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Localization of Region-Based Active Contours

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***Abstract*—In this paper, we propose a natural framework that allows any region-based segmentation processed in local way. We consider local rather than global image statistics and evolve a contour based on local information. Localized contours are capable of segmenting objects with heterogeneous feature profiles that would be difficult to capture correctly using a standard global method. The presented technique is versatile enough to be used with any global region-based active contour energy. We describe this framework and demonstrate the localization of three well-known energies in order to illustrate how our framework can be applied to any energy. We then compare each localized energy to its global counterpart to show the improvements that can be achieved. Next, study of the behaviours of these energies in response to the degree of localization is given. Finally, we show results on challenging images to illustrate the robust and accurate segmentations that are possible with this new class of active contour models.**

***Index Terms*—Active contours, level set methods, curve evolution, image segmentation, partial differential equationsn.**

I. INTRODUCTION

ACTIVE contour methods have become very popular in recent years, and have found applications in a wide range of problems including visual tracking and image segmentation; see [1]–[4] and the references therein. The basic idea is to allow a contour to deform so as to minimize a given energy functional in order to produce the desired segmentation; see [5]–[9]. Two main categories exist for active contours: edge-based and region-based.

Edge-based active contour models utilize image gradients in order to identify object boundaries, e.g., [10], [11]. This type of highly localized image information is adequate in some situations, but has been found to be very sensitive to image noise and highly dependent on initial curve placement. One benefit of this type of flow is the fact that no global constraints are placed on the image. Thus, the foreground and background can be heterogeneous and a correct segmentation can still be achieved in certain cases.

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More recently, work in active contours has been focused on region-based flows inspired by the region-competition work of Zhu and Yuille [12]. These approaches model the foreground and background regions statistically and find an energy optimum where the model best fits the image. Some of the most well-known and widely used region-based active contour models assume the various image regions to be of constant intensity [13]–[16]. More advanced techniques attempt to model regions by known distributions, intensity histograms, texture maps, or structure tensors [17]–[20].

There are many advantages of region-based approaches when compared to edge-based methods including robustness against initial curve placement and insensitivity to image noise. However, techniques that attempt to model regions using global statistics are usually not ideal for segmenting heterogeneous objects. In cases where the object to be segmented cannot be easily distinguished in terms of global statistics, region-based active contours may lead to erroneous segmentations. Consider the synthetic image in Fig. 1. Here, we see a situation where the foreground and background are heterogeneous and share nearly the same statistical model. The construction of this image causes it to be segmented improperly by a standard region-based algorithm [13], but correctly by an edge-based algorithm [11]. Heterogeneous objects frequently occur in natural and medical imagery. To accurately segment these objects, a new class of active contour energies should be considered which utilizes local information, but also incorporates the benefits of region-based techniques.

(a) (b) (c)

Fig. 1. Synthetic image of a blob with heterogeneous intensity on a background of similar heterogeneous intensity. (a) Initial contour. (b) Unsuccessful result of region-based segmentation. (c) Successful result of edge-based segmentation technique.[33]

There have been several methods in the literature which are relevant to the present work. Paragios and Deriche [21] presented a method in which edge-based energies and region-based energies were explicitly summed to create a joint energy which was then minimized. In [22] and [23], Sum and Cheung take a

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similar approach and minimize the sum of a global region-based energy and a local energy based on image contrast. The idea of incorporating localized statistics into a variational framework begins with the work of Brox and Cremers [24] who show that segmenting with local means is a first order approximation of the popular piecewise smooth simplification [25] of the Mumford-Shah functional [26]. This focus on the piecewise smooth model is also presented in several related works as we now describe.

Piovano *et al.* [28] focus on fast implementations employing convolutions that can be used to compute localized statistics quickly and, hence, yield re- sults similar to piecewise-smooth segmentation in a much more efficient manner. The effect of varying scales is noted, but not discussed in detail. The work of An *et al.* [29] also notes the efficiency of localized approaches versus full piecewise smooth estimation. That work goes on to introduce a way in which lo- calizations at two different scales can be combined to allow sen- sitivity to both coarse and fine image features. The authors pro- pose a similar flow in [30] based on computing geodesic curves in the space of localized means rather than an approximating a piecewise-smooth model. Lankton *et al.* also propose the use of localized energies in 3-D tensor volumes for the purpose of neural fiber bundle segmentation. All of these works focus on a localized energy that is based on the piecewise constant model of Chan and Vese [13].

In the present work, we make three main contributions. First, we present a novel framework that can be used to localize any region-based energy. Second, we provide a way for localized active contours to interact with one another to create n -ary segmentations. Third, we study in depth the effect of the localization radius on segmentation results. The localization framework we present allows any region-based energy to be localized in a fully variational way. The significant improvement of localization within this framework is that objects which have heterogeneous statistics can be successfully segmented with localized energies when corresponding global energies fail. We go on to use the framework to derive three localized energies. The first, presented in Section III-A, is similar to those in the works mentioned above. Two additional region-based segmentation energies and their localized counterparts are formulated in Sections III-B and III-C. To best of our knowledge localization of energies other than the Chan and Vese energy have never been shown. We provide these as examples to demonstrate how any energy can be localized in a similar manner. Our key claim is that localization in our variational framework can improve the segmentations provided by any globally defined energy in certain circumstances. We do not suggest that one of the proposed localized energies is superior to the others, just that in many cases localizing a global energy in the manner suggested in this work will improve performance.

Additionally, because binary segmentation is often insufficient for higher-level vision problems, we also include a novel method that allows n localized active contours to naturally compete in an image while segmenting different objects that may or may not share borders. This new method extends the work of Brox and Weickert [31], so that it can be successfully utilized with localized active contours.

We also study the significance of a parameter common to all localized statistical models, namely, the degree of localization to use. This scale-type parameter has been mentioned by other authors, but choosing it correctly is crucial to the success of localized energy segmentations. We provide experiments that explain its effect and give guidelines to assist in choosing this parameter correctly. Additional experiments are also presented to analyze the strengths and limitations of our technique.

We now briefly summarize the contents of the remainder of this paper. In the following section, we present our general framework for localizing region-based flows. In Section III, we introduce several energies implemented in this framework.In Section IV, we discuss some of the key implementation details. We go on to show numerous experiments in Section V. Here, we compare the proposed flows with their corresponding global flows, analyze key parameters, discuss limitations of the technique, and show several examples of accurate segmentations on challenging images. In Section VI, we make concluding remarks and give directions for future research.

II. LOCAL REGION-BASED FRAMEWORK

In this section, we describe our proposed local region-based framework for guiding active contours. Within this framework, segmentations are not based on global region models. Instead, we allow the foreground and background to be described in terms of smaller local regions, removing the assumption that the foreground and background regions can be represented with global statistics.

We will see that the analysis of local regions leads to the construction of a family of local energies at each point along the curve. In order to optimize these local energies, each point is considered separately, and moves to minimize (or maximize) the energy computed in its own local region. To compute these local energies, local neighborhoods are split into local interior and local exterior by the evolving curve. The energy optimization is then done by fitting a model to each local region.

We let I denote a given image defined on the domain Ω, and let C be a closed contour represented as the zero level set of a signed distance function ϕ, i.e.,$C = \{x \mid \phi(x) = 0\}$ [8], [9]. We specify the interior of C by the following approximation of the smoothed Heaviside function:

$$\mathcal{H}\phi(x) = \begin{cases} 1, & \phi(x) < -\epsilon \\ 0, & \phi(x) > \epsilon \\ \frac{1}{2}\left\{1 + \frac{\phi}{\epsilon} + \frac{1}{\pi}\sin\left(\frac{\pi\phi(x)}{\epsilon}\right)\right\}, & \text{otherwise.} \end{cases} \quad (1)$$

Similarly, the exterior of C is defined as $(1 - \mathcal{H}\phi(x))$.

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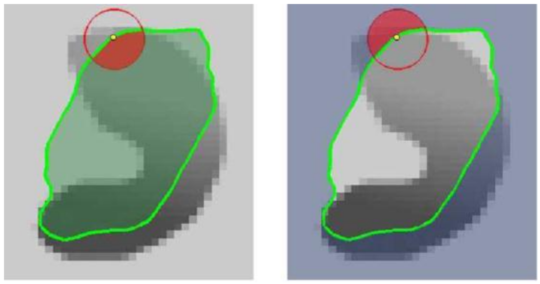

(a) (b)

Fig. 2. Ball is considered at each point along the contour. This ball is split by the contour into local interior and local exterior regions. In both images, the point is represented by the small dot. The neighbourhood is represented by the larger red circle. In (a), the local interior is the shaded part of the circle and in (b), the shaded part of the circle indicates the local exterior.[33]

To specify the area just around the curve, we will use the derivative of $\mathcal{H}\phi(x)$, a smoothed version of the Dirac delta

$$\delta\phi(x) = \begin{cases} 1, & \phi(x) = 0 \\ 0 & |\phi(x)| < \epsilon \\ \frac{1}{2\epsilon}\left\{1 + \cos\left(\frac{\pi\phi(x)}{\epsilon}\right)\right\}, & \text{otherwise.} \end{cases} \quad (2)$$

We now introduce a second spatial variable y. In the remainder of this paper, we will use x and y as independent spatial variables each representing a single point in Ω. Using this notation, we introduce a characteristic function in terms of a radius parameter r

$$\mathcal{B}(x, y) = \begin{cases} 1, & \|x - y\| < r \\ 0 & \text{otherwise.} \end{cases} \quad (3)$$

We use $\mathcal{B}(x, y)$ to mask local regions. This function will be

1

when the point y is within a ball of radius r centered at x, and 0 otherwise. The interaction of $\mathcal{B}(x, y)$ with the interior and exterior regions is illustrated in Fig. 2. Using $\mathcal{B}(x, y)$, we now define an energy functional in terms of a generic force function, F. Our energy is given as follows:

$$E(\phi) = \int_{\Omega_x} \delta\phi(x) \int_{\Omega_y} \mathcal{B}(x, y) \cdot F(I(y), \phi(y))\, dy\, dx \quad (4)$$

The function, F is a generic internal energy measure used to represent local adherence to a given model at each point along the contour. In Section III, we examine several possible candidates for F and show how any region-based energy can be modified and rewritten as an F to be included in this framework.

In computing E, we only consider contributions from the points near the contour. By ignoring inhomogeneity that may arise far away, we give ourselves the ability to capture a much broader range of objects. In (4), we accomplish this with multiplication by the Dirac function, $\delta\phi(x)$ in the outer integral over x. Note that this term ensures that the curve will not change topology by spontaneously developing new contours, although it still allows for contours to split and merge. For every point x selected by $\delta\phi(x)$, we mask with $\mathcal{B}(x, y)$ to ensure that F op- erates only on local image information about x. Thus, the total contribution of the first term of the energy is the sum of F values for every $\mathcal{B}(x, y)$ neighborhood along the zero level set.

Finally, in order to keep the curve smooth, we add a regularization term as is commonly done. We penalize the arc length of the curve and weight this penalty by a parameter λ. The final energy is given as follows:

$$E(\phi) = \int_{\Omega_x} \delta\phi(x) \int_{\Omega_y} \mathcal{B}(x, y) \cdot F(I(y), \phi(y))\, dy\, dx$$
$$+\lambda \int_{\Omega_x} \delta\phi(x) \|\nabla\phi(x)\|\, dx \quad (5)$$

By taking the first variation of this energy with respect to ϕ we obtain the following evolution equation (see Appendix):

$$\frac{\partial\phi}{\partial t}(x) = \delta\phi(x) \int_{\Omega_y} \mathcal{B}(x, y) \cdot \nabla_{\phi(y)} F(I(y), \phi(y))\, dy$$
$$+\lambda\delta\phi(x)\, \mathrm{div}\left(\frac{\nabla\phi(x)}{|\nabla\phi(x)|}\right). \quad (6)$$

Notice that the only restriction on the internal energy, F is that its first variation with respect to ϕ can be computed. This ensures that nearly all region-based segmentation energies can be put into this framework.

III. VARIOUS INTERNAL ENERGY MEASURES

Having formulated our framework in terms of a generic internal energy measure F, we will introduce three specific energies that can be inserted: the *uniform modeling energy*, the *means separation energy*, and the *histogram separation energy*. We present these energies as examples of how any energy can be improved by localization, and make no claim that one energy out performs the others in all cases. In this section, we briefly describe each global energy, give an intuitive description of its behavior, and then show how it can be incorporated into the generic framework described above.

Two well known techniques [13], [16] make use of global mean intensities of the interior and exterior regions which we as denote u and v, respectively

$$u = \frac{\int_{\Omega_y} \mathcal{H}\phi(y) \cdot I(y)\, dy}{\int_{\Omega_y} \mathcal{H}\phi(y)\, dy} \quad (7)$$

$$v = \frac{\int_{\Omega_y} (1 - \mathcal{H}\phi(y)) \cdot I(y)\, dy}{\int_{\Omega_y} (1 - \mathcal{H}\phi(y))\, dy}. \quad (8)$$

In Sections III-A and III-B, we will discuss internal energy functions that rely on local mean intensities to separate regions. In these sections we make use of localized equivalents of u and v defined in terms of the $\mathcal{B}(x, y)$ function. The localized versions of the means, u_x and v_x

$$u_x = \frac{\int_{\Omega_y} \mathcal{B}(x, y) \cdot \mathcal{H}\phi(y) \cdot I(y)\, dy}{\int_{\Omega_y} \mathcal{B}(x, y) \cdot \mathcal{H}\phi(y)\, dy} \quad (9)$$

$$v_x = \frac{\int_{\Omega_y} \mathcal{B}(x, y) \cdot (1 - \mathcal{H}\phi(y)) \cdot I(y)\, dy}{\int_{\Omega_y} \mathcal{B}(x, y) \cdot (1 - \mathcal{H}\phi(y))\, dy} \quad (10)$$

represent the intensity means in the interior and exterior of the contour localized by $\mathcal{B}(x, y)$ at a point x. These localized statistics are needed to determine local energies at each point along the cur

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IV. IMPLEMENTATION DETAILS

We have introduced energies in terms of a signed distance function, ϕ. This makes it very natural to implement flows in a level set framework as proposed by [6] and [8]. In order to improve efficiency, we only compute values of in a narrow

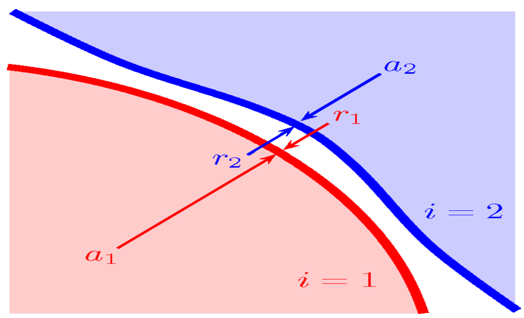

Fig. 3. Advance, and retreat, forces are shown as they affect two interacting contours.[33]

band around the zero level set [8]. Consequently, we re-initialize ϕ every few iterations using a fast marching scheme [6].

The proposed local region-based method begins by initializing every pixel in the narrow band with the local interior and exterior statistics. The nature of this operation varies depending on the energy implemented. Computation of local means, for instance, is simpler than computation of local histograms. An additional cost occurs whenever the narrow band moves to include an uninitialized pixel. In this case, the local statistics of this new pixel must be initialized as well. The number of initialization operations performed is, therefore, dependent on how far from its final position the contour is initialized. The initialization operation is only performed once for each pixel and, therefore, adds a constant complexity increase. However, depending on the size of the local radius, these computations can be significant.

The update step occurs when any initialized pixel is crossed by the contour moving it from the interior to the exterior or vice versa. In our implementation we keep local statistical models in memory for every initialized pixel. When the interface crosses a pixel, the statistical models of all pixels within the $\mathcal{B}(x,y)$ neighborhood are updated. When local means are used, each pixel must maintain the number of pixels in the local regions both inside and outside of the curve as well as the sums of pixel intensities in those two regions. Updating this model consists of transferring values from the “inside” groups the “outside” groups or vice versa. For the histogram separation energy, we keep a full histogram of the local interior and exterior regions for each initialized pixel. Although this requires significantly more memory to maintain than the means model, updates are just as simple: pixel intensities are subtracted from bins of the interior histogram and added to the same bin of the associated exterior histogram or vice versa.

Compared to global methods, local methods incur a linear increase in update computation to manage all of the local statistics. Assume that at each iteration, pixels are crossed by the moving contour, and require an update of their statistics. A global region-based method would perform statistical updates (one for each pixel), whereas the corresponding local region-based flow would perform updates where is the number of pixels that exist within the $\mathcal{B}(x,y)$ neighbourhood. Our experiments confirm this linear increase.

V. EXPERIMENTS

In order to demonstrate the strengths and limitations of the proposed localized active contours, we performed several experiments. First, we compare the three presented localized energies with their global counterparts to show the improvements offered by localization. We follow this with a demonstration of the multiple region segmentation methodology discussed in Section IV. Next, we continue with a study of the effects of local radius selection and contour initialization. Finally, we examine the speed and convergence properties of the proposed method.

A. Comparison With Global Energies

In Section III, we presented three global energies and showed how they could be localized using the framework described in this work. Here, we demonstrate the improvements that are offered by such a localization. As with all segmentation techniques, these three global techniques behave somewhat differently from one another. This is due to differences in the underlying assumptions about the given image inherent in each energy. Likewise, there are differences in the behavior of the corresponding localized energies. The purpose of the experiments given below is to demonstrate that localization can improve the performance of a given global energy, not to specifically compare the original global energies themselves.

In Fig. 4, we compare the global means separation energy from Section III-B and its corresponding localization. Notice that the global energy finds only the brightest parts of the image while the localization comes to rest on object boundaries. Both the HUG image and the MONKEY image show objects and backgrounds which are multimodal, but that have intensities that change smoothly and quickly. In the HUG image in Fig. 4, the proposed method is initialized with two ellipses that correspond to a single level set. The contour changes topology as the two ellipses merge to capture both animals. The initial position of the contour (chosen to be between the two animals) is necessary in order for it to segment these holes.

VI. CONCLUSION

In this work, we proposed a novel framework based on localizing region-based active contours, which in certain cases has resulted in significant improvement in accuracy for segmenting heterogeneous images. We introduced several energies of this localized type and presented the steps required to localize any global region-based energy.

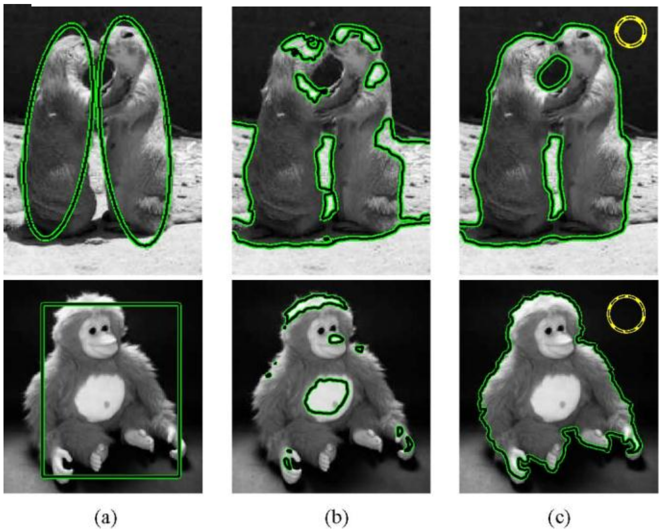

Fig. 4. Segmentations of the HUG and MONKEY images. (a) Shows the initialization; (b) and (c) show segmentation using the global and local versions of the means separation energy respectively. The dashed yellow circle in (c) represents the localization scale.

We went on to draw important conclusions from our experiments. First, we showed several illustrative examples where global region-based energies failed while the localized versions gave very reasonable segmentations. Our experiments with varying the size of the local radius demonstrated how local radii should be chosen in order to correspond to the size of salient objects and the proximity of nearby clutter. We also pointed out how convergence time decreases as radius size increases.

Next, we analyzed the limitations of the technique including its increased sensitivity to initialization compared to global methods. Finally, we performed experiments on the execution time of the proposed techniques and their global counterparts to show that while the proposed methods are slower in some cases, the speed difference is not significant for most applications.

Future work includes altering the size of the radius automatically which will remove the added parameter and allow the technique to be used with less tuning by the user.

Finally, the ability of this type of flow to capture heterogeneous objects makes it ideal for use in some tracking applications. This segmentation approach in combina- tion with existing contour trackers may allow these algorithms to keep track of an entire object rather than one region of homogeneous intensity.

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