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Formation control of multi-vehicle system using PID-like consensus algorithm

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Abstract— **Formation control of multi-vehicle system has been studied as a combination of estimation problem and tracking problem. Virtual structure based formation control strategy is applied which solves the problem of formation maintenance and movement using a control architecture in hierarchical manner. The control architecture is distributed in nature and requires only local neighbor to neighbor communication which consists of three layers: formation state estimator, formation control module and the physical agent. In particular, a PID-like discrete-time consensus algorithm is applied on group level to estimate the time varying group trajectory information. Based on the estimated group trajectory information, a PID-like discrete-time consensus algorithm based tracking controller is applied on vehicle level. Numerical solution presented in the end, verifies the effectiveness of proposed approach**.

Keywords—*formation control, consensus, virtual structure, multi-vehicle system*

I. Introduction

Formation control of multi-vehicle system has received significant attention among researchers in the field of cooperative control. Due to constraints on communication bandwidth and communication range, distributed algorithms are developed which requires only local neighbor to neighbor information exchange. A number of formation control strategies have been studied in recent years, which includes leader- follower, virtual leader, artificial potential [1] and virtual structure based approach.

Much of research have been focused on the decentralized or distributed cooperative control strategy, which overcomes the problem of single point failure in case of centralized scheme. The distributed control law for each agents are coupled and the states of each agent evolves according to the states of its neighbor (e.g.,[2]-[5]).

The formation control approach based on virtual leader and virtual structure [6], [7], [8], relies on the fact that a virtual leader or a virtual coordinate frame located at the virtual center of formation is specified as a reference for whole group such that each vehicle's desired state can be defined with respect to the virtual leader or the virtual coordinate frame. This approach facilitates single vehicle path planning and trajectory generation technique for the virtual leader or the virtual coordinate frame which ensures formation movement, while single agent path planning or trajectory tracking strategy can be employed for each agent to maintain formation. Similar approach has been used alongwith a virtual structure based distributed formation control architecture is reported in [9]. The hierarchical architecture consists of three main layers i.e. consensus based formation state estimator, consensus based formation control module and the physical vehicle. On the formation state estimation level, each vehicle estimates the center of the virtual coordinate frame via a continuous time consensus algorithm and all the vehicles are made to track the estimated trajectory using consensus based tracking algorithm. Consensus algorithms used, requires instantaneous measurements of derivatives of the local neighbor's state, which may not be realistic in practical application.

Main contribution of this manuscript is twofold. First, PID-like discrete time consensus algorithm is proposed at formation state estimation level which drives the formation state estimation error to zero. Second, at physical vehicle level PID-like discrete time consensus algorithm based controller is employed to track the estimated group trajectory information. Simulation results are presented to verify the effectiveness of proposed methodology.

II. Graph theory preliminaries

Information exchange among vehicles can be modeled by directed or undirected graph. A directed graph (digraph) consists of a pair (N,E) where N is a finite nonempty set of nodes and $E \in N \times N$ is set of ordered pairs of nodes, called edges. An edge (i,j) in a directed graph, denotes that vehicle j can obtain information from vehicle i but not necessarily vice versa. If there is an edge from node i to node j in a digraph, then i is the parent node and j is the child node. On the other hand, the pairs of nodes in an undirected graph are unordered, where an edge (i,j) denotes that vehicle i and j can obtain information from one another.

The adjacency matrix $A=[a_{ij}] \in R^{n\times n}$ of a directed graph is defined as $a_{ii}=0$ and $a_{ij}>0$ if $(j,i)\in E$ where $i \neq j$. The adjacency matrix of an undirected graph is defined analogously except that $a_{ij}=a_{ji}$ $\forall i \neq j$ since $(j,i)\in E$ implies $(i,j)\in E$ .Let matrix $L \in R^{n\times n}$ be defined

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as $l_{ii} = \sum_{j \neq i} a_{ij}$ and $l_{ij} = -a_{ij}$, where $i \neq j$. The matrix L satisfies the following condition

$$l_{ij} \leq 0, i \neq j, \sum_{j=1}^{n} l_{ij} = 0 \quad i = 1, \ldots\ldots, n. \tag{1}$$

For an undirected graph, L is called Laplacian matrix [11], which is symmetric positive semidefinite. However, L for a directed graph does not have this property.

III. Virtual structure approach

The formation control approach based on virtual structure [7, 8] relies on the fact that a virtual coordinate frame located at the virtual center of formation is specified as a reference for whole group such that each vehicle's desired state can be defined with respect to the virtual coordinate frame. A virtual leader is specified which has the knowledge of the virtual center of formation. This approach facilitates single vehicle path planning and trajectory generation technique for the virtual leader or the virtual coordinate frame which ensures formation movement, while single vehicle path planning or trajectory tracking strategy can be employed for each vehicle to maintain formation.

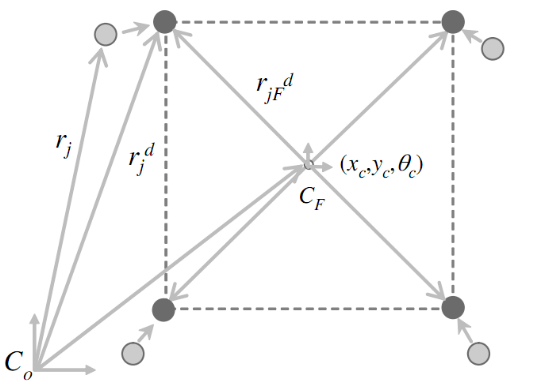


Fig.1. A formation composed of four vehicles

Fig.1 shows an illustrative example of the virtual structure approach with a formation composed of four vehicles with planner motion, where C_O represents the inertial frame and C_F represents a virtual coordinate frame located at a virtual center (x_c, y_c) with an orientation θ_c relative to C_O. In Fig.1, $r_j = [x_j, y_j]^T$ and $r_j^d = [x_j^d, y_j^d]^T$ represent, respectively, the j^{th} vehicle 's actual and desired position. $r_{jF}^d = [x_{jF}^d, y_{jF}^d]^T$ represent's the desired deviation of the j^{th} vehicle relative to C_F.

The desired formation shape can be maintained accurately if each vehicle can track its desired position accurately. In Fig.1, it is assumed that each vehicle exactly knows the state of the virtual coordinate frame, known as formation state.

$$\begin{bmatrix} x_j^d \\ y_j^d \end{bmatrix} = \begin{bmatrix} x_c \\ y_c \end{bmatrix} + \begin{bmatrix} \cos\theta_c & -\sin\theta_c \\ \sin\theta_c & \cos\theta_c \end{bmatrix} \times \begin{bmatrix} x_{jF}^d \\ y_{jF}^d \end{bmatrix} \tag{2}$$

However, due to unreliable communication link and limited information exchange, the vehicles may have different understanding or knowledge of formation state.

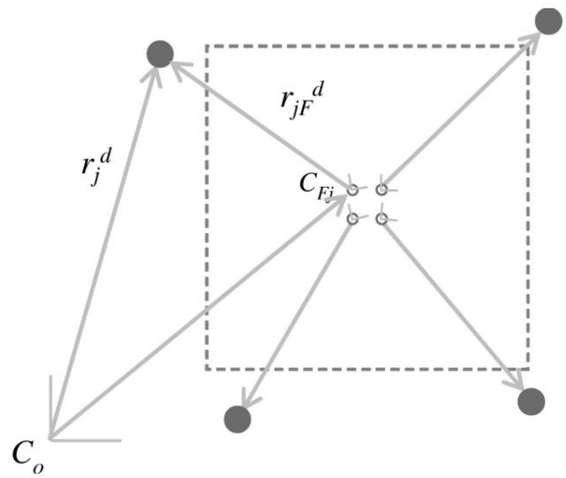


Fig.2 A formation composed of four vehicles with different understanding of virtual coordinate frame

Where C_{Fj} denotes j^{th} vehicle understanding of center of virtual coordinate frame. In this case, the desired position of each vehicle is given as [10]

$$\begin{bmatrix} x_j^d \\ y_j^d \end{bmatrix} = \begin{bmatrix} x_{cj} \\ y_{cj} \end{bmatrix} + \begin{bmatrix} \cos\theta_{cj} & -\sin\theta_{cj} \\ \sin\theta_{cj} & \cos\theta_{cj} \end{bmatrix} \times \begin{bmatrix} x_{jF}^d \\ y_{jF}^d \end{bmatrix} \tag{3}$$

IV. Distributed formation control architecture

A coordination architecture for formation control is reported in [9], the architecture used in this manuscript is taken from [10] and included here for completeness. The architecture can accommodate an arbitrary number of group leaders and ensures accurate formation maintenance through information exchange between local neighbors. The hierarchical architecture consists of consensus-based formation state estimator, consensus-based formation control module and the physical vehicle layer.

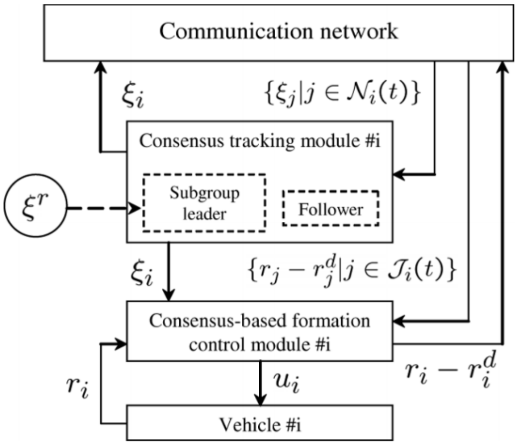


Fig.3 Formation control architecture

The objective of the formation state estimator is to drive $\xi_{ci} = [x_{ci}, y_{ci}, \theta_{ci}]^T$ to $\xi^r = [x_c, y_c, \theta_c]^T$, which represents the desired state of the virtual coordinate frame available only to the group leaders. The local control law u_i for each vehicle is based on its formation state estimate and the position tracking errors of its local neighbors. The architecture is distributed in the sense that it requires only local interaction. Advantage of this scheme lies in the fact that it can accommodate more than one group leader make it free from any failure caused by a single group leader. The task of formation state estimation is basically to track the position and orientation information of the centre of the virtual coordinate frame.

On the formation state estimation level, each vehicle estimates the state of the virtual coordinate frame using a PID-like discrete time consensus algorithm given as

$$\xi_{ci}[k+1] = \xi_{ci}[k] + \frac{T a^c_{i(n+1)}}{\sum_{j=1}^{n+1} a_{ij}} \left(\frac{\xi^r[k] - \xi^r[k-1]}{T} - \gamma e^r_i[k] - \beta \sum_{p=1}^{k} e^r_i[p] \right) + \frac{T}{\sum_{j=1}^{n+1} a^c_{ij}} \sum_{j=1}^{n} a^c_{ij} \left(\frac{\xi_{cj}[k] - \xi_{cj}[k-1]}{T} - \gamma e^j_i[k] - \beta \sum_{p=1}^{k} e^j_i[p] \right) \quad (4)$$

where $a^c_{ij}, i = 1,.......n$, $j = 1,........,n+1$, is the (i, j) entry of the adjacency matrix at the estimation level topology and

$$e^r_i[k] = \xi_{ci}[k] - \xi^r[k],$$
$$e^j_i[k] = \xi_{ci}[k] - \xi_{cj}[k]$$

with γ, β being positive scalars.

Suppose that the vehicle have single integrator dynamics given by

$$\dot{r}_i = u_i, \qquad i = 1,........,n. \quad (5)$$

where $r_i \in R^m$ is the state and $u_i \in R^m$ is the control input of the i^{th} vehicle. Using the first order forward difference approximation, (5) can be written in discrete time form as

$$r_i[k+1] = r_i[k] + Tu_i[k] \qquad i = 1,........,n \quad (6)$$

On vehicle control level, each vehicle uses following PID-like discrete time consensus algorithm to track the desired position $r^d_i[k] = [x^d_i, y^d_i]^T$.

$$u_i[k] = \left(\frac{r^d_i[k] - r^d_i[k-1]}{T} \right) - \alpha_i \left(r_i[k] - r^d_i[k] \right) - \beta_i \sum_{p=1}^{k} a^v_{ij} \left(r_i[p] - r^d_i[p] \right) - \frac{1}{\sum_{j=1}^{n} a^v_{ij}} \sum_{j=1}^{n} a^v_{ij} \left(\left(r_i[k] - r^d_i[k] \right) - \left(r_j[k] - r^d_j[k] \right) \right) \quad (7)$$

where α_i, β_i are positive scalars, and a^v_{ij} is the (i, j) entry of the adjacency matrix defined by vehicle control level topology. The distributed nature of (4) and (7) ensures robustness of group to failure of follower vehicles. In addition, each vehicle simply exchanges information with its local neighbors without the need to identify the group leader.

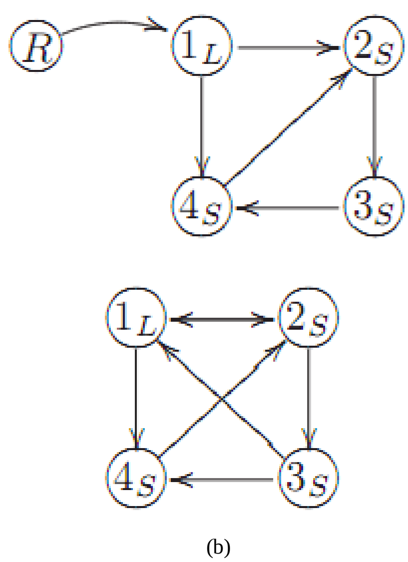


Fig. 4(a) estimation level topology, (b) vehicle control level topology

V. Numerical Simulation

Consider the case where a team of four vehicles are required to maintain a square formation with a lateral length of 0.85m. The motion of the i^{th} vehicle is described by (6). The desired state of the virtual coordinate frame evolves as follows

$$\begin{aligned} x_c[k+1] &= x_c[k] + Tv_r \cos\theta_c \\ y_c[k+1] &= y_c[k] + Tv_r \sin\theta_c \\ \theta_c[k+1] &= \theta_c[k] + T\omega_r \end{aligned} \tag{8}$$

Let $x_{jF} = l_j \cos(\phi_j)$ and $y_{jF} = l_j \sin(\phi_j)$,where $l_j = 0.6m$ and $\phi_j = \pi - \frac{\pi}{4} j$ rad; $j = 1,2,3,4.$ The virtual coordinate frame is initially located at $(0,0)$ m with an orientation 0 rad. Each vehicle applies equation (4) to estimate the formation state and equation (7) to compute u_i .

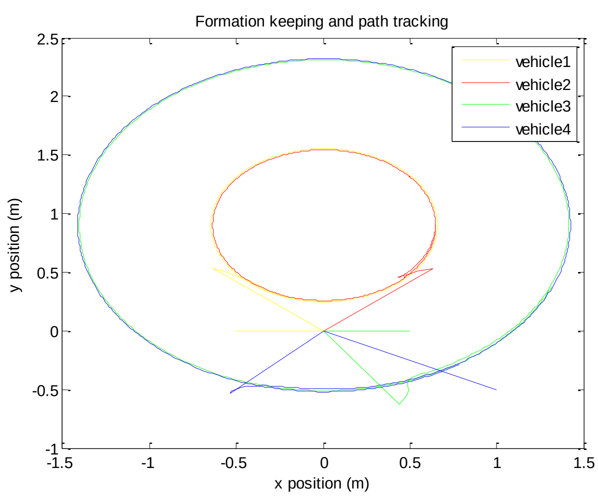
Fig. 3(a) shows the information exchange topology at formation state estimation level and Fig. 3(b) vehicle control level.

Fig.4 shows the simulation result with a single group leader and three followers where the virtual coordinate frame traces the circular trajectory.

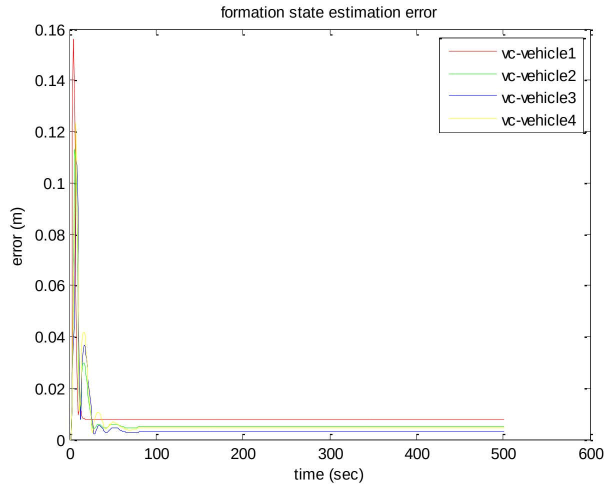
In particular, Fig. 5(a) shows the trajectory of the four vehicles. Fig. 5(b) shows the virtual centre position estimation error defined as $\sqrt{(x_c - x_{ci})^2 + (y_c - y_{ci})^2}$, and Fig.6 shows the inter vehicle distance within the formation denoted by d_{ij} and defined as

$$d_{ij} = \sqrt{\left(x_i(k) - x_j(k)\right)^2 + \left(y_i(k) - y_j(k)\right)^2}$$

where $i, j = 1,2,3,4.$ Plot of inter vehicle distance confirms tight formation.



(a)



(b)

Fig. 5: (a) vehicle trajectory, (b) formation state estimation error (vc-virtual centre)

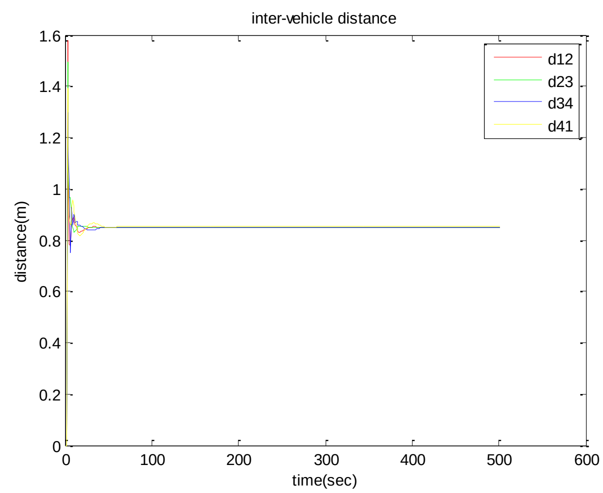


Fig.6. Inter vehicle distance within formation

V. conclusion

Virtual structure based formation control strategy have been used alongwith a distributed formation control architecture. Vehicles are made to agree on time varying formation state of virtual center using a PID-like discrete time consensus-based estimation algorithm.proposed algorithm ensures zero error in the estimation of virtual center. Shape of formation is preserved by employing a consensus based tracking controller to track the desired position of vehicles. Numerical simulations show effectiveness of the proposed algorithm.

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