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Kalman Filter Based Channel Estimation Method For MIMO-OFDM System with Low Complexity

Ganesh Miriyala, Vindhya Kanigiri, G Ravindar Reddy and B Praveen Kumar

***Abstract*— In multi input multi output orthogonal frequency division multiplexing (MIMO-OFDM) systems, a low complexity subspace based time domain channel estimation method is studied. This method works based on parametric channel model, where the response of the time varying channel is considered as a collection of sparse propagation paths. Translate the estimation of channel parameters into an unconstrained minimized problem by considering the channel correlation matrix. To solve this problem, Kalman filter based method which tracks the subspace is proposed, which provides the constant subspace to construct state equation and measurement equation. This method represents a low complexity subspace schemes. The approach can be extended to MC-CDMA systems. The simulation results prove that the Kalman filter method can track faster fading channel, and is more accurate with low complexity.**

***Keywords*— Channel estimation, Kalman filter, low rank adaptive filter, subspace tracking and time varying channel.**

I. Introduction

Combination of Multi-input Multi-output (MIMO) with the orthogonal frequency division multiplexing (OFDM) has received a great extent attention in recent times to combat multi path delay spread, and increases the system capacity. MIMI-OFDM has turn out to be one of the most promising technique for the 4G (fourth generation) of wireless communication systems. For the reason that the necessity of high mobility for 4G systems, the signals would experience fast time varying channel environment at MIMO-OFDM receivers and strong Doppler frequency spread interface. This requires the receivers which are exactly estimate, real time track the variation of multipath channels with low complexity.

Ganesh Miriyala
National Institute of Technology Hamirpur, HP
India
gani.india@gmail.com

Vindhya Kanigiri
Jawaharlal Nehru Technological University, Hyderabad, AP
India
kanigiri.03@gmail.com

G. Ravindar Reddy
Teegala Krishna reddy Engineering College, Hyderabad, AP
India
ravinder.497@gmail.com

B. Praveen Kumar
Netaji Institute of Engineering and Technology, Nalgonda, AP
India
boorasmail@gmail.com

In recent years, parametric channel estimation methods have been studied widely in OFDM systems. These methods model the radio channels by a few dominant sparse paths [1]. Hence, parameters of channel paths are estimated before the stage of interpolating over the entire frequency time grid, which are including complex amplitudes and delay. In [2], [3], subspace analysis methods are presented. These methods obtain the channel parameters. But, cumulating the large amount of samples for these methods are very complex. A subspace tracking method is presented in [4]. This method employs to track the delay subspace by using a QR decomposition based Recursive Least Square filter. This QR decomposition should operate in the algorithm flow. Givens rotation and Householder transform are the conventional approaches to QR decomposition with a complexity of $O(N^3)$, for $N \times N$ matrix. Hence, the complexity of the QR-Recursive Least Square (QR-RLS) method is high as $O\left(N_p r^2\right)$, where r is the number of channel paths and N_p is the number of pilot subcarriers. Because of low complexity to track the channel variation compared with decomposition or matrix inverse Kalaman filter is presented in [5]-[7]. In this method, it won't consider the low dimensional signal subspace, i.e., they operate upon high dimensional signal subspace channel matrix, which leads to weaker performance at low signal to noise rate (SNR).

A time varying pilot-aided parametric channel estimation method by tracking the subspace in MIMO-OFDM systems with Kalman filter has proposed. For time varying multi path channels, variation of delay is slow, but not of amplitudes, that gives the guarantee to the valid of this adaptive filter. Accuracy of estimation is significantly increased through the adaptive tracking of a low dimensioned signal subspace. The applications of Kalman filter provide fast convergence with low complexity $O\left(N_p r\right)$. This method is compatible in low complexity and low dimension compared with existing methods.

II. System Model se

A. *Channel Model*

Consider a time varying MIMO-OFDM system with m_T transmit, m_R receive antennas with K subcarriers. The baseband impulse response of wireless channel between j^{th} transmit antenna and the k^{th} receive antenna is given as

$$h_{jk}(t,\tau) = \sum_{m=0}^{N_p-1} a_m^{(jk)}(t)\delta(\tau - \tau_m^{(jk)}(t)) \tag{1}$$

Where N_p the number of paths, $\tau^{(jk)}(t)$ is m^{th} path delay and $a_m^{(jk)}(t)$ is corresponding amplitude. By consider the constant delays in M symbols provided that variations in M symbols are much smaller than the resolution of the OFDM system. Because of Doppler frequency shift, inter-carrier interference is neglected, $a_m^{(jk)}(t)$ assumed to be quasi-fixed over one OFDM symbol.

By considering the pulse shaping filter is used in both transmitter and receiver, then the sampled channel response can be expressed as

$$h_{jk}(n,l)=\sum_{m=0}^{N_p-1} a_m^{(jk)}(n)g(l-\tau_m^{(jk)}(n)),\ l=0,1,...,L-1 \tag{2}$$

Because of the time delays are unknown, Consider L as maximum length of the channel. g (.) is the convolution of the pulse shaping filter and the matched filter at the receiver. $\tau^{(jk)}(t)$ are the sampled time delays. These delays are not integers in general so the channel tap power will leak to other samples [8]. The frequency response of the channel in the n^{th} OFDM symbol can represented as

$$H_{jk}=W_{jk}h_{jk}(n) \tag{3}$$

Where $h_{jk}(n)=[h_{jk}(n,0),h_{jk}(n,1),...,h_{jk}(n,L-1)]^T$,

$$[W_{jk}]_{s,l}=\frac{1}{\sqrt{K}}e^{-\frac{j2\pi sl}{K}} \quad (s=0,1,...,S-1,l=0,1,...,L-1) \tag{4}$$

B. *Delay-Subspace*

In (2), if the number of time delays and multi-paths are known, then the equation can be written as

$$h(n)=\phi_n\alpha_n \tag{5}$$

Where

$$\phi_n=\begin{bmatrix} g(-\tau_1(n)) & g(-\tau_2(n)) & ... & g(-\tau_{N_p}(n)) \\ g(1-\tau_1(n)) & g(1-\tau_2(n)) & ... & g(1-\tau_{N_p}(n)) \\ ... & ... & ... & ... \\ g(L-1-\tau_1(n)) & g(L-1-\tau_2(n)) & ... & g(L-1-\tau_{N_p}(n)) \end{bmatrix}$$

ϕ_n is matrix of $L\times N_p$, which is decided by pulse shaping filter and multipath delays. $\alpha_n=[a_1(n)\ a_2(n)\ \cdots\ a_{N_p}(n)]^T$ is complex gain of the time varying channel. Because of channel estimation method for each transmit antenna is same, the transmit antenna index j will be omitted. We define $r_n=rank(\phi_n)\le\min(L,N_p)$ as rank of matrix ϕ_n, compare with the time resolution $(1/B)$ of the system. The r_n -dimensional subspace spanned by the columns of the ϕ_n is called delay subspace and it's denoted by $L\times r_n$ orthonormal basis as U_n $(L\times r_n)$. By using the concept of singular value decomposition (SVD) $\phi_n=U_n\Lambda_n V_n^H$. Then the equation (5) can be written as

$$h(n)=U_n d_n \tag{6}$$

Where $r\times 1$ vector

$$d_n=\Lambda_n V_n^H\alpha_n \tag{7}$$

This method reduces the total number of delay vector to be estimated and tracked from N_p to r_n.

III. Channel Estimation Based On Pilots

A. *LS Channel Estimation*

Consider the same pilot pattern for each transmit antenna. $P=[p_1\ p_2\cdots p_T]$ being set of T pilot tones in each OFDM symbol. In (4), by considering only pilot-sub-carriers, we can write

$$\tilde{Y}_p(n)=\tilde{X}_p(n)\tilde{H}_p(n)+\tilde{\xi}_p(n)=\tilde{X}_p(n)\tilde{W}_p\tilde{h}_p(n)+\tilde{\xi}_p(n) \tag{8}$$

Where $\tilde{W}_p=diag(\tilde{W}_{p1}\ \tilde{W}_{p2}\cdots\tilde{W}_{pm_T})$, $[\tilde{W}_{pi}]_{s,l}=\frac{1}{\sqrt{K}}e^{-j2\pi\frac{p_s l}{K}}$

$s=1,2,\cdots,S,\ i=1,2,\cdots,m_T,\ l=1,2,\cdots,L$ and $\tilde{X}_p(n),\tilde{H}_p(n)$ are the pilot position values of $\tilde{X}(n)$ and $\tilde{H}(n)$.

The Least Square time varying channel estimated of $h_{LS}(n)$ are given by

$$\hat{h}_{LS}(n)=(\tilde{W}_p^H\tilde{X}_p^H(n)\tilde{X}_P(n)\tilde{W}_P)^{-1}\tilde{W}_p^H\tilde{X}_p^H(n)\tilde{Y}_p(n) \tag{9}$$

And then we get for all antennas as $\hat{h}_{i,LS}(n),(i=1,2,\cdots,m_T)$.

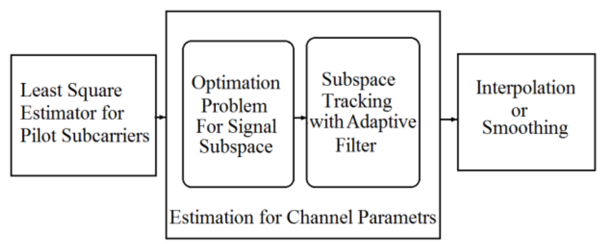

Figure 1. Block diagram of parametric channel estimation

IV. Channel Estimation Based on Subspace Tracking

As shown in Fig. 1, the complete procedure of the parametric channel estimation for $H(n)$ consists of three steps. First, the time varying channel response at each pilot-subcarrier is computed with Least Square (LS) method. Then, the time varying channel parameter of each path are estimated and tracked to improve the accuracy of estimates of Least Square results. Finally, the time varying channel response at all of the N subcarriers is obtained by smoothing or interpolation.

The presented method with adaptive algorithm is used in second step, and they works based on the consideration given below.

Step1: For low-dimensional signal subspace, translate the estimation of time varying channel parameters into an unconstrained minimization problem.

Step2: Solve the step1 problem via subspace tracking with the adaptive algorithm, which is Kalmn filter.

By the assumption of that the total number of channel paths L is obtained from the minimum description length (MDL) method as mentioned in [4].

A. *Unconstrained Minimization Problem for Subspace*

The Least Square estimation for time varying channel with pilot-sub-carriers $\tilde{h}_p(n)$ is $\tilde{h}_{LS,p}(n) = \tilde{Y}_p(n)/\tilde{X}_p(n)$ [4]. Omit subscript $\boldsymbol{p}$, and let $\tilde{h}_{LS}(n) = [\tilde{h}_{LS}(0)\ \tilde{h}_{LS}(1)\ \cdots\ \tilde{h}_{LS}(N_p-1)]^T$, then

$$\tilde{h}_{LS}(n) = \phi_{p,n}\alpha_n + \zeta_n \tag{11}$$

Where $\phi_{p,n}$ is Vander monde matrix of size $\mathrm{N}_p \times L$, and it is a collection of rows corresponding to pilot-sub-carriers in ϕ_n. ζ_n is a vector of AWGN noise with variance σ_ζ^2.

From (6), $h_{LS}(n)$ can be represented as $h_{LS}(n) = U_n d_n$. By defining correlation matrix as

$$\begin{aligned} R &= E[h_{LS}(n)h_{LS}^H(n)] \\ &= U_n \Lambda V^H E[\alpha_n \alpha_n^H] V \Lambda^H U_n^H + \sigma_\zeta^2 I_{N_p} \\ &= U_n \Sigma U_n^H + \sigma_\zeta^2 I_{N_p} \end{aligned}$$

Where U_n consists of r eigenvectors corresponding to main Eigen values of diagonal matrix Σ. So, the estimation of ϕ_n is equal to tracking the signal subspace U_n, for given least square results of each $h_{LS}(n)$.

In order to get the least mean square error (MSE) of estimation, the cost function is given by

$$J(U_n, d_n) = E\left[\left\| h_{LS}(n) - U_n d_n \right\|^2\right] \tag{12}$$

Minimizing the equation (12), we will get the estimator of d_n, which is nothing but the projection of $h_{LS}(n)$ to the signal subspace U_n. As given in [4], $\hat{d}_n = U_n^H h_{LS}(n)$. To get the estimator for subspace U_n, we define the new cost function

$$\begin{aligned} \hat{U}_n &= \arg_{U_n} J(U_n) \\ &= \arg_{U_n} E\left[\left\| h_{LS}(n) - U_n U_n^H h_{LS}(n) \right\|\right] \end{aligned} \tag{13}$$

By observing the equation (13), we can state that the signal subspace tracking as a solution to the minimization problem. The estimates of α_n and delays are converted into optimization problem for U_n. We can solve this problem by three following methods to analyze their computational complexity.

B. *Kalman filter for Subspace Tracking*

In (13), $J(U_n)$ has a stationary point at $\breve{U}_n$, it is proved in [10]. Iff $\breve{U}_n = U_{n,r}\psi$ where $U_{n,r} \in \mathbb{C}^{N\times r}$ any r distinct number of eigenvectors of R, $\psi \in \mathbb{C}^{r\times r}$ is an arbitrary unitary matrix, and $J(U_n)$ has global minima where U_n is equal to signal subspace. Tracking the signal subspace of R is guaranteed by solving the global convergence of $J(U_n)$. Therefore, a unique point $\hat{U}_n^* \in \mathbb{C}^{N\times r}$ is exist, such that $\hat{U}_n^* = \arg\min_{U_n} J(U_n)$. By expanding $J(\hat{U}_n^*)$

$$\begin{aligned} J(\hat{U}_n^*) &= E[h_{LS}^H(n)h_{LS}(n)] - 2E[h_{LS}^H(n)\hat{U}_n^*\hat{U}_n^{*H}h_{LS}(n)] \\ &\quad + E[h_{LS}^H(n)\hat{U}_n^*\hat{U}_n^{*H}\hat{U}_n^*\hat{U}_n^{*H}h_{LS}(n)] \end{aligned} \tag{14}$$

Observe that $h_{LS}(n) = \hat{U}_n^* d_n + \zeta_n$, $\hat{U}_n^{*H}\hat{U}_n^* = I_r$, ζ_n and d_n are uncorrelated, then

$$J(\hat{U}_n^*) = E\left[\zeta_n^H \zeta_n\right] - E\left[\zeta_n^H \hat{U}_n^* \hat{U}_n^{*H} \zeta_n\right] < N_p \sigma_\zeta^2 \tag{15}$$

Using Kalman filter to reach $\hat{U}_n^*$ adaptively, $x(n) = h_{LS}(n)$, $y(n) = \hat{U}_n^*(n)x(n)$, then construct the state equation

$$\hat{U}_n^*(n+1) = \hat{U}_n^*(n) \tag{16}$$

and the measurement equation

$$x(n) = \hat{U}_n^*(n)y(n) + \tilde{e}(n) \tag{17}$$

Where $\tilde{e}(n)$ is minimal error vector when adapting $\hat{U}_n^*$, and

$$\begin{aligned} E\left[\tilde{e}^H(n)\tilde{e}(n)\right] &= E\left[\left\| x(n) - \hat{U}_n^*(n)\hat{U}_n^{*H}(n) \right\|^2\right] \\ &= J(\hat{U}_n^*) < N_p \sigma_\zeta^2 \end{aligned} \tag{18}$$

The channel estimation algorithm based on subspace with Kalman filter is given as

Initialization:

$$K(1,0) = I_r,\ U_n^*(0) = \begin{bmatrix} I_r \\ 0 \end{bmatrix},\ \zeta_e(0) = c_1$$

For each OFDM symbol $n = 1, 2, \cdots$

$x(n) = h_{LS}(n);\ y(n) = \hat{U}_n^*(n-1)x(n);$

$g(n) = \left[K(n,n-1)y(n)\right] / \left[y^H(n)K(n,n-1)y(n) + \zeta_e(n-1)\right];$

$K(n+1,n) = K(n,n-1) - g(n)y^H(n)K(n,n-1);$

$\hat{U}_n^*(n) = \hat{U}_n^*(n-1) + \left[x(n) - \hat{U}_n^*(n-1)y(n)\right]g^H(n);$

Estimator Output:

$h_{Kalman}(n) = \hat{U}_n^*(n)\hat{U}_n^{*H}(n)x(n);$

$\zeta_e(n) = c_2 \cdot norm\left(h_{Kalman}(n) - x(n)\right).$

In order to get the better convergence, update $\zeta_e(n)$ for each OFDM symbol. $c_1 = 10$ and $c_2 = 0.75$ are adjustable constant factors.

C. *Comparison of Kalman Filter and QR-RLS Methods*

Kalman filter based method is used to obtain the signal subspace, can be achieved by minimizing the (13), and mainly focused on low-dimensional subspace and to avoid the matrix inverse or decomposition, apply the tracking technique. For this method, the tracking operation of each OFDM symbol is low as $O(N_p r)$ when compare to QR-RLS with $O(N_p r^2)$ given in [4], [11]. With the estimator output of the time varying channel $h_{Kalman}(n)$, we can interpolate the frequency time grid.

V. Simulation Results

The performance of the proposed time varying channel estimation method has been done by conducting the simulation for MIMO-OFDM systems with two transmit antennas and two receive antennas. The system consists of 256 subcarriers occupying the bandwidth of 2 MHz, $L = 16$ is the maximum length of channel, QPSK modulation scheme, each OFDM symbol consists of $K_p = 32$ pilots, and optimal pilot design is used. The time varying channel model is COST 207 typical urban (TU) environment with multi-paths $N_p = 6$.

Fig 2 and Fig 3 gives the performance of mean square error (MSE) and bit error rate (BER) of channel estimation with respect to the signal noise ratio of QR-RLS estimator and Kalman filter with optimal pilots, the maximum Doppler frequency shift is 40Hz. The curves of RLS and Kalman filter are almost super imposed with same error flow and convergence speed. But complexity of Kalman filter is low.

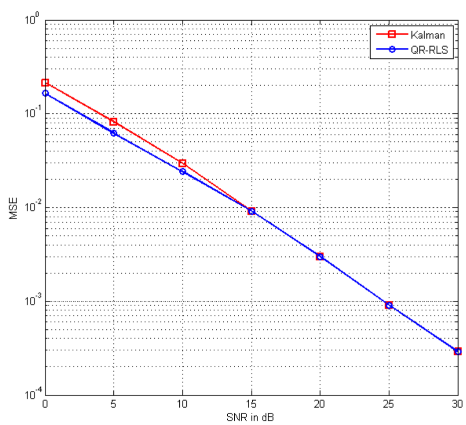

Figure 2. The MSE of channel estimation versus SNR at f_{dmax} = 40Hz.

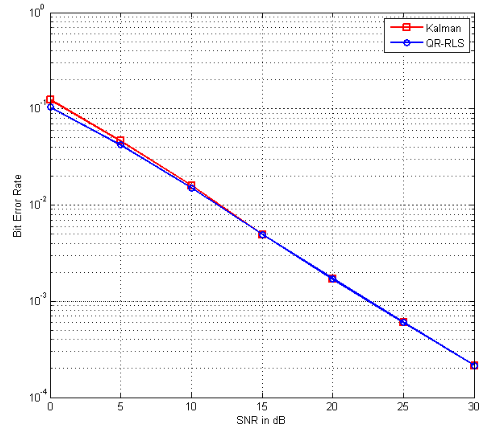

Figure 3. The BER of channel estimation versus SNR at f_{dmax} = 40Hz.

Fig 4 and Fig 5 gives the performance of MSE and BER of channel estimation with the variation of signal noise ratio of QR-RLS and Kalman filter respectively with maximum Doppler frequency shift f_{dmax} = 40Hz and 200Hz. Here, ζ_n in (11) consists of ICI terms and AWGN noise. If f_d increases, ICI tends to be serious. RLS and LMS methods cannot find the AWGN noise and ICI, so that once there exists Doppler frequency spread. But, proposed Kalman filter method has capability to suppress the ICI and gives the better performance in this channel.

The simulation is performed in Doppler frequency shift of 40Hz and 200Hz with vehicular velocity of 14.4 km/h and 72km/h respectively. Kalman filter method reduces the MSE and BER of channel estimation almost one order of magnitude less than QR-RLS method. But Kalman filter method consumes lower complexity.

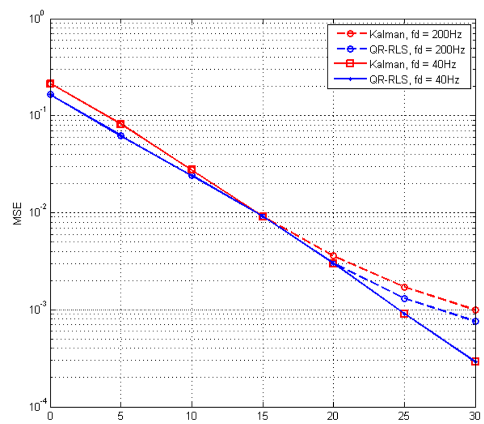

Figure 4. The MSE of channel estimation versus SNR at f_{dmax} = 40Hz and 200Hz.

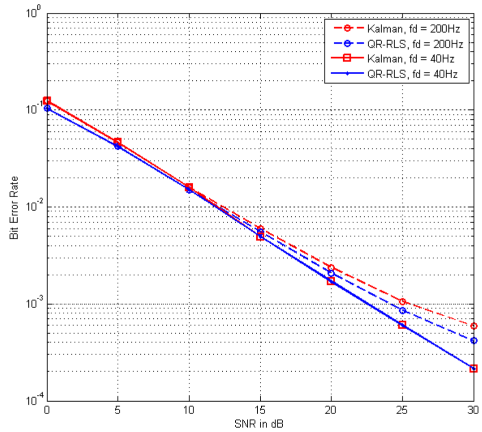

Figure 5. The BER of channel estimation versus SNR at f_{dmax} = 40Hz and 200Hz.

VI. Conclusion

A Kalman filter based time domain pilot aided parametric channel estimation method is presented. The signal subspace of the time varying channel correlation matrix is tracked by the Kalman filter. The key of this method lies in the idea that the estimate of time varying channel parameters can be translated into an unconstrained minimization problem. To solve this problem, apply this method via subspace tracking. This method consumes low complexity. For Doppler frequency spread channels, Kalman filter method is best one.

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About Authors:

Ganesh Miriyala received the graduation in Electronics and communication engineering from Swami Ramananda Tirtha Institute of Science and Technology, Affiliated to Jawaharlal Nehru Technological University, Hyderabad, India.

He is currently doing Masters in Signal Processing in Electrical Department in National Institute of Technology Hamirpur, India. He published an IEEE paper on MIMO-OFDM Channel Estimation. Research interests lie in the field of signal processing for digital communications, with emphasis on MIMO systems, multicarrier modulation, and channel estimation.

Vindhya Kanigiri received the graduation in Electronics and communication engineering from Swami Ramananda Tirtha Institute of Science and Technology, Affiliated to Jawaharlal Nehru Technological University, Hyderabad, India.

She is currently doing Masters in Systems and Signal Processing in Department of Electronics and communication in Jawaharlal Nehru Technological University Hyderabad, India. Research interests are Image and Signal Processing.

B. Praveen Kumar received the Masters in VLSI System Design from Netaji Institute of Engineering and Technology, Affiliated to Jawaharlal Nehru Technological University, Hyderabad, India.

He is currently working as a Assistant professor in Netaji Institute of Engineering and Technology, Affiliated to Jawaharlal Nehru Technological University, Hyderabad, India.

G. Ravindar Reddy received the Masters in Embedded System from CVSR Engineering college, Affiliated to Jawaharlal Nehru Technological University, Hyderabad, India.

He is currently working as a Assistant professor in Teegala Krishna reddy Engineering college, Affiliated to Jawaharlal Nehru Technological University, Hyderabad, India.