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Implementation Of Linear Quadratic Regulator For CSTR Tank

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Abstract - In this article, an optimal controller design for a continuous stirred-tank reactor (CSTR) is proposed. CSTR is widely used in chemical processes as a major processing unit. Firstly, we started with extracting a linearized model for CSTR. This model is used as the basis point for the controller design. To control, we designed a Linear Quadratic Regulator (LQR) controller in order to suppress the effects of disturbances in the inflow liquid. Our main aim is to maintain the concentration and volume of liquid in the CSTR to the given set point. The expectations agree with the simulation results.

Keywords- Continuous Stirred Tank Reactor, LQR, Optimal Control.

I. INTRODUCTION

The applications of control in an optimization problem involve non-linear functions. The main lacking point of linear systems modeling a given system or to implement a control action on it, is that the linear system approximates the actual system only around the operating points. The non-linear systems represent a dynamic behavior of any process better. [2, 4].

The CSTR involves the above mentioned nonlinearities and it also has time varying characteristics. The reactors are generally the hardest parts of any given chemical process to control [5]. The nonlinearities are difficult to model and it is even harder to use that complicated model in designing of a controller.

In this work, in order to model the CSTR, the study started with obtaining the nonlinear state functions. After that, the functions are linearized in order to obtain the linear state space representation of CSTR.

With the state space representation of CSTR, the work continued with applying the LQR optimum control techniques in order to find the LQR controller's parameters [4]. With the help of MATLAB, these parameters are experimented and the simulated results are obtained [6, 8].

The rest of the paper is organized as follows. Section II is dedicated for review of related works. In Section III, mathematical modeling of CSTR is conducted. Section IV is dealing with the LQR Optimum Controller design techniques. Section V is devoted for the simulations and their results. The conclusion will be drawn in Section VI.

II. DESCRIPTION AND ANALYSIS OF CSTR

In the figure given, a typical process flow for CSTR is shown. There are two time varying inlets to the tank with flow rates $F_1(t)$ and $F_2(t)$. The dissolved material concentrations of both the inlets are different, viz. c_1 and c_2 respectively. The outgoing flow has a flow rate F(t). It is assumed that the tank is continuously stirred and mixed well so that the concentration of the outlet equals the concentration in the tank i.e. c(t).

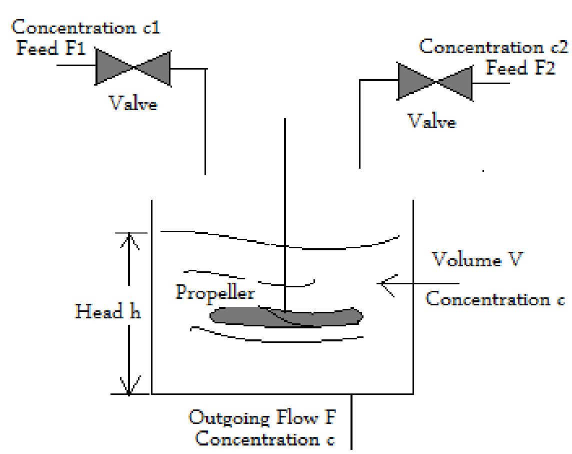

Figure 1: Schematic model of continuous stirred tank reactor

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The mass balance equations are:

$$\frac{\partial v(t)}{dt} = F_1(t) + F_2(t) - F(t) \quad (1)$$

$$\frac{\partial}{\partial t}[c(t)V(t)] = c_1F_1(t) + c_2F_2(t) - c(t)F(t) \quad (2)$$

where V(t) is the volume of the fluid in the tank. The outgoing flow rate W(t) depends upon the head h(t) as follows,

$$F(t) = \sqrt[k]{h(t)} \quad (3)$$

where k is an exponential constant. If the tank has constant cross-sectional area S, we can write

$$F(t) = \sqrt[k]{V(t)/S} \quad (4)$$

So the mass balance equations are :

$$\frac{\partial v(t)}{dt} = F_1(t) + F_2(t) - \sqrt[k]{V(t)/S} \quad (5)$$

$$\frac{\partial}{\partial t}[c(t)V(t)] = c_1F_1(t) + c_2F_2(t) - c(t)\sqrt[k]{V_0/S} \quad (6)$$

In the steady-state situation, all quantities are assumed to be constant, say W_{10}, W_{20} and W_0 for the flow rates, V_0 for the volume and c_0 for the concentration in the tank. Then the following equations will hold:

$$F_{10} + F_{20} - F_0 = 0 \quad (7)$$

$$c_1F_{10} + c_2F_{20} - c_0F_0 = 0 \quad (8)$$

$$F_0 = \sqrt[k]{\frac{V_0}{S}} \quad (9)$$

For the given F_{10} and F_{20}, these equations can be solved for F_0, V_0 and c_0. Let us now assume that only small deviations from the steady-state conditions occur, so we write

$$F_1(t) = F_{10} + \mu_1(t) \quad (10)$$

$$F_2(t) = F_{20} + \mu_2(t) \quad (11)$$

$$V(t) = V_0 + \alpha(t) \quad (12)$$

$$c(t) = c_0 + \beta(t) \quad (13)$$

where μ_1 and μ_2 are considered as input variables and α and β are considered as state variables. By assuming that these four quantities are small and applying linearization method:

$$\alpha(t) = \mu_1(t) + \mu_2(t) - \frac{k}{2V_0}\sqrt{\frac{V_0}{S}}\alpha(t) \quad (14)$$

$$\beta(t)V_0 + c_0\alpha(t) = c_1\mu_1(t) + c_2\mu_2(t) - c_0\frac{k}{2V_0}\sqrt{\frac{V_0}{S}}\alpha(t) - k^2\sqrt{\frac{V_0}{S}}\beta(t) \quad (15)$$

Substitution of eqn. (4) into these equations yields the following,

$$\alpha(t) = \mu_1(t) + \mu_2(t) - \frac{1}{2}\frac{F_0}{V_0}\alpha(t) \quad (16)$$

$$\beta(t)V_0 + c_0\alpha(t) = c_1\mu_1(t) + c_2\mu_2(t) - \frac{1}{2}c_0\frac{F_0}{V_0}\alpha(t) - F_0\beta(t) \quad (17)$$

In this case study, we have considered the output variables η_1 and η_2 and the output equations are as given below,

$$\eta_1 = F(t) - F_0 \cong \frac{1}{2}\frac{F_0}{V_0}\alpha(t) \quad (18)$$

$$\eta_2 = c(t) - c_0 = \beta(t) \quad (19)$$

III. MATHEMATICAL MODELING OF CSTR

Considering Table- 1, as given, we have the state model as below,

Sl. No.	Parameter Name	Parameter Value	Unit of Parameter
1.	F_0	0.020	m^3/sec
2.	F_{10}	0.015	m^3/sec
3.	F_{20}	0.005	m^3/sec
4.	c_0	1.250	$kmol/m^3$
5.	c_{10}	1.000	$kmol/m^3$
6.	c_{20}	2.000	$kmol/m^3$
7.	V_0	1.000	m^3
8.	Total Time Span T	50	seconds

$$\alpha(t) = \mu_1(t) + \mu_2(t) - 0.01\alpha(t) \quad (20)$$

$$\beta(t) = -0.02\beta(t) - 0.25\mu_1(t) + 0.75\mu_2(t) \quad (21)$$

Choosing the state variable as given below:

$x_1 = \alpha,\quad \dot{x_1} = \dot{\alpha},\quad x_2 = \beta, \dot{x_2} = \dot{\beta}$

And $u_1 = \mu_1(t)$, $u_2 = \mu_2(t)$

The state vector looks like $X = \begin{bmatrix} \dot{x_1} \\ \dot{x_2} \end{bmatrix}$

And the state representation of the CSTR are given by

$$\dot{X} = Ax + Bu = \begin{bmatrix} 1 & 0 \\ 0 & 0.02 \end{bmatrix}\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ -0.25 & 0.75 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

(22)

Considering the Table 1, the linearized output equations are as follows:

$$\eta_1 = 0.01\alpha(t) \quad (23)$$

$$\eta_2 = \beta(t) \quad (24)$$

$$Y = CX = \begin{bmatrix} 0.01 & 0 \\ 0 & 1 \end{bmatrix} X \quad (25)$$

IV. CONTROLLER DESIGN FOR THE CSTR AND SIMULATION

A. Introduction to LQR:

For a given system whose state space equations are as below.

$$\dot{X} = Ax + Bu \quad (26)$$

$$Y = CX + Du \quad (27)$$

To design an optimal controller, one should design an input to make the J parameter, which can be found by the given equation, as minimal:

$$J = \frac{1}{2}\int_0^{\infty}[x^T(t)Q(t)x(t) + u^T(t)R(t)u(t)]d(t) \quad (28)$$

where Q and R denote the weighting matrix of state matrix and input variable. In any condition of disturbance, where the system is shifted to another state point, the LQR controller can take the system to the zero state conditions where the J parameter is again minimized [1]. The output value of LQR controller is defined as the optimal control. The control signal is equal to:

$$u(t) = -R^{-1}B^T P(t)x(t) = -Kx(t) \quad (29)$$

In the above equation, P(t) denotes the solution of Riccati equation, K is the linear optimal feedback matrix. The last part of design consists of solution of the Riccati equation.

$$PA + A^T P - PBR^{-1}B^T + Q = 0 \quad (30)$$

The values for P and K are found by

$$K = -R^{-1}B^T P = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \quad (31)$$

B. Controller Design for CSTR & Simulation and Analysis:

The selection of Q and R determines the optimality in the optimal control law. The choice of these matrices depends only on the designer. Generally, preferred method for determining the values for these matrices is the method of trial and error in simulation. As a rule of thumb, Q and R matrices are chosen to be diagonal. In general, for a small input, a large R matrix is needed. For a state to be small in magnitude, the corresponding diagonal element should be large. Another correlation between the matrices and output is that, for a fixed Q matrix, a decrease in R matrix's values will decrease the transition time and the overshoot but this action will increase the rise time and the steady state error. In the other condition, where R is kept fixed but Q decreases, the transition time and
overshoot will increase, in contrast to this effect the rise time and steady state error will decrease.

In the simulation, we have chosen the Q and R matrices as follows.

Q= [125 0; 0 25]

R= [0.01 0; 0 0.01]

With these Q and R matrices and with the MATLAB function lqr (A,B,Q,R), we found the K matrix for solving the optimal problem. The found K matrix is given below.

K = [85.9133, -31.9726; 71.5351, 38.4049].

After finding the K matrix, we applied state feedback and the closed loop system's state space equations became as (A-BK, B, C-DK, D).

The Simulink controller model, used to simulate the model of CSTR we have found, is given below.

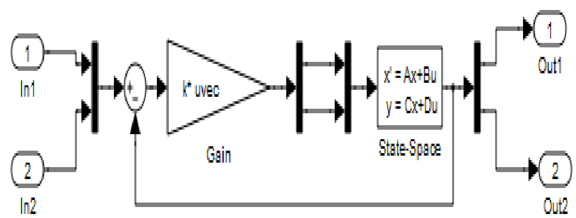

Figure1: LQR Simulink Model for CSTR Model

V. SIMULATION RESULTS

The responses for several different disturbances are simulated and are given as below.

Firstly, we will give the general response of the system for the desired set points. The controller speeds up the settling time of volume and concentration significantly. The system response, the volume and concentration values are given below.

The first figure shows the response of volume versus time of the system. The obtained result is satisfactory for any implementation of this kind of controller. The second figure shows the response of fluid concentration versus time of the system. Although there is a high overshoot, the system's settling time is small which is improving the general effectiveness of the system.

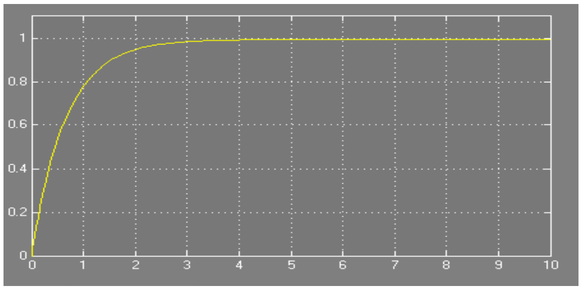
Figure2: The response of the system's volume vs. time

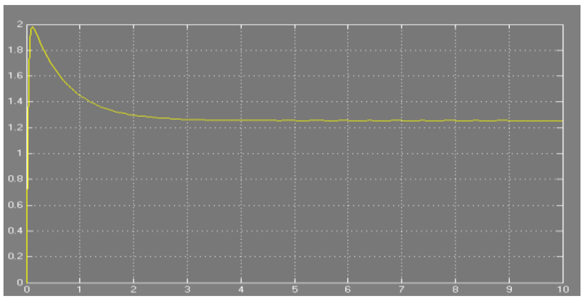
Figure3: The response of the system's concentration vs. time

Secondly, we disturbed the system by an initial value. The response to this type of disturbance converges to zero quickly, which is a highly desired action, and cancel the effects of this type of disturbance. The results obtained at this step are provided below.

Figure 4 shows the response of controller for an impulsive disturbance in the volume of liquid. The controller suppresses the disturbance of magnitude 1 (one) in a time interval of 0.11885 seconds and the output of the controller returns to zero state conditions. Figure 5 shows the response of controller for an impulsive disturbance in the concentration of liquids. The LQR controller cancels the effects of the disturbance of magnitude 1.25 in 0.1915 seconds and the system returns to zero-state conditions.

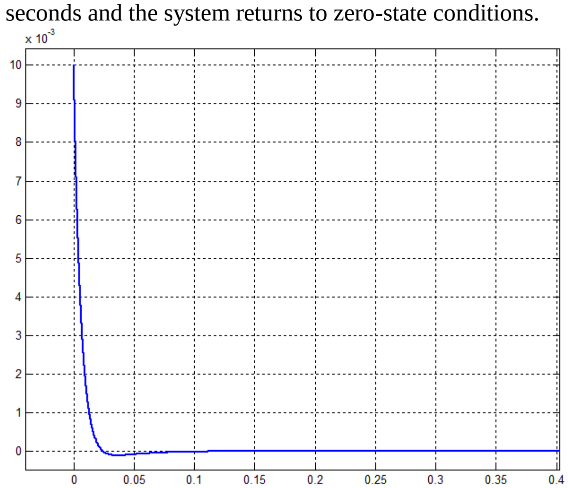

Figure4: The response of controller after the disturbance of mag. 1 is given to the volume.

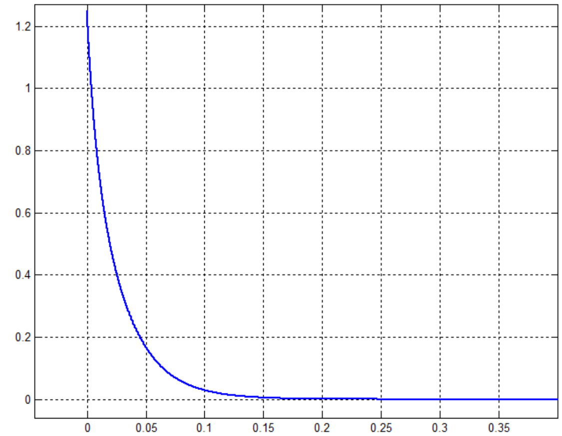
Figure5: The response of controller after the disturbance of mag. 1.25 is given to the concentration.

The third simulation done is by disturbing the system with a step disturbance. In this simulation, we have used step disturbances in order to observe the effects on the controlled variables viz. the volume of liquid, and the concentration of liquid. The obtained results are given below.

Figure 6 shows the variations of first state variable for a step disturbance in the fluid volume in the tank versus time. The result shows that the controller comes to a steady state error value after 0.2339 seconds with a magnitude of 0.005732. This response indicates a very small deviation from the expected result. Figure 7 plots the variations of second state variable for a step disturbance in the fluid concentration in the tank versus time. The simulation shows that the concentration value deviates from the desired value by a negligibly small amount. The obtained results from the simulation gives that the concentration reaches a steady state value in 0.2258 seconds with a magnitude of 0.01537.

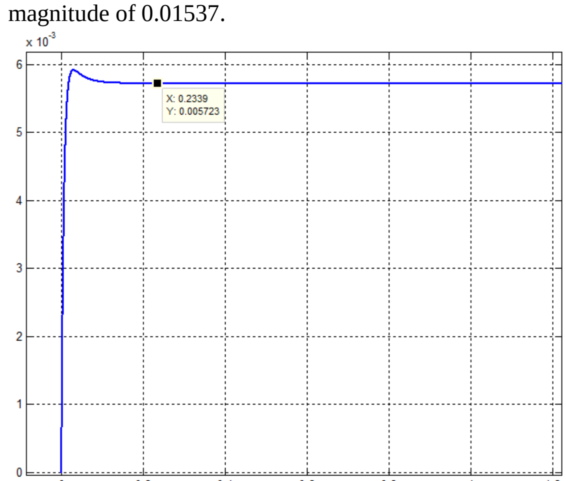

Figure6: The response of first state variable, volume of liquid, to a step disturbance.

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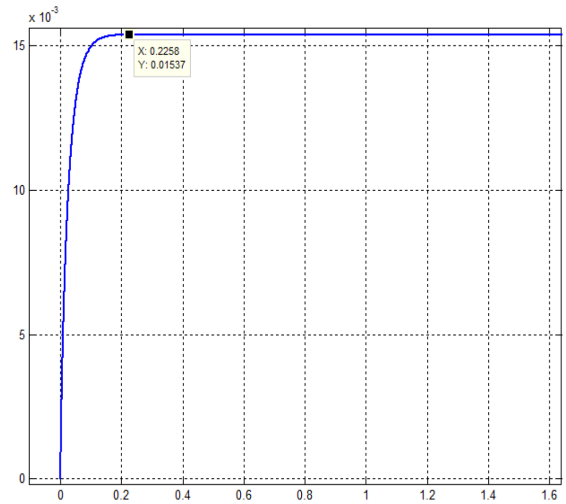

Figure7: The response of second state variable, the concentration of liquid, to a step disturbance.

Figure 8 is an accretion of all the above simulated results under different conditions of disturbances introduced to the volume and concentration of fluid respectively, from which the final conclusion has been drawn in the following section.

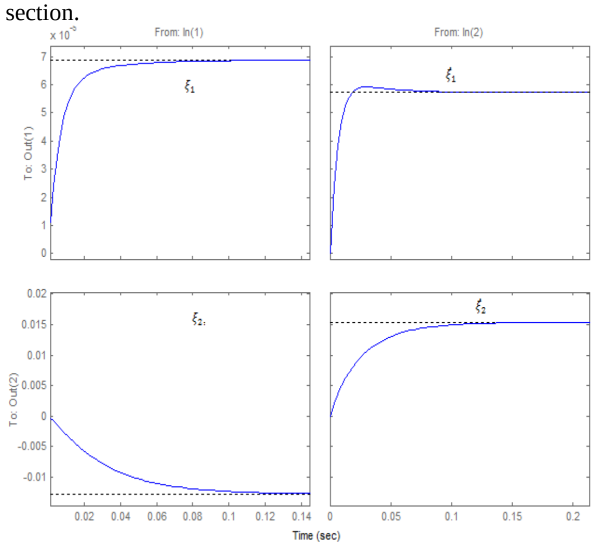

Figure 7: LQR system step response.

VI. CONCLUSION

In this case study, a controller for volume of liquid and concentration for liquid in a CSTR tank is studied. LQR controller is used in optimum control design. Firstly, CSTR model is extracted, since the model contains nonlinearities, the model is linearized. Thus we obtained the state space equations for the CSTR tank. After this point, optimum control design principles are used in order to implement the LQR controller design. The theoretical results are used in MATLAB and tabulated in the preceding sections.
The obtained results indicate that :

- The response of overall system is highly satisfactory for the control of state variables, namely the volume and the concentration of liquid respectively.
- The response to an impulsive disturbance is improved greatly. The obtained results indicate that the controller suppresses the disturbance significantly, in time and in the magnitude.
- The response to a step disturbance is satisfactory. The results show that the steady state error is negligibly small compared to the steady state values of the state variables. Also the controller drives the system to a DC error value which can be eliminated by several other control methods and also can be subtracted from the inlet flow, which is compensating the error caused by this type of step disturbance.
- By optimizing the values of Q and R, the steady state error value is decreased to negligibly small amounts. Thus these parameters can be changed for the desired values of rise time settling time overshoot and steady state error values for any given application

The obtained work is not only limited to the kind of disturbances introduced but can also be put into effect for any other type of disturbances as well and any other kind of system as well other than CSTR.

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