

# Comparative Study of Pitch Angel Control using QFT & PID tuning

[Chandrima Roy<sup>1</sup>, Kalyankumar Datta<sup>2</sup>]

**Abstract**— The main application area of QFT ([1],[2],[3]) includes the Power system stabilizers, Hydraulic Actuators, Distillation Columns, Power Electronics Wastewater Treatment system, Flight Control System, Robotics, Different process control system etc. Recent application of QFT has been developed in CDMA, FPGA related domains as well. In this paper QFT is used for controlling the pitch angle and a comparative study with PID tuning is also given.

**Keywords**— QFT,PID, Parametric Gridding methods, Horowitz –Sidi method, Loopshaping, Prefilter design.

## I. Introduction

In QFT, controllers are designed to satisfy the uncertain plant dynamics and external disturbance. In [3], QFT based controller is designed for Boeing-707 aircraft with the aim of controlling pitch angle with reference to elevator deflection. The same plant and problem specification has been taken for designing a QFT based controller in this chapter. In [3], methods of template and bound generation are not explicitly stated. To generate templates, parametric gridding method [4] is used & to generate bounds conventional method [5] is followed to compute the results. After loop shaping & Prefilter design using QFT toolbox in MATLAB environment [6], the final results are verified for performance specifications given in [3].

## II. Problem Definition

### A. Plant description

The plant model in the simplest form with parametric uncertainties is represented in [3] by the following transfer function:

$$P(s) = \frac{25a}{s(s+25)(s+b)} \dots\dots\dots (1)$$

Where,  $a \in [0.309, 0.883]$ , and  $b \in [0.684, 1.558]$  .... (2)

The variations in the plant parameters produce a uncertainty and also decreases its robustness.

Chandrima Roy<sup>1</sup> /Assistant Professor, Dept of ECE  
Heritage Institute of Technology  
Kolkata, India  
chandrimaa.roy@gmail.com

Kalyankumar Datta<sup>2</sup>/Professor Emeritus (Hons.), Dept of EE

Jadavpur University  
Kolkata, India  
kalyankumardatta@gmail.com

### B. Block diagram of the system

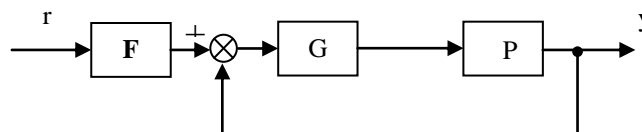


Fig 1. QFT design structure of the Plant.

F=prefilter G=controller P=plant

## III. Performance Specification

### Tracking specification:

- Overshoot < 2%;
- Settling time < 3sec
- Steady state error: nil

### Stability specification:

Gain margin > 4.95 dB; Phase margin>45.12 deg

Taking in to account the time domain specifications, the corresponding tracking models are calculated as,

$$T_{RU}(s) = \frac{6.25}{s^2 + 3.9s + 6.25} \dots\dots\dots(3)$$

An additional zero is given (at  $s=-20$ ) to the model to flare the tracking bounds in the high-frequency range. The resulting upper tracking bound is represented by the following transfer function,

$$T_{RU}(s) = \frac{6.25(0.05s+1)}{s^2 + 3.9s + 6.25} \dots\dots\dots(4)$$

The lower tracking bound is chosen initially as a first order model,

$$T_{RL}(s) = \frac{1.4}{s+1.4}$$

Two poles are added (at  $s=-3$  and  $s=-5$ ) to fulfill the requirement that the upper and lower bounds diverge above the frequency at which the upper bound crosses through 0dB. The resulting lower tracking bound transfer function is as follows,

$$T_{RL}(s) = \frac{1.4}{(s+1.4)(s+3)(s+5)} \dots\dots\dots(5)$$

The acceptable tracking bounds range, defined by these two bound transfer functions is depicted in Fig (2), in the time domain (step response).

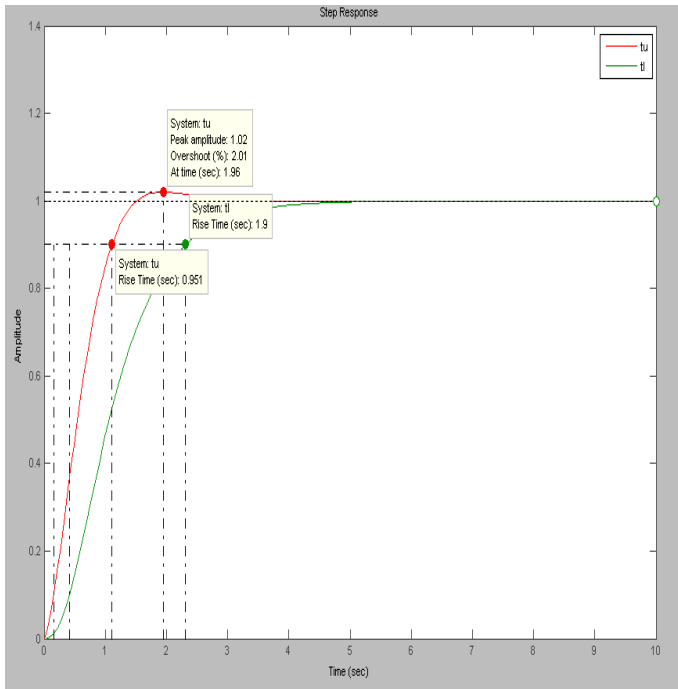


Fig 2: step response of the upper and lower bounds

A. Selection of design parameters

design process is to select an adequate and finite set of frequencies. This set is determined by the bandwidth of the system and by the frequencies of interest, for which the different desired behaviour specifications are defined. In this case, the discrete operating frequency range as;  $\omega = [0.01, 0.1, 0.5, 2, 8, 20, 60]$  is taken from reference [3].

III. Design procedure

This process is carried out by Template and Bound Generation. When the system is not defined by a single model due to the parametric uncertainty, the frequency responses of the system for a given frequency is represented by a set of points, as many different models are there. All of these points define a region of uncertainty known as *Template*. There will be as many templates as frequencies in the set  $\omega$ . The most common way to calculate a template is to perform a sweep of the values that the model parameters can take. As mentioned earlier, the plant model consists of four uncertain parameters with uncertainty range defined

earlier have to be defined. Do not use abbreviations in the title or heads unless they are unavoidable.

A. Choice of Nominal Plant & Template Generation

The selected nominal plant is given below which corresponds to  $a=0.309, b=0.684$  & the nominal plant is indicated in Fig. (3) by a star (\*)

$$Po(s) = \frac{7.725}{s^3 + 25.68s^2 + 17.1s} \dots\dots\dots(6)$$

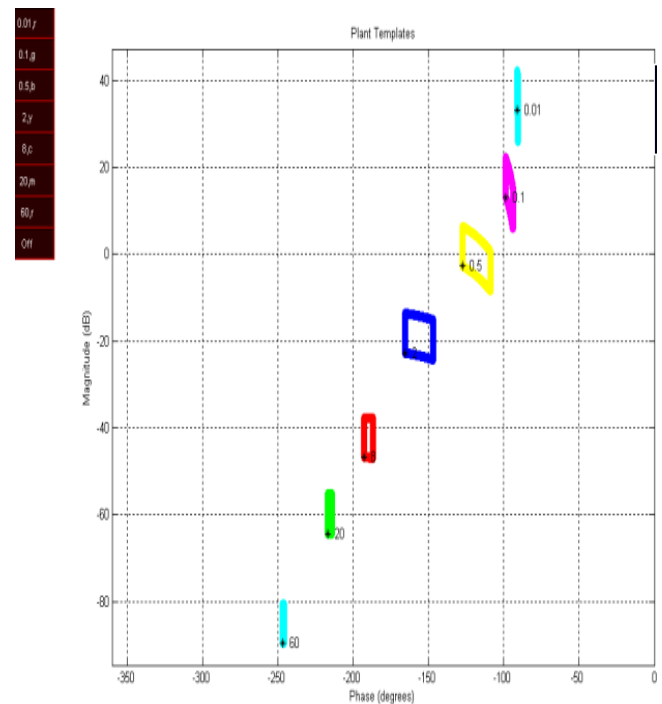


Fig 3: Plant Templates using Parametric Gridding method

**B. Bound generation**

In the bound generation step of Horowitz’s quantitative feedback theory (QFT) design procedure [5], the plant template is used to translate the given robustness specifications into frequency domains in the Nichols chart where the controller gain-phase values are allowed to lie. These domains define what are commonly known as QFT controller bounds. The next step in the design is to define, in QFT terminology, the desired behavior restrictions. The specifications given, combined with the uncertainty of the system, form what are termed bounds. They are represented on the magnitude-phase plane, and there is one for each frequency and specification; they are denoted as  $B(\omega)$ .

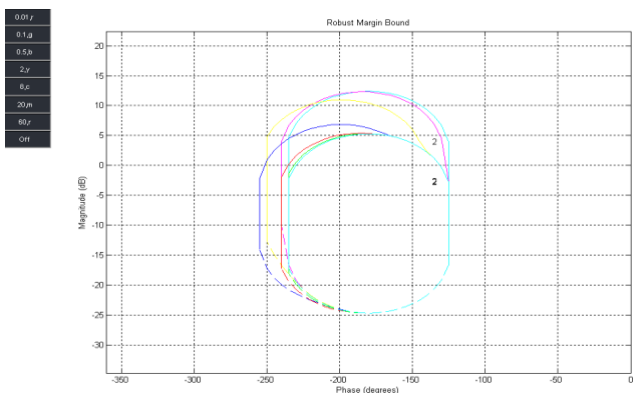
Out of various types of bounds, robust stability bounds and robust tracking bounds are of our interest as per the problem requirement. Relative stability is normally expressed in terms of certain desired gain margins and phases. These are related with a value in decibels  $\gamma$  known as the M-circle because it takes this shape if represented in a magnitude-phase diagram. This circle identifies an exclusion zone around the point  $[-180^\circ, 0\text{dB}]$  in NC, which the loop function must not cross in order to ensure the margin of minimum stability. The specification of robust stability is written as:

$$\left| \frac{P(s)G(s)}{1+P(s)G(s)} \right| \leq W_{SI} = \gamma = 1.3. \text{ For all } \omega \geq 0$$

The tracking specification is established by means of lower,  $T_{RL}(s)$ , and upper,  $T_{RU}(s)$  bounds in the system response. In order to apply the QFT technique, this specification is defined in the frequency domain as follows

$$T_{RL}(s) \leq \frac{F(s)G(s)P(s)}{1+P(s)G(s)} \leq T_{RU}(s)$$

In order to obtain robust tracking bounds, a sub set of  $\omega$  is considered combining the specification and the uncertainty; the robust tracking bounds are obtained.

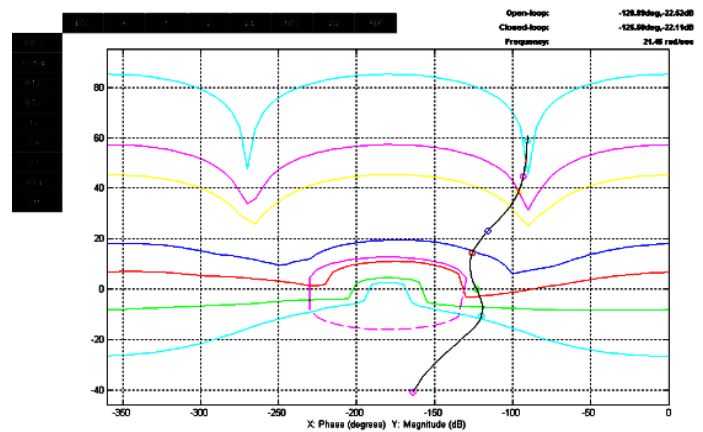


**Fig 4: Robust margin bounds by Horowitz –Sidi method**

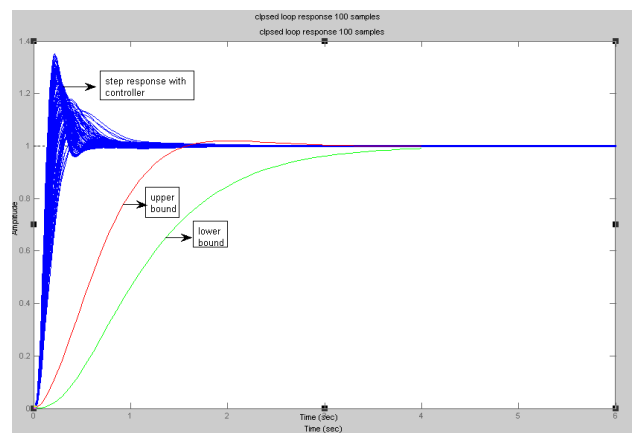
**C. Proposed design of QFT controller and prefilter**

The adjustment is made using the Matlab QFT Toolbox[6], shifting the loop curves vertically and horizontally on the magnitude-Phase plane, until it is situated in such a way as not to violate the bounds and also to have the lowest possible gain. The representation of the loop function  $L_o(s)$  is a curve with several points. These points correspond to the response of the loop for the various frequencies defined in  $\omega$ . From Fig: (6), the open loop frequency response is located below the appropriate tracking performance bounds at each trial frequency. Thus an appropriate control gain should be introduced to push the open loop frequency response upwards. Now as the open loop frequency response has no intersection with the stability bounds, no dynamic compensator is required to change the shape of the open loop frequency response. The controller (a simple gain controller) is finally designed as:

$$G(s) = 29$$



**Fig 5: Loop Shaping in Nichols Chart**



**Fig 6: step response of the closed loop plant family with controller along with the upper and lower bounds**

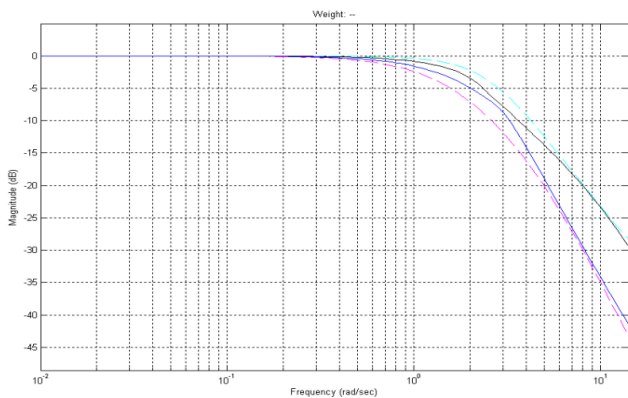
The controller design has reduced the variations in the closed loop frequency response to the desired range. A Prefilter is now required to achieve the required shape of the closed loop frequency response.

The Prefilter that makes the frequency response to be a desired one is obtained as:

$$F(s) = \frac{\frac{s}{6.5} + 1}{\frac{s}{1.8} + 1} \dots\dots\dots(7) \text{ Here it has to be}$$

checked if the system with Controller G(s) and Prefilter F(s) satisfies all the required design specifications or not. Both in time and frequency domain validity of the design is checked. Here, the closed loop frequency response of the uncertain system with Controller and Prefilter satisfies the design specifications, as shown in the following figures. These steps are referred as frequency and time domain validation.

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**Fig 6. Closed Loop Frequency response with Prefilter F(s)**

**D. Response with PID controller**

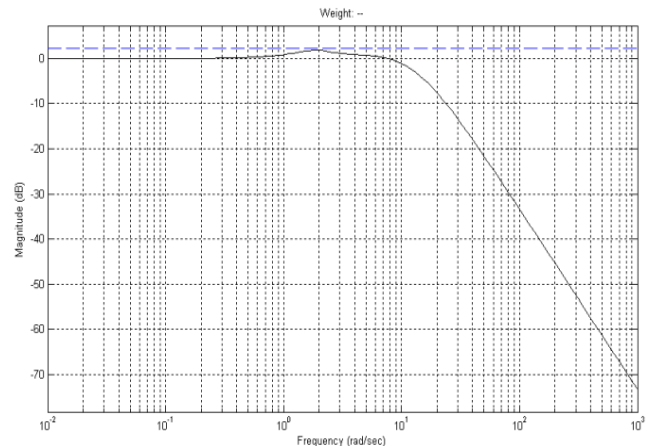
PID controller in parallel form :->>  $K_p + K_i/s + K_d s$  where the followings are the value set  $K_p = 2.31, K_i = 3.73, K_d = 0.228$ .

The design with this PID controller results in Crossover freq: 3.4451rad/sec, PM= 60 dB, GM= 28dB. And the tracking specifications remain as

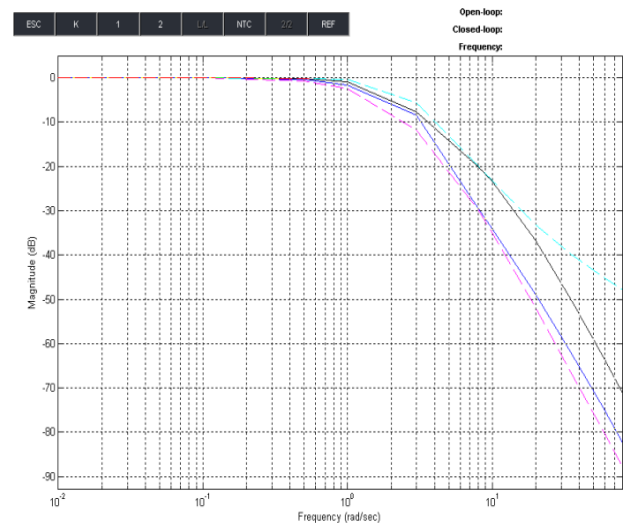
Overshoot: 0%,  $T_s = 15$ sec, steady state error is nil. Only the setting time specification could not be matched.

**E. Design validation**

Headings, Here it has to be checked if the system with Controller G(s) and Prefilter F(s) satisfies all the required design specifications or not. Both in time and frequency domain validity of the design is checked. Here, the closed loop frequency response of the uncertain system with Controller and Prefilter satisfies the design specifications, as shown in the



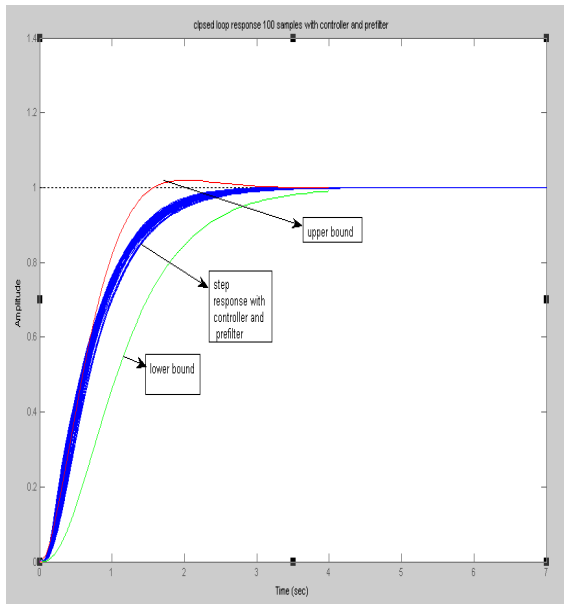
**Fig 7: Analysis of Robust Stability Margins with Controller G(s)**



**Fig 8: Analysis of Robust tracking frequency response with Controller & Prefilter**

This figure illustrates the tracking performance results where the maximum variation of the closed loop system frequency response is drawn (the area between black & blue line) together with the design specifications (the cyan & magenta line). The resultant closed loop system has met all the design specifications in the operating range.

The response of the plant with the PID controller and QFT designed prefilter looks as the figure given below:



**Fig 9 step response of the closed loop plant family along with PID controller and QFT Prefilter**

## IV. Conclusion

Template generation is carried out with only Parametric Gridding method [4] which is very common in use. QFT Bounds are computed by conventional algorithm using MATLAB QFT toolbox[6].The desired specification is achieved only with The help of gain controller and an appropriate Prefilter is Designed.Then ,the design validation is verified and it is observed That the specification given in [3] are met satisfactorily.

## References

- [1] J.W. Kim, D.J. Xuan and Y.B. Kim, "Robust control application for a three-axis road simulator," *Journal of Mechanical Science and Technology*, vol. 25, no.1, pp. 221-231, 2011J.
- [2] K.K.Ahn and Q.T. Dinh "Self-Tuning of Quantitative Feedback Theory for Force Control of an Electro-Hydraulic Test Machine," *Control Engineering Practice*, vol. 17, pp. 1291–1306, 2009.
- [3] L.I. Qiang and C. Jian, "Design of Robust Controller in Electro Hydraulic Load Simulator," in Proc. of the 5<sup>th</sup> Int. Conf. on Machine Learning and Cybernetics, pp. 779-784, August 2008.
- [4] P.S.V Natraj, "Robust Controller Design for Aircraft using QFT,"
- [5] D.J. Balance and G. Hughes, "A survey of Templates generation method for Quantitative Feedback Theory," *UKACC Int. Conf. on Control*, in Proc. of IEEE, vol. 1, pp. 172-174, 1996.
- [6] Horowitz, "Quantitative Feedback Theory," in *IEE Proc*, vol. 129, No 6, Nov. 1982.
- [7] C. Borghesani, Y. Chait, O. Yaniv, Quantitative Feedback Theory Toolbox *User's Guide*, The Math Works Inc., Massachusetts, USA, 1994.