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Quotient Space and Canonical Mapping with Application in Computer Science

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Abstract: **In this paper we have to studied about a closed linear space of dual nuclear beally conver space, we have to considered that quotient spaces of dual nuclear locally convex space, by itself closed linear space, we have to proved that this quotient space is also dual nuclear. We have also studied about a given canonical mapping from one canonical bounded space to another bounded space. We have prove that the locally convex space in this connection is dual nuclear. Finally we have discussed about the given canonical mapping from one pre-compact canonical sub space into another pre-compact canonical sub space. We have shown that the concerning locally convex space is dual nuclear. Canonical Mapping may also be applied in visualizing Dynamic data with maps for this purpose we have:-**

- **Creating maps from Graph Data.**
- **Maps of Dynamic Data**
- **Maps of Readability.**

Key words: **- convex space, bounded set, linear space, dual nuclear, close linear space**

Notions and Definition:

Let E be a locally convex space. Then by h(E) we denote the collection of each pre-compact subset H of E. By f(E) we denote the fundamental system of bounded sub set in E.

Let M be a linear sub space of E then by E/M we denote the quotient space of E by M. Let I be the indexed set such that all x϶E correspond uniquely to the element i of index set I. Then by $[x_i,I]$ we denote a family of element x϶E .We denote I1 (E) for the linear space which is the collection of all weakly summable families $[x_i,I]$ from the ocallly convex space E.We denote $\hat{x}$ = x+m the equivalance class of E/M for all x϶E.

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Definition 1: Let G be a locally convex space and H its linear subspace Also $\hat{x}$=x(H)=x+H be the equivalence classes or closets for each element x϶G. Then the collection of their equivalence classes defined by the equation X+H={x+z:z϶H} is the quotient space of G by H denoted by G/H.

Definition 2: A subset H of the locally convex space G is bounded if the relation H ⊂ g ∪ is valid for zero neighborhoods U in G and for a positive number g.

Definition 3: A bounded set P of the locally convex Hausorff space E is pre-compact of the relation P.

Result & Discussion:

Theorem: Let Q be a closed subspace of a dual nuclear locally convex space P, then the quotient space P/Q is also a dual nuclear locally convex space.

Proof:

Since P is locally convex space and Q is closed space of P then by hypothesis P/Q is also locally convex space. Also since P is dual nuclear, there exists a fundamental system β f(P) of bounded subsets A_n in with P

$A_1 \subset A_{2} \subset$…………………….. $\subset A_n$ …… (1)

Let there exists a bounded subset Dεβ F (p) for each bounded subset Mεβ f(P) with M<D such that the canonical mapping from P (M) into P(D) nuclear ……………… (2)

Let us consider a weakly sum able family $[X_1(Q),I]$ from the locally convex space P/Q such that for each continuous linear form $d\left(\frac{P}{Q}\right)'$ and for an index set I inequality .

Σ|<xi (Q), d>|< + ∞ ……………… (3)

Holds, for all xi , ε P , let these exists linear form L ε Q' such that the relation .

Σ|<x(Q),d>=Σ|<x_i, 1>|………….. (4)

Holds the relation (3) and (4) imply that Σ|<x_i.b>|<+∞…………………… (5)

for xiεP.

The relation (5) implies that $[x_i, I]$ is a weekly sum able family from corresponding to each weakly sum able family $[x_i(Q),I]$ from P/Q. we can construct a weakly bounded subset D^=D+Q of [P/Q] with the help of a weakly sum able family $[x_i,(Q),I]M$ defined by the equation. $\hat{D}=D+Q=\{\Sigma(\lambda_i,x_i+z:|\lambda_i|\le 1,\lambda_i\varepsilon K,F\varepsilon f(I),z\varepsilon Q\}$ (6)

Where f(I) is the collection of all finite subsets F of an index set I. the relation (6) implies that….

$\hat{D}=D+Q=\{\Sigma(\lambda_i,x_i)+z:|\lambda_i|\le 1,\lambda_i\varepsilon K,F\varepsilon f(I), z\varepsilon Q\}$ (7)

Where K is the field of scalars.

We can define the closed linear subspace Q f(P) as follows:

$Q=\{z\varepsilon P,Q\subset P\}$…………… (8)

Relation (7) and (8) imply that

$B=\{\Sigma(\lambda_i x_i):|\lambda i|\le 1,\lambda_i\varepsilon K,i\varepsilon I \text{ and } F \varepsilon f(I)\}$ (9)

Relation (8)n and (9) imply that…..

$B= \{\Sigma (\lambda_i x_i):|\lambda i|\le 1, \lambda i\varepsilon K, i\varepsilon I \text{ and } F \varepsilon f(I)\}$ (10)

Let the images of bounded subsets M, Dεβf(P) with M<D be the boundary subsets M^, D^εβ(P/Q) with M^< D^. Then canonical mapping P(M,D) from P(M) into P(D) is determined by the equation P(M,D)x=x ………… (11)

For xε(M) with P(M) ⊂ P(D)

In the same way the canonical mapping P/Q (M^, D^) is obtained by equation

P/Q(M^,D^)x(Q)=x(Q)=x+Q=x …… (12)

For x(Q)εP/Q(M^)with P/Q(M^) ⊂ P/Q(D^).

On the basis of equation (12), the canonical mapping obtained from equation (11) is nuclear, and then we have

$P(M,D)x=x$

$=\sum\langle x, K_n\rangle Y_n$……… (13)

For x ∈ P(M), Kn ∈ P(M) and Yn ∈ P(D), equation (13) implies that

$X= \sum K_n(x)Y_n$

$= \sum a_nY_n,$ where we put $K_n(x)=a_n$

$\Rightarrow x=a_1y_1+a_2y_2+\ldots\ldots+a_ny_n+\ldots$ (14)

For n ∈N

Now,

$\| P(M,D)x \|= \| \sum \langle x,K_n\rangle Y_n \|$

$\Rightarrow \|P(M,D) \| \|x\| \le \| x\| \sum \|K_n\| \|Y_n\|$

$\Rightarrow \|P(M,D) \| \le \sum \|K_n\| \|Y_n\|$

$\Rightarrow \|P(M,D) \| =\text{in f}\{ \sum \|K_n\| \|Y_n\|$ (15)

Further,

$\sum \|K_n\| \|Y_n\|< + \infty$…………………………… (16)

as P(M,D) is nuclear.

$= \sum[K_n(Q)x(Q)]Y_n(Q)$

$= \sum\langle x (Q), K_n(Q)\rangle Y_n(Q)$

$=\sum\langle x(Q),K_n(Q)\rangle Y_n(Q)$ ……(17)

Where we put, $K_n(Q)=tn\ \varepsilon\ (P/Q)$

=The topological dual of P/Q.

With x(Q) ε P/Q(M) and $Y_{n\ \varepsilon P/Q(D)}$

Now, $\|p/q(M^\wedge,D^\wedge)x(Q) \|=\|\sum\langle XQ, tn\rangle Y_n(Q) \|$

Now, $\|P/Q(M^\wedge,D^\wedge)\| \| x(Q) \|\le \|x(Q) \|\sum\| tn \| \| Y_n(Q) \|\|\|P/Q(M^\wedge, D^\wedge) \le \sum\| tn \| \| Y_n(Q) \|$………………… (18)

Also, $\sum\| tn \| \| Y_n(Q) \|=\sum\| kn(Q) \| \| Y_n(Q) \|$

$\le \| kn \| \| Y_n \| + \| Z \|$ for z ε Q

$<+ \infty + \| Z \|$ for z ε Q

$\sum \|kn \| \| Y_n(Q) \|$ (19)

From 17,18, and 19 it is obvious that canonical mapping P/Q(M^,D^) is nuclear. Hence for every fundamental system βf(P) of bounded subset $D^{\wedge}_m$ with D^1 ⊂ D^2 ⊂…………D^m, there exists a bounded subset Dεβf(P/Q) for each bounded mapping P/Q(M^, D^) is nuclear. Consequently, the quotient space P/Q is dual nuclear which completes the proof of the therom.

In Continuations of Quotient Space and Canonical Mappings:

From the equation 17, 18 & 19 we have shown that the Canonical mapping P//Q (M, D) is nuclear. This map may also be applied visualizing Dynamic data with maps. For this purpose we have:

1. Creating maps from Graph Data:

We have summary of the G Map algorithm (G maps means Geographic map) for static graphs. The input to the algorithm is a relational data set, from which a graph, G= (V, E) is extracted. The set of vertices V corresponds to the objects in the data for example artist and the set of edge E corresponds to the relationship between pairs of objects i.e. similarity between a pair of artists. In its full generality, The graphs is vertex weighted and edge weighted, with vertex weights, corresponding to some notation of the importance of a vertex and the edge weights corresponding to some notation of the closeness between a pair of vertices. In the case of music the importance of a vertex can be determined by the popularity of an artist derived from the total number of listeners, or by the total number of songs played in a given time of period. The weight of an edge can be defined by the strength of the similarity between a pair of artist.

Maps of Dynamic data

1) Canonical map embedded by MDS using modified edged weights and smaller label font size.

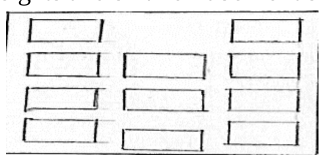

Fig.1

2) For each pair of daily crawl files that are D days, the top 250 artists are expected super set of which is consider as Host artists.

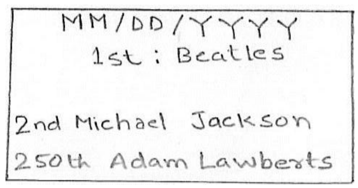

Fig. 2

3) Host artists are extracted from the Canonical map.

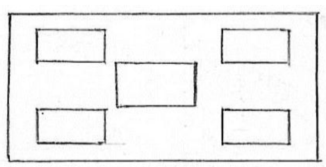

Fig. 3

4) All label font size is set to the Average size with same position information which could result in label overlaps.

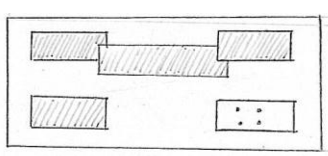

Fig.4

5) Label overlap removal is applied for readability.

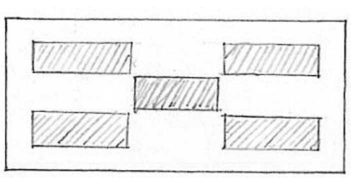

Fig. 5

Algorithm pipeline to create a base map from a Canonical map

1) Canonical map is created with all crawled artist by embedding them using MDS and small size label font.
2) From each pair of daily crawled files that are D days apart, 250 artists with highest increase in play count are extracted.
3) Position data of the host artists are extracted from the canonical map.
4) All label font size are set to the average size.
5) Overlap removal is applied and resulting layout is used as a base map.

Map Readability: Our initial attempt at obtaining a canonical map with G map of the 10,000 artist crawled from the top artist in last fm immediately exposed a problem with this approach. We use modularity & MDS for clustering & embedding respectively. The pairing & seemed applicable given that in the underlying graph the strength of an edge corresponding to the measure of the similarity between the two naturally interpreted as a distance. MDS can determine a layout that matches the underlying clustering. If the relation $H \subset g\ U$ is valid g for zero neighborhood U in G & for a positive number g. $P \subset U\ \{x_r+v\}$ is valid for a finite of elements x_1, x2……..x_n ε E & for a suitable zero neighborhood V in E. For each family $[x_i I]$ from P. we have shown that $[x_i\ I]$ is a weekly sum able family corresponding to a weakly family [xi (Q) , I] from P/Q . Hence D is a weekly bounded subset of P corresponding to each weakly bounded subset.

Dˆ=D =Q of P/Q

In locally convex space all weakly bounded subset are bounded consequently, we have bounded subset D of P corresponding to each bounded subset.

Dˆ=D =Q of P/Q

Significant fragmentation even in a map created for the top 250 artist (in term of number of listener). Using a force directed layout or ling long layout in place of MDS resulted in even more fragmentations.

Conclusions:

One possibility is that fragmentation problem is to some extent caused by the independent nature of the clustering and the embedding steps. Therefore, we combined the two steps using the clustering result and additional input parameters of the embedding process. We increase the edge length, between artists that belongs to different clusters; leading to a much better canonical map, ref. [fig 2] fragmentation is significantly reduced although there are irregularities near some country boundaries. When the canonical map generated in this fashion. There is no fragmentation in a map of 250 artists.

It is worth mentioning that G map used a label overlap removal routine..[…..] to ensure that vertex labels are readable. This is accomplish by moving apart vertices with overlapping lables, but can potentially lead to a vertex near a border between two countries jumping into the wrong country. By strengthening the edges between vertices in the same cluster, we help vertices stay in their own countries. Even though such edge length modification distorts the underlying similarity information, most of the resulting layout changes are local. Since a smaller number of host artists will be extracted out of canonical map in a later step, we need to determine node position in such a way to prevent this we simply use smaller label font size that are proportional to the popularity of artists [ref.fig 1].

About authors:

Prof M. Aslam received PhD in mathematics from Magadh University, Bodh Gaya in 1988. He has more than 35 years of teaching experience at different colleges of Magadh University; Bodh Gaya on different profile. He has many research papers in his credit in various national journals. Also, he has presented many research papers in different conferences sponsored by AICTE and UGC. He is guiding 6 research scholars on different topics of mathematics and computer science. His area of interest is functional analysis and topology. His paper has also been accepted for All India Science Congress on Nuclear Mapping.

I,Fahim Uddin (Sr. Lecturer,Computer Science) have an experience of 6 plus years in teaching at different colleges of Magadh University and Uttarakhand Technical University. I have done M.Sc (Physics), MCA and MPhill (Computer Science). I have to my credit two research papers, published in national journals on Wireless Sensor Networks. I have also presented research papers in many national seminars sponsored by AICTE and UGC. In one of the national seminars sponsored by AICTE, my paper was adjudged and awarded as the '*BEST PAPER'*. Presently, I am pursuing my PhD from Uttarakhand Technical University, Dehradun on 'Wireless Sensor Networks' which also happens to be my core area of interest. I may please be contacted at: fahim4tec@sify.com

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