

# Impact of Distributed Generators on Voltage Stability Margin of Distribution Networks

[Deblina Maity, Sumit Banerjee C K Chanda ]

**Abstract**— The paper presents voltage stability margin (VSM) of radial distribution network by considering a unique reactive loading index without and with distributed generator (DG). It is shown that the branch, at which the value of reactive loading index is minimum, is considered to be the weakest branch of the system. Then, the voltage stability margin (VSM) of the feeder is selected heuristically. It is the product of reactive loading indices of all branches of the feeder. The VSM of all feeders can be evaluated. The feeder which has the smallest value of VSM can be considered as the weakest feeder of the system and is at the proximity of voltage collapse. So, for multiple feeders, the voltage stability margin of a system may be considered as the VSM of the feeder which has the smallest value. The effectiveness of the proposed VSM without and with considering DG has been successfully tested on a 12.66 kV radial distribution network consisting of 33 nodes and the results are found to be in very good agreement.

**Keywords**— Distributed generators; Reactive loading index; radial distribution network; voltage collapse; voltage stability margin; weakest branch.

## I. INTRODUCTION

In the deregulated energy market, electric power utilities are continuously searching new technologies to provide acceptable power quality and reliability to their valuable customers. So, electric power utilities are concerned with distributed generators (DGs) which include fuel cells, wind farm, microturbine, photovoltaic, internal combustion engine generators etc.

DG is a small generator which can operate stand-alone or in connection with distribution networks and can be installed at or near the load unlike large central power plants. DG ratings range from 5 kW up to 100 MW. Loss reduction and voltage improvement are two most important benefits of DG installation in distribution networks.

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Voltage stability [1, 2] is one of vital criteria that dictate the maximum permissible loading of a distribution system [3].

The loads generally play an important role in voltage stability analysis and therefore the voltage stability is also known as load stability.

Voltage collapse [4, 5] may occur in a power system due to lost in voltage stability in the system. Voltage collapse is the phenomenon of voltage instability that can appear in a transmission or distribution system operating under the heaviest loading conditions, in which the voltage decreases monotonically leading the system to be blackout. While in normal operating conditions, small loads increase causes a small voltage drop augment, if the entire network or a particular node is over a certain critical load level; further loads increase causes a fast decrease to zero of the voltage which suddenly leads the system to the collapse. Therefore voltage stability analysis is important in order to identify critical nodes [6] in a power system i.e. nodes which are closed to their voltage stability limits and thus enable certain measures to be taken by the control engineer in order to avoid any incidence of voltage collapse.

So far the researchers have paid very little attention to develop a voltage stability indicator [7-10], for a radial distribution network [11-15] in power system. The problem of voltage stability [1] may be explained as inability of the power system to provide the reactive power [16] or non-uniform consumption of reactive power by the system itself. Therefore, voltage stability is a major concern in planning and assessment of security of large power systems in contingency situation, specially in developing countries because of non-uniform growth of load demand and lacuna in the reactive power management side [16]. Most of the low voltage distribution systems [17-20] having single feeding node and the structure of the network is mainly radial with some uniform and non-uniform tapings.

Radial distribution systems [21-22] having a low reactance to resistance ratio, which causes a high power loss. Hence, the radial distribution system is one of the power systems, which may suffer from voltage instability. For a low voltage distribution system, the conventional Newton-Raphson method normally suffers from convergence problems due to low  $\frac{X}{R}$  ratio of the branches.

The current article has been developed a novel and simple theory to identify the weakest branch and weakest feeder of a radial distribution system without and with considering distributed generator (DG) at optimal location. The

effectiveness of the proposed idea is then tested on 33 node radial distribution system.

## I. BASIC THEORY

We consider a simple 2-node system as shown in Fig. 1 [23].

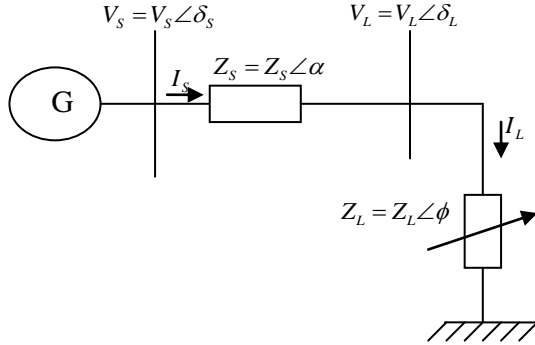


Fig 1: A simple 2-node system.

Here

- $I_s$  sending end line current
- $I_R$  receiving (load) end line current
- $V_s$  magnitude of source end voltage in per unit
- $\delta_s$  phase angle of source end voltage in degree
- $V_L$  magnitude of receiving (load) end voltage in per unit
- $\delta_L$  phase angle of receiving (load) end voltage in degree
- $P_L$  real power demands at receiving (load) end bus
- $Q_L$  reactive power demands at receiving (load) end bus
- $S_L$  complex power
- $Z_s$  magnitude of line impedance in per unit
- $\alpha$  phase angle of line impedance in degree
- $Z_L$  magnitude of load impedance in per unit
- $\phi$  phase angle of load impedance in degree
- $L_q$  reactive loading index of the branch

Here a load having an impedance of  $\vec{Z}_L = Z_L \angle \phi$  is connected to a source having an impedance of  $\vec{Z}_s = Z_s \angle \alpha$ . If line shunt admittances are neglected, the current flowing through the line equals the load current;

From Figure 1,

$$\frac{\vec{V}_s - \vec{V}_L}{\vec{Z}_s} = \frac{P_L - jQ_L}{\vec{V}_L^*} \quad (1)$$

Using simple calculation, we can write load reactive power  $Q_L$  as

$$Q_L = \frac{V_s V_L}{Z_s} \sin(\delta_L + \alpha - \delta_s) - \frac{V_L^2}{Z_s} \sin \alpha \quad (2)$$

The load voltage  $V_L$  can be varied by changing the load reactive power  $Q_L$ . The load reactive power  $Q_L$  becomes maximum when the following condition is satisfied.

$$\frac{dQ_L}{dV_L} = 0 \quad (3)$$

Now, from (2) and (3)

$$2 \frac{V_L}{V_s} \sin \alpha - \sin(\delta_L + \alpha - \delta_s) = 0 \quad (4)$$

Now, at no load,  $V_L = V_s$  and  $\delta_L = \delta_s$ . Therefore at no load, the left hand side (LHS) of (4) will be  $\sin \alpha$ . However, at the maximum reactive power  $Q_L$ , the equality sign of (4) hold and thus the LHS of (4) becomes zero.

Hence the LHS of (4) is considered as a reactive loading index,  $L_q$  of the system that varies between  $\sin \alpha$  (at no load) and zero (at maximum reactive power).

Thus,

$$L_q = 2 \frac{V_L}{V_s} \sin \alpha - \sin(\delta_L + \alpha - \delta_s) \quad (5)$$

$$\text{Here, } \sin \alpha \geq L_q \geq 0 \quad (6)$$

## II. Voltage stability margin and distflow technique of radial distribution NETWORK

A distribution network consists of N number of nodes. Normally, a number of branches are series connected to form a radial feeder in low voltage distribution network. Consider branch  $i$ , which is connected between nodes  $p$  and  $q$  (where node  $p$  is closer to the source or generator node).

Now, the impedance  $Z_s$  of a branch or line is connected between the source and the load nodes for a two node system. In this paper  $L_q$  is defined as the reactive loading index of the branch.

reactive loading index  $(L_q)_i$  of branch  $i$  can be written as

$$(L_q)_i = 2 \frac{V_q}{V_p} \sin \alpha - \sin \alpha \quad (7)$$

Similarly, the reactive loading index of all other branches of the feeder can be determined from (7).

Here, the voltage stability margin (VSM) of the feeder is selected heuristically. It is the product of reactive loading indices of all branches of the feeder.

$$VSM = \prod (L_{q_i}) \quad \text{where } i = 1, 2, \dots, n \quad (8)$$

Where n is the set of branches constituting the feeder starting from source node to ending at end node.

A typical radial distribution network consisting of root node, main line, lateral line, sub lateral line and minor line. So a practical radial distribution system may consist of more than one feeder. Hence, the VSM of all feeders can be evaluated using equation (8) and the feeder which has the smallest value of VSM can be considered as the weakest feeder of the system and is at the proximity of voltage collapse. So, for multiple feeders, the voltage stability margin of a system may be considered as the VSM of the feeder which has the smallest value.

In radial distribution system the power flow problem can be solved by distflow technique. Consider that the branch *i* is connected between nodes *p* and *q*. Now the branch *i* has a series impedance of  $Z_S = (R_S + jX_S)$ . The active and reactive power flow through the branch near node *p* (at point m) is  $P_i$  and  $Q_i$  respectively and the active and reactive power flow through the branch near node *q* (at point n) is  $P_{i+1}$  and  $Q_{i+1}$  respectively. The active and reactive loss of branch *i* is given by

$$P_{loss} = \frac{P_{i+1}^2 + Q_{i+1}^2}{V_q^2} R_S \quad (9)$$

$$Q_{loss} = \frac{P_{i+1}^2 + Q_{i+1}^2}{V_q^2} X_S \quad (10)$$

Hence we can write

$$P_i = P_{i+1} + P_{loss} = P_{i+1} + \frac{P_{i+1}^2 + Q_{i+1}^2}{V_q^2} R_S \quad (11)$$

$$Q_i = Q_{i+1} + Q_{loss} = Q_{i+1} + \frac{P_{i+1}^2 + Q_{i+1}^2}{V_q^2} X_S \quad (12)$$

Here,  $(P_{i+1} + jQ_{i+1})$  is the sum of complex load at node *q* and all the complex power flow through the downstream branches of node *q*.

Now, the voltage magnitude at node *q* is given by

$$V_q^2 = V_p^2 - 2(P_i R_S + Q_i X_S) + \frac{(P_i^2 + Q_i^2)(R_S^2 + X_S^2)}{V_p^2} \quad (13)$$

The power flow solution of a radial distribution feeder involves recursive use of (9) to (13) in reverse and forward direction. Now beginning at the last branch and finishing at the first branch of the feeder, we determine the complex power flow through each branch of the feeder in the reverse direction using (9) to (13). Then we determine the voltage magnitude of all the nodes in forward direction using (13).

### III. RESULTS AND DISCUSSIONS

The effectiveness of the proposed idea is tested on 12.66 KV radial distribution systems consisting of 33-nodes. The single line diagram of the 33-node system is shown in Fig. 2 and its data is given in [23].

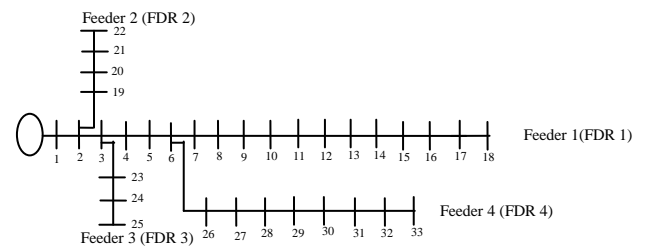


Fig 2: Single line diagram of a main feeder.

The 33-node system (Fig. 2) has four radial feeders, FDR 1, FDR 2, FDR 3 and FDR 4. Each feeder (starting from source node to ending at end node) has a set of branches which is given below.

FDR 1: (1-2-3-4-5-6-7-8-9-10-11-12-13-14-15-16-17-18)

FDR 2: (1-2-19-20-21-22)

FDR 3: (1-2-3-23-24-25)

FDR 4: (1-2-3-4-5-6-26-27-28-29-30-31-32-33)

The reactive loading index of all branches of 33-node system are evaluated using equation (5) and then they are shown in Fig. 3 (nominal loading condition only). The investigation reveals that the value of reactive loading index  $L_q$  to be minimum in branch 5 (connected between nodes 5 and 6). Thus branch 5 can be considered as the weakest branch of the system.

The voltage stability margin of all feeders are then evaluated using equation (8) and then they are shown in Fig. 4 (under nominal loading condition only) and it indicates that FDR 1 has the lowest voltage stability margin. Thus FDR 1 is considered as the weakest feeder of 33-node system.

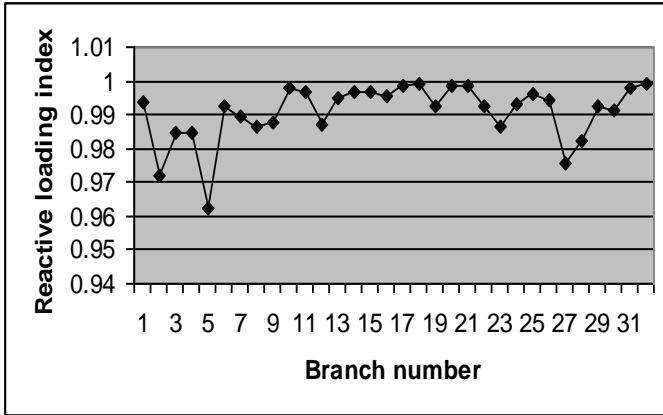


Fig 3: Reactive loading index of all branches of the 33-node system under nominal loading conditions.

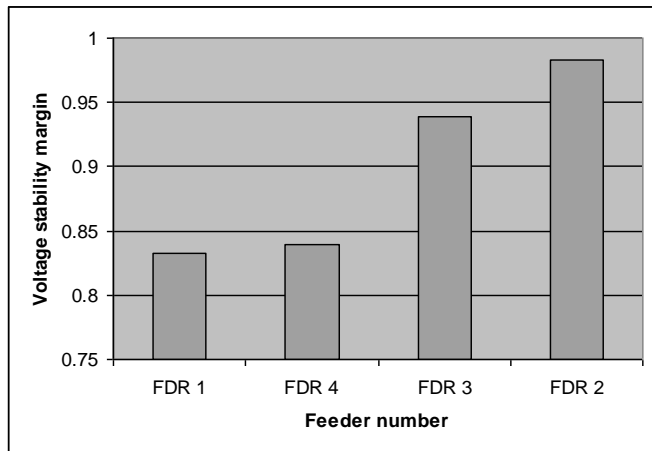


Fig 4: Voltage stability margin of all feeders of the 33-node system under nominal loading conditions.

Now, active and reactive loads of all the nodes are increased {i.e.,  $PL(i) = \alpha.PL_o(i)$  and  $QL(i) = \alpha.QL_o(i)$  for  $i = 2,3,4,\dots,33$  and  $\alpha$  is increased from zero to a critical value where voltage collapses.}. When the load of all nodes is successively increased, the power flow algorithm successfully converged for a load multiplier factor of up to 3.62071. This point is considered to be the critical loading point beyond which a small increment of load causes the voltage collapse.

Here also branch 5 is considered as the weakest branch of the system.

Fig. 5 shows the plot of reactive loading index Vs. load multiplier factor ( $\alpha$ ) of the weakest branch (branch 5) of 33-node network under critical loading condition.

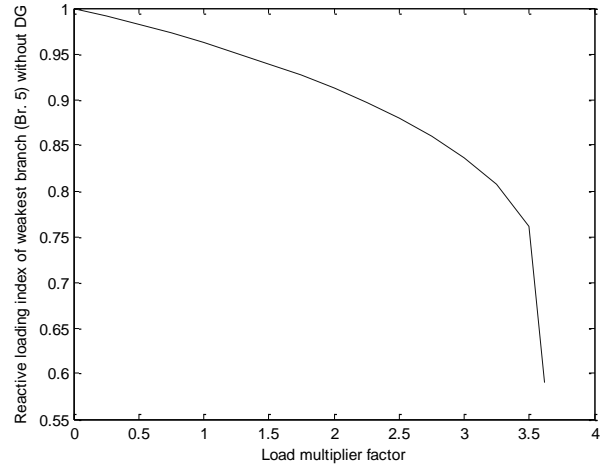
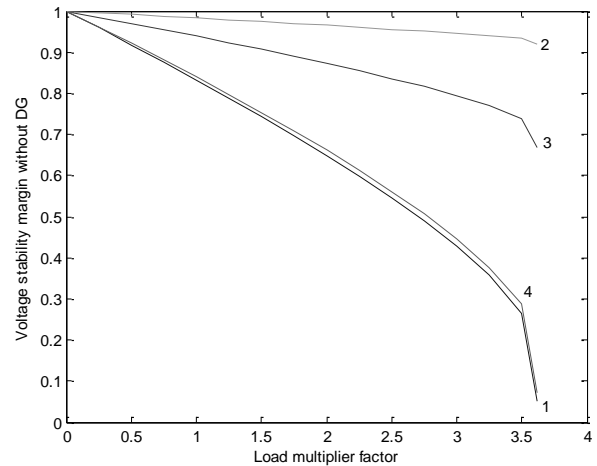


Fig 5: Reactive loading index of the weakest branch of 33 node system under critical loading condition without DG.

Fig. 6 shows the plot of voltage stability margin Vs. load multiplier factor ( $\alpha$ ) of all feeders of 33-node network under critical loading condition. From Fig 6, it indicates that FDR 1 has the lowest voltage stability margin. Thus FDR 1 is considered as the weakest feeder of 33-node system.



1 → Feeder 1 2 → Feeder 2 3 → Feeder 3 4 → Feeder 4

Fig 6: Voltage stability margin of all feeders of 33 node system under critical loading condition without DG.

Now a 400 kW unity power factor distributed generator (DG) is inserted at node 15 (optimal location). Then, active and reactive loads of all the nodes are increased {i.e.,  $PL(i) = \alpha.PL_o(i)$  and  $QL(i) = \alpha.QL_o(i)$  for  $i = 2,3,4,\dots,33$  and  $\alpha$  is increased from zero to a critical value where voltage collapses.}. When the load of all nodes is successively increased, the power flow algorithm successfully converged for a load multiplier factor of up to 4.04434. This point is considered to be the critical loading

point beyond which a small increment of load causes the voltage collapse. Here also branch 5 is considered as the weakest branch of the system.

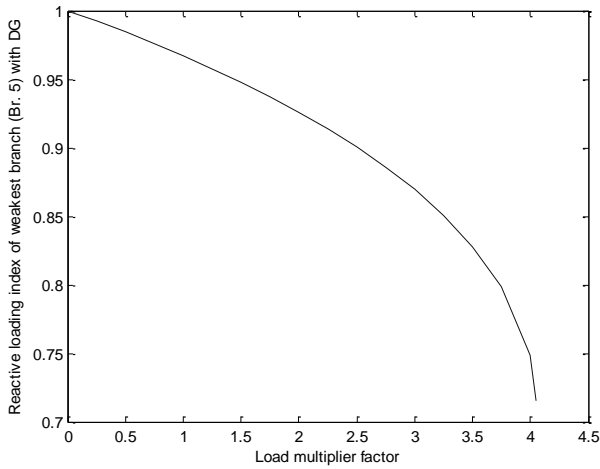
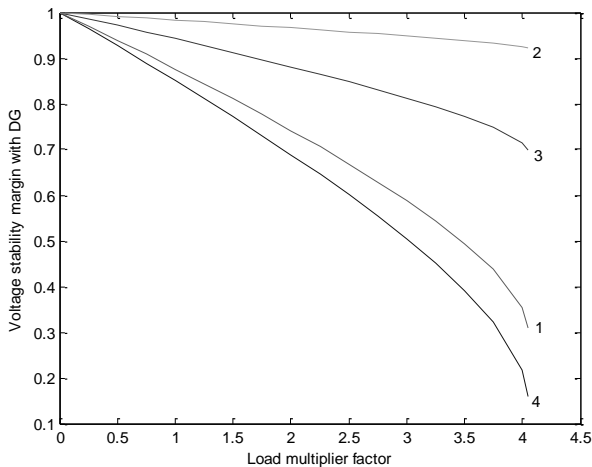


Fig 7: Reactive loading index of the weakest branch of 33 node system under critical loading condition with DG.

Fig. 7 shows the plot of reactive loading index Vs. load multiplier factor ( $\alpha$ ) of the weakest branch (branch 5) with DG at optimal location of 33-node network.

Fig. 8 shows the plot of voltage stability margin Vs. load multiplier factor ( $\alpha$ ) of all feeders of 33-node network with DG at optimal location. From Fig 8, it indicates that FDR 4 has the lowest voltage stability margin. Thus FDR 4 is considered as the weakest feeder of 33-node system. From Fig. 8, it is seen that with the insertion of distributed generator (DG) at node 15, load capability limit of the feeder has increased.



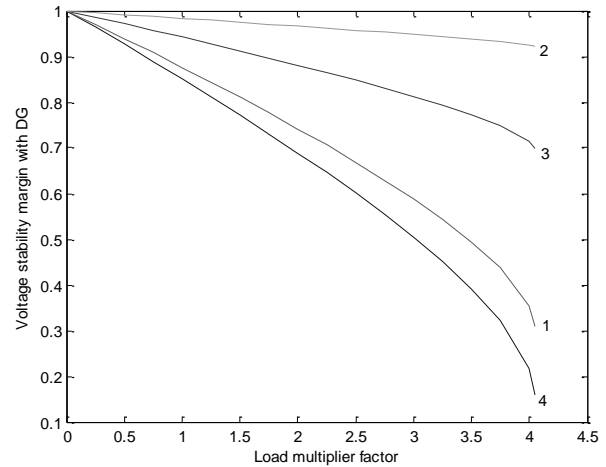
1 → Feeder 1 2 → Feeder 2 3 → Feeder 3 4 → Feeder 4

Fig 8: Voltage stability margin of all feeders of 33 node system under critical loading condition with DG.

Fig. 7 shows the plot of reactive loading index Vs. load multiplier factor ( $\alpha$ ) of the weakest branch (branch 5) with DG at optimal location of 33-node network.

Fig. 8 shows the plot of voltage stability margin Vs. load multiplier factor ( $\alpha$ ) of all feeders of 33-node network with DG at optimal location. From Fig 8, it indicates that FDR 4 has the lowest voltage stability margin. Thus FDR 4 is considered as the weakest feeder of 33-node system.

From Fig. 8, it is seen that with the insertion of distributed generator (DG) at node 15, load capability limit of the feeder has increased.



1 → Feeder 1 2 → Feeder 2 3 → Feeder 3 4 → Feeder 4

Fig 8: Voltage stability margin of all feeders of 33 node system under critical loading condition with DG.

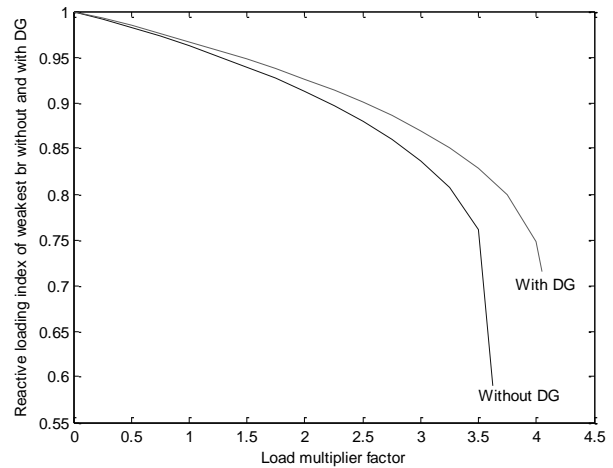


Fig 9: Reactive loading index of the weakest branch of 33 node system under critical loading condition without and with DG.

Fig 9 shows the comparison of reactive loading index of the weakest branch (branch 5) without and with DGs. From Fig 9, it is seen that the profile of the weakest branch has improved after inserting DG at node 15.



Analysis also reveals that with the insertion of DG, there is a significant reduction of power loss in the distribution network. (Table I)

Table I: Active power loss of the system without and with DG (nominal loading condition)

Without DGs	With DGs (unity power factor DGs) DG at node 15= 400 kW
203 kW	159 kW

#### IV. CONCLUSIONS

From the above discussion we conclude that voltage stability margin (VSM) by considering a unique reactive loading index has been proposed for radial distribution system without and with distributed generator (DG). Using this VSM, it is possible to compute the weakest feeder in the system. The investigation also reveals that, branch 5 can be considered as the weakest branch of 33-node system. Hence, with the knowledge of reactive loading index, the operating personnel can have a sufficient knowledge regarding the weakest branch and weakest feeder of the power network. Analysis also reveals that with the insertion of DGs, there is a significant reduction of power loss in the distribution network. The effectiveness of the proposed VSM has been demonstrated on relatively large radial distribution systems.

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