t^3}{6}\dddot{X}_{t-\Delta t} + \cdots \quad (12)$$

$$\dot{X}_t = \dot{X}_{t-\Delta t} + \Delta t\ddot{X}_{t-\Delta t} + \frac{\Delta t^2}{2}\dddot{X}_{t-\Delta t} + \cdots \quad (13)$$

Newmark truncated these equations and expressed them in the following form:

$$X_t = X_{t-\Delta t} + \Delta t\dot{X}_{t-\Delta t} + \frac{\Delta t^2}{2}\ddot{X}_{t-\Delta t} + \beta\Delta t^3\dddot{X} \ldots \quad (14)$$

$$\dot{X}_t = \dot{X}_{t-\Delta t} + \Delta t\ddot{X}_{t-\Delta t} + \gamma\Delta t^2\dddot{X} \quad (15)$$

If the acceleration was assumed to be linear within the time step, the following equation can be written:

$$\dddot{X} = \frac{(\ddot{X}_t - \ddot{X}_{t-\Delta t})}{\Delta t} \quad (16)$$

The substitution of *equation (15)* into *equation (14)* produced Newmark's equations in standard form

$$X_t = X_{t-\Delta t} + \Delta t\dot{X}_{t-\Delta t} + \left(\frac{1}{2} - \beta\right)\Delta t^2\ddot{X}_{t-\Delta t} + \beta\Delta t^2\ddot{X}_t \quad (17)$$

$$\dot{X}_t = \dot{X}_{t-\Delta t} + (1-\gamma)\Delta t\ddot{X}_{t-\Delta t} + \gamma\Delta t\ddot{X}_t \ldots.. \quad (18)$$

Newmark used *equations (18, 17 and 11)* iteratively, for each time step, for each displacement DOF of the structural system. The term $\ddot{u}_t$ was obtained from *equation (11)* by dividing the equation by the mass associated with the DOF.

V. MODAL ANALYSIS METHOD:

An Eigen value analysis of the structure results in the Eigen modes which can be used to study the response of the structure[20][21]. The dynamic response is obtained by transforming the system to modal coordinates by using modal transformation $x^d = \phi y$. By using the Eigen modes of the structure, the stiffness and damping matrices are decoupled and it can be analyzed as a single degree of freedom system.

$$\ddot{y}_i + 2\zeta_i\omega_i\dot{y}_i + \omega_i^2 y_i = \sum_{k=1}^{m}\beta_{ki}\ddot{u}_k(t),\ i = 1 \ldots..n \quad (19)$$

where k represents the degrees of freedom attached to the supports, n represents mode number.

The modal participation factor is given by

$$\beta_{ki} = \frac{\phi_i^T(Mr_k + M_c i_k)}{\phi_i^T M\phi_i} \quad (20)$$

Where r_k is the kth column of influence matrix , i_k is the kth column of n x n identity matrix.

VI. RAYLEIGH RITZ METHOD:

This method can be used for reducing the number of degrees of freedom by finding approximate modes which are orthonormal to each other. In this method the structural displacement is represented by $x^d = \psi y$ where Ψ are the Ritz modes, which are used to reduce the system to single degree of freedom system and can be used to calculate an approximate value of the fundamental frequency[22][23]. In Rayleigh Ritz method the displacements are represented as linear combination of Ritz vectors

$$x^d(t) = \sum_{j=1}^{J} y_j(t)\psi_j = \Psi\, y(t) \quad (21)$$

Where $y_j(t)$ are called the generalized coordinates. The Ritz vectors ψ_j are linearly independent vectors but they satisfy the geometric boundary conditions. The Ritz vectors are generated from the vector 's' which describes the spatial distribution of the forces. It can be obtained by solving the equation

$$ky_1 = s \quad (22)$$

The obtained vector is mass normalized to get the first Ritz vector, thus

$$\psi_1 = \frac{y_1}{(y_1^T m y_1)^{1/2}} \quad (23)$$

The next vector $\psi_2$is determined from the first Ritz vectors. The vector $y_2$is obtained by solving

$$ky_2 = m\psi_1 \quad (24)$$

The vector $y_2$contains a component of the previous vector ψ_1. It can be expressed as

$$y_2 = \hat{\psi}_2 + a_{12}\psi_1 \quad (25)$$

$$a_{12} = \psi_1^T m y_2 \quad (26)$$

where $\hat{\psi}_2$is a pure vector which doesn't contain any influence of the previous vector. This vector is mass normalized to get the second Ritz vector. The advantage of using the Ritz vector is that the exact behavior of the structure can be predicted by using fewer modes. It also reduces the computational efforts to calculate the Eigen modes.

VII. NUMERICAL EXAMPLE:

A simplified cable stayed bridge [24] as shown in the *figure 1* is assumed to demonstrate the various methods describes above. The geometric non-linearity in the pretension cable of the bridge is neglected.

The stiffness matrix K_{ss}, K_{sg}, K_{gg} corresponding to super structure degrees of freedom, coupling term and support degrees of freedom are extracted. The results obtained from various methods described earlier by subjecting the support to El Centro earthquake without time lag are shown in *figures 2,3, 4, 5* and with a time lag at various supports in *figures 6,7, 8, 9*

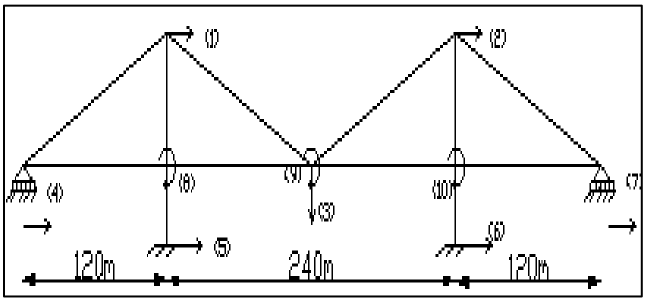

Figure 1 Simplified model of a cable stayed bridge

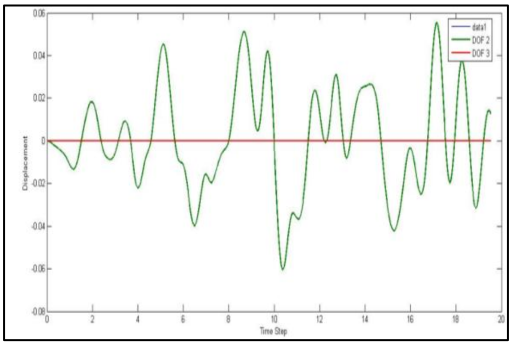

Figure 2 Response of Superstructure DOF by Direct Time Integration Method for SSE

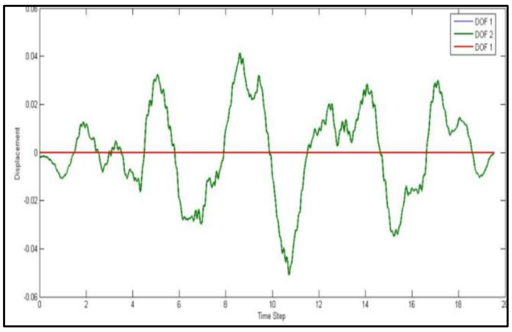

Figure 3 Response of Superstructure DOF by Pseudostatic Method for SSE

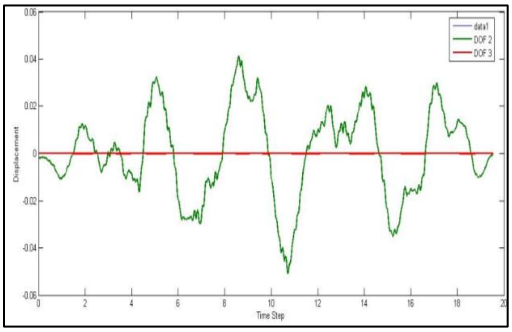

Figure 4 Response of Superstructure DOF by Modal analysis method for SSE

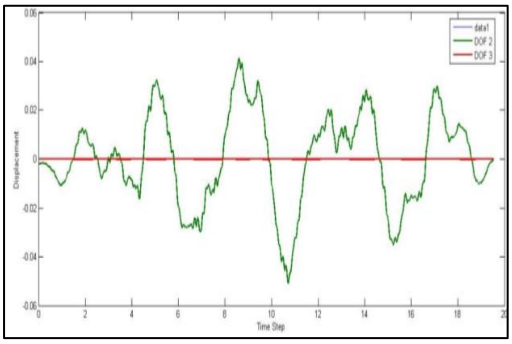

Figure 5 Response of Superstructure DOF by Ritz Vectors for SSE

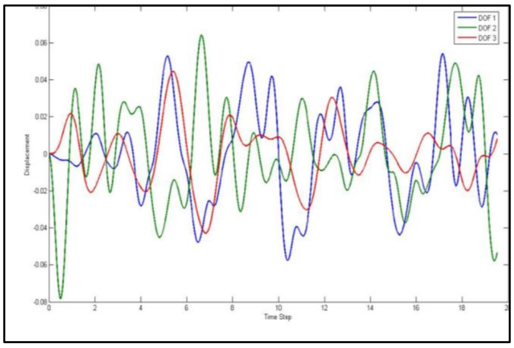

Figure 6 Response of Superstructure DOF by Direct Time Integration Method for MSE

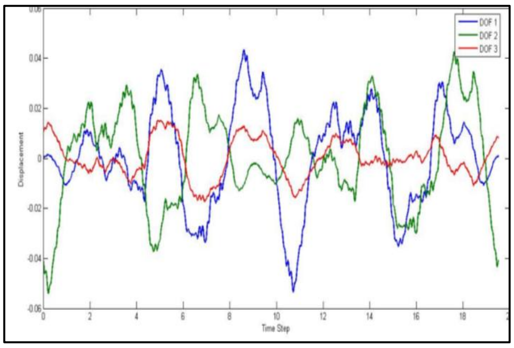

Figure 7 Response of Superstructure DOF by Pseudostatic Method for MSE

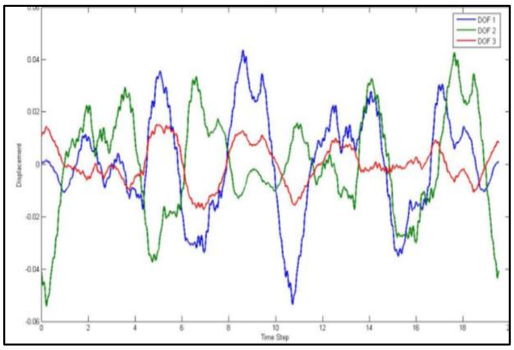

Figure 8 Response of Superstructure DOF by Modal analysis method for MSE

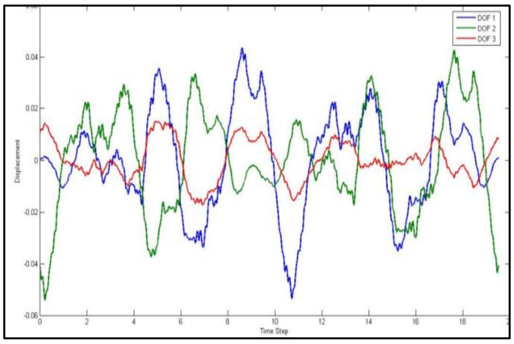

Figure 9 Response of Superstructure DOF by Ritz Vectors for MSE

VIII. Results and Discussion

From the *figure 3-10* it can be observed that the direct time integration algorithm resulted in smoother responses as the output response is not divided into dynamic and static part. The responses from Pseudo-static method, modal analysis method and Ritz vectors are similar since in all these methods the response was divided into static and dynamic part and the dynamic response was calculated by various methods described earlier and added up to the static component which remained constant. When the supports were given uniform excitations it can be seen that the degrees of freedom 1 and 2 coincide exactly with each other. But when the supports were given different excitations the responses obtained in degree of freedom 1 and 2 are entirely different and they are out of phase indicating the potential risk in using asynchronous motion and the need to consider multi support excitations. Asynchronous motion also imparts rotation in the structure which is not present in uniformly excited motion. From *figure 9 and 10* it can be seen that the force dependent Ritz vectors gave similar responses as that of the modal vectors. The advantage of using Ritz vector is that the computational time is greatly reduced and fewer modes are enough to extract the responses of the structures..

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