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Synthesis of Digitally Controlled Cosecant Squared Pattern in Time-Modulated Linear Antenna Arrays using Differential Evolution

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Abstract- **In this paper, the additional dimension 'time' is used to synthesize cosecant squared shaped, beam pattern in linear antenna arrays with digitally controlled static amplitude and the phase shift of the array elements. The static amplitude and the phase shift are assumed to be obtained from the output of a five bit digital converter. For the desired shaped beam pattern, normalized on-time duration and the quantized value of the static amplitudes and phase shift are obtained by using Differential evolution (DE) algorithm. The search space of the static amplitudes, phase shift and switch on-time durations are considered as (0.05, 1), (0, 2π) and (0.04, 1) respectively. Thus with relatively low dynamic range ratio of static amplitude and sufficiently high value of on-time duration, the feed network of the antenna array can be designed easily. The proposed approach is demonstrated with a 30 element time modulated linear antenna array (TMLAA) by synthesizing the shaped beam pattern for the angular beam width of 30^0 and reducing the maximum side lobe level (SLL_{max}) and sideband level (SBL_{max}) to -22.92 dB and -27.29 respectively.**

Key words— **Time modulated Linear Antenna Array (TMLAA), Sideband Level (SBL), Side lobe level (SLL), Cosecant Squared Pattern (CSP), Differential Evolution (DE).**

I. Introduction

In 1962, Shanks and Bickmore [1] first utilized the time modulation technique in an eight element slotted linear antenna arrays to synthesize low/ultra-low side lobe patterns. To control the power pattern of the array by using an additional dimension 'time', ON–OFF switching devices is used in the feed network to switch on the different array element for their stipulated on-time duration at regular interval of time. The fourth dimension 'time' reduces the stringent error in designing the antenna array of ultralow side lobe pattern. Since, the time modulation technique can be employed to obtain the pattern of low/ ultra low side lobe levels (SLLs) with the dynamic range ratio of static amplitude that required for obtaining the pattern of ordinary SLLs in conventional antenna arrays. However because of time modulation, the time modulated antenna arrays (TMAAs) generate unwanted harmonics or sidebands spaced at multiples of the switching frequency as detailed in [2]. The sideband radiation causes the energy losses and hence reduces the efficiency and directivity of the array [3]. In [4], the differential evolution (DE) algorithm is applied to synthesize the power patterns in time modulated linear antenna arrays with low value of dynamic range ratio of static amplitude and sideband level (SBL). Also, genetic algorithm (GA) in [5], simulated annealing (SA) technique in [6], is successfully utilized to optimize the "switch-on" time sequence to control the sideband power by reducing the side lobe level of uniformly excited linear arrays. It is further observed that other optimization algorithm such as artificial bee colony (ABC) [7] and particle swarm optimization (PSO) [8] is used to optimize the time modulated antenna arrays.The synthesis of time-modulated antenna arrays with the desired pattern at fundamental radiation is a multi-objective optimization problem [9]. The multiple objectives for synthesizing the Cosecant Squared Pattern (CSP) in time modulated linear antenna arrays (TMLAAs) are the beam width of the desired shaped beam pattern, side lobe level (SLL), ripple and the maximum sideband level (SBL_{max}). The CSP is preferred due to the fact that, this pattern can provides high gain near the horizon [10] where the range is higher but low gain at high-elevation angles where range is lower. Thus, this pattern is useful for an aircraft system because when the aircraft is near the horizon its range is higher whereas at high elevation angle its range is lower. As a result when it moves from the horizon towards the higher elevation, the corresponding aircraft receiver gets almost constant power at different elevation angles although its range is changing. Also antennas with cosecant squared radiation pattern are used for costal and air surveillance radars because of its efficient scanning ability in the space. A cosecant squared radiation pattern can be achieved by using multiple feeders and a reflector antenna. The analytical methods such as Taylor series, Dolph–Chebyshev, Binomial, etc are easily trapped in local optima to produce the pattern at fundamental radiation but there is no control in sideband radiation. In this work, our objective is to synthesize digitally controlled cosecant square pattern in linear time modulated antenna arrays by suppressing the SLL and SBL to a sufficiently low value. The array elements are assumed to excite with the static amplitude and phase shift which are obtained from the output of a five bit digital converter. The digitally controlled voltage or phase are preferred over analog value due to the fact that application of the digital converter in the array feed network reduces the design complexity and hence cost. The differential evolution (DE) algorithm is used to optimize the multiple objectives of the optimization

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problem. DE is a population based, stochastic evolutionary algorithm. The algorithm uses a fewer control parameters and also has the ability to find the true minima not depending upon the initial parameters values [11]. In the DE [11-13] algorithm the vector differences of floating-point parameters are found out instead of the uniform or point crossovers on binary strings which are followed in the genetic algorithm (GA). The most significant advantage of the DE compared to the GA is that it helps us in optimizing large antenna arrays with more computational efficiency as compared to the GA.

II. Time modulated antenna array

A linear array antenna having N number of isotropic elements with uniform spacing equal to half of the operating wavelength as shown in Fig.1, where each element is controlled by a high speed RF switch. A switch on–off time function, U_n (t), n = 1, 2, . . ., N, represents the on and off time duration of the n-th element in every modulation period Tp. The far-field pattern of the time-modulated linear array is given by

$$E(\theta,t) = e^{j2\pi f_0 t}\sum_{n=0}^{N-1} A_n U_n(t) e^{j(n\pi\cos\theta+\varphi_n)} \quad (1)$$

Where A_n and φ_n are the static amplitude and phase shift of the n-th element; f_0 is the centre frequency; θ is angle between the line joining the observing point and origin with the axis of the array. The switch on–off time function of the n-th element, U_n(t) is a periodic function with time period T_p, and is on for the time T_{on}^n $(0 \le T_{on}^n \le T_p)$ in each period T_p, as shown in Fig.2. The periodical excitation of the array elements can be decomposed by applying Fourier series technique and the resulting array factor expression at k-th harmonics can be written as in (1), [2].

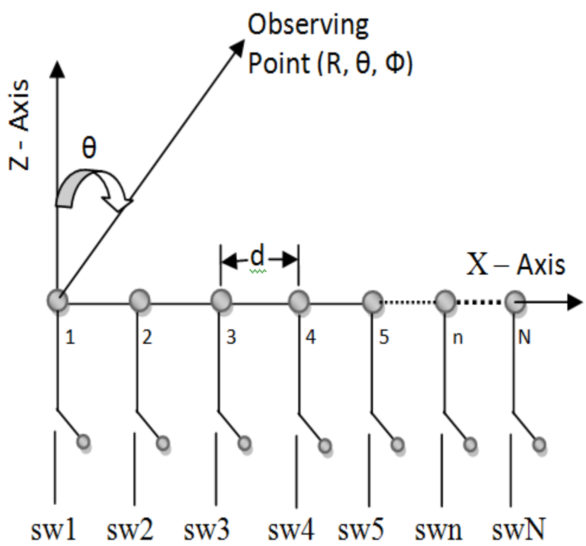

Fig.1. Linear array having N-element, controlled by RF switches

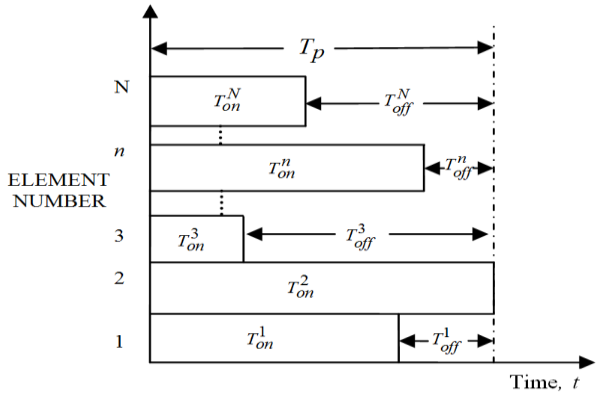

Fig.2. On-Off time duration of each element during one time modulation period T_p

$$E_k = e^{j(\omega_0+k\omega_p)t}\sum_{n=1}^{N} A_n\tau_n \frac{\sin(k\pi\tau_n)}{k\pi\tau_n} e^{-j[\pi(k\tau_n - n\cos\theta)+\varphi_n]} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

Where $\omega_p = 2\pi/T_p$ is the periodic switching frequency in rad/sec, $\tau_n = T_{on}^n / T_p$ is the normalized on-time duration of the n-th element. From (2), it is seen that, in addition to the operating frequency, $\omega_0 = 2\pi f_0$; TMLAA also radiates the signal at different harmonics of ω_p. At the operating frequency, the desired pattern can be synthesized by using an additional dimension 'time'. However the unwanted harmonic radiation (called sideband radiation) power may be high which reduces the antenna efficiency and gain. In this paper DE is used to synthesize the desired cosecant squared pattern by reducing the sideband level to a sufficiently low value.

III. DE Algorithm

In 1995, Price and Storn commenced the Differential Evolution (DE) algorithm which is based upon differential mutation operator. It is a robust statistical method for the minimization of the cost function as the algorithm does not make use of a single nominal parameter vector but it uses a population of equally important vectors. The flowchart of DE algorithm is given in Fig.3. In the algorithm the objective function is sampled by a set of initial points which are chosen randomly from the entire search space. Then in the next step the algorithm adds the weighted difference between the two randomly selected population vectors to the third random population vector to generate a new parameter vector. This process of generating the new parameter vector is called mutation. Now this parameter vector is further mixed with the predefined parameters to produce the trial vector and this process is called crossover. Then in the last step, called selection in which trial vector is replaced by the target vector if the trial vector reduces the values of the cost function than that obtained due to the target vector. To realize the algorithm let the problem is a function of D number of independent parameters. In this work for N element array, the number of independent parameters is 3N in which the first N parameters are the normalized amplitude coefficient of the N array elements; next N parameters are to represent the static phase of each element; and remaining N to represent the on-time

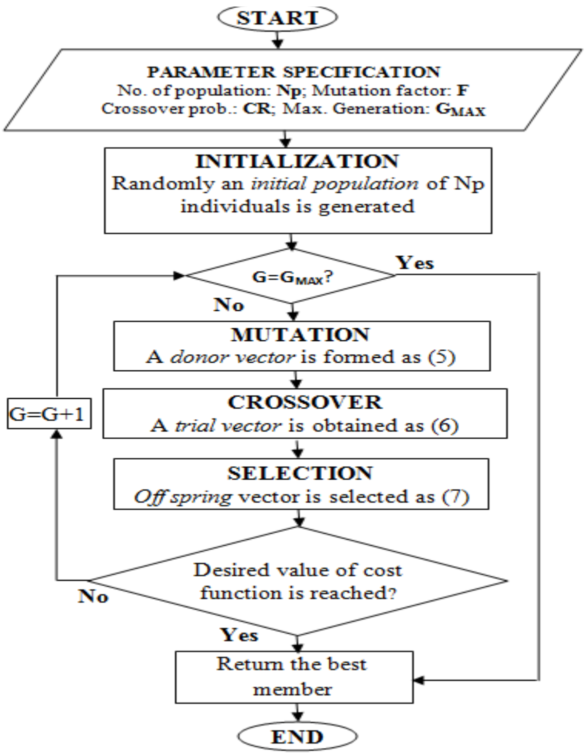

Fig.3. Flowchart of DE Algorithm

duration of the array elements. Hence, If NP is the population size then the parameter vectors are represented as

$$X_{k,G} = [x_{1,k,G}, x_{2,k,G}, \ldots\ldots x_{D,k,G}]k \quad (3)$$

Where k = 1, 2, 3 ...NP and G is the generation number

The four basic steps of DE algorithm are as follows-

A. *Step-1 Initialization*:

In this step the initial parameter $x_{i,k,1}$ which is the i[th] parameter of k[th] vector for 1[st] generation is chosen randomly to initiate searching on an interval $[x_i^l, x_i^u]$ where x_i^l and x_i^u are the upper and lower bounds for each parameter respectively and are defined as in (4).

$$x_{i,k,1} = rand_j(0,1).(x_i^l - x_i^u) + x_i^l \quad (4)$$

Where $rand_j(0,1)$, gives a uniformly distributed random number in a range (0,1).

B. *Step-2 Mutation*:

In this step the search space is expanded from its initial location. Two vectors $X_{r1,G}$ and $X_{r2,G}$ are selected randomly for a k[th] target vector $X_{k,G}$. Let $X_{best,G}$ is the best vector of the current population. Hence the donor vector $V_{k,G}$ is formed as:

$$V_{k,G} = X_{best,G} + F(X_{r1,G} - X_{r2,G}) \quad (5)$$

Where, F is a positive real number called the mutation factor and the range of F is [0, 2].

C. *Step-3 Crossover:*

This step employs a uniform crossover to build up trial vectors $U_{k,G}$ which is obtained by exchanging the elements of the target vector $X_{k,G}$ and the donor vector $V_{k,G}$ with a crossover probability CR [0, 1] as given in (6).

$$U_{k,G} = u_{i,k,G} = \begin{cases} v_{i,k,G}, if\,(rand_j(0,1) \le Cr \\ x_{i,k,G}, otherwise \end{cases} \quad (6)$$

D. *Step-4 Selection*:

In this step the target vector compared with the trial vector and the minimum value is admitted to the next generation. The best value for the next generation is selected as in (7).

$$X_{k,G+1} = \begin{cases} U_{k,G}, if \quad f(U_{k,G}) \le f(X_{k,G}) \\ X_{k,G}, if \quad f(U_{k,G}) > f(X_{k,G}) \end{cases} \quad (7)$$

For k=1, 2, 3 ... NP

The above steps are continued until the predefined number of generation is reached or the desired value of the cost function is obtained.

In the cost function, the multiple objectives are added to form a single fitness function of the optimization problem and the cost function is defined as in (8).

$$f_{\cos t} = H_1 W_1 \delta_1^2 + H_2 W_2 \delta_2^2 + H_3 W_3 \delta_3^2 + \Delta_{\max} \quad (8)$$

Where $\delta_1 = |SLL_d| - |SLL_{\max}|$, $\delta_2 = |R_d| - |R_{\max}|$ and $\delta_3 = |SBL_d| - |SBL_{\max}|$. $SLL_{\max}$ $R_{\max}$ and SBL_{max} are the maximum value of the SLL and ripple and SBL at each iteration of the searching process whereas SLL_d, R_d and SBL_d represent their corresponding desired value. H_1, H2 and H_3 are the Heaviside step function and W_1, W_2 and W_3 are the weighting factors of terms respectively. $\Delta_{\max}$ is the sum of all the sampled ripples within the angular width of the shaped region.

IV. Result and Discussion

The TMLAA with number of element 30 and inter element 0.5 λ is considered. It is desired that the array pattern should follow the cosecant squared function for the angular width of 30^0 starting from 100^0 to 130^0. For the desired shaped beam pattern, the normalized static amplitude (A_n), the static phase shifts (ϕ_n) and on-time duration (τ_n) of the array elements are taken as the optimizing parameters of the DE algorithm. The search range for A_n, ϕ_n and τ_n are chosen as (0.05, 1), (0, 2π) and (0.04, 1) respectively. For the static amplitude and phase shift, only 32 equally stepped values obtained from the search range of A_n and ϕ_n are considered so that these can be controlled by using a five bit digital converter. However, any value of the normalized on-time from its search is the accepted values. The DE parameters are set as follows. As the number of array element is

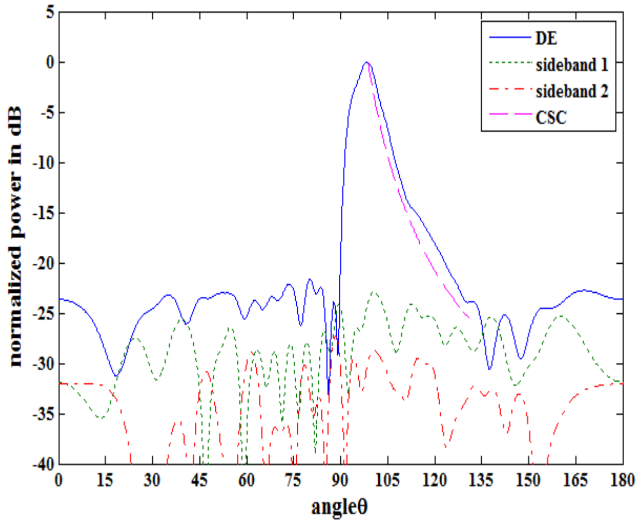

Fig.4. DE optimized power pattern of the 30 element TMLA where the static amplitude and phase are digitized and continuous value of on-time duration is used.

considered as 30, the dimension of the optimization problem becomes 90. The initial population size is taken as 50 and these are randomly generated by a [50 × 90] matrix. The mutation factor and the crossover probability are preferred as 0.5 and 0.85 respectively. Now DE is used to minimize the cost function as expressed in (8) by optimizing the values of parameters A_n, ϕ_n and τ_n from

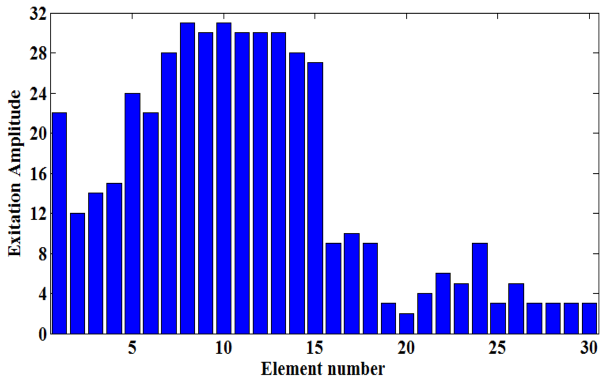

Fig.5. Element wise digitized static amplitudes of the pattern shown in Fig. 2.

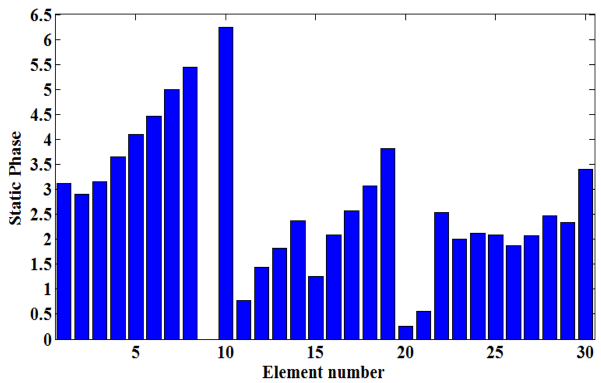

Fig.6. Element wise static phase shift of Fig. 4.

Table.1. Element wise numerical values of the digitized static amplitude and phase and continuous on-time duration

Element number (n)	Normalized amplitude (A_n)	Static phase shift, ϕ_n (Rad)	Normalized On-time (τ_n)
1	0.6875	3.1416	0.0960
2	0.3750	2.9452	0.5858
3	0.4375	3.1416	0.8532
4	0.4688	3.7306	0.9882
5	0.7500	4.1233	0.8883
6	0.6875	4.5160	0.9675
7	0.8750	5.1051	0.9621
8	0.9688	5.4978	0.9864
9	0.9375	0	0.9720
10	0.9688	6.2832	0.9628
11	0.9375	0.7854	0.9629
12	0.9375	1.3744	0.8761
13	0.9375	1.7671	0.5627
14	0.8750	2.3562	0.0406
15	0.8438	1.1781	0.0479
16	0.2813	2.1598	0.9178
17	0.3125	2.5525	0.9820
18	0.2813	3.1416	0.9392
19	0.0938	3.7306	0.8671
20	0.0625	0.1963	0.1380
21	0.1250	0.5890	0.7483
22	0.1875	2.5525	0.0568
23	0.1563	1.9635	0.2669
24	0.2813	2.1598	0.2851
25	0.0938	2.1598	0.6373
26	0.1563	1.9635	0.3344
27	0.0938	2.1598	0.9266
28	0.0938	2.5525	0.9560
29	0.0938	2.3562	0.8658
30	0.0938	3.3379	0.5901

their corresponding search ranges. The DE optimized pattern is shown in Fig. 4 and the corresponding optimum values of normalized amplitude (A_n), static phase shift (ϕ_n) and normalized on-time durations (τ_n) are shown in Table-1. From Fig.4, it can be observed that the DE optimized CSP approximately follows the actual cosecant square plot (dashed line) for the angular width of 30^0 with the range (100-130) degrees. The maximum value of the side lobe level is reduced to −20.64 dB and almost without any ripple in the desired region. From table-1, it is observed that the dynamic range ratio (DRR) of amplitude is 15.5. The maximum values of the sideband level at first two sidebands are found to be −22.92 dB and −27.29 dB respectively. As can be seen from Table-1 that, the excitation amplitudes and static phase shifts are digitized which can be controlled by a five bit digital converter. As for example, let us consider the normalized static amplitude and phase of the first element i.e., A_1 and φ_1 = 0.6875 and 3.1416. These are obtained as (1/32)*22 and $(2\pi/32)*16$. Thus the binary equivalent of 22 (10110) and 16 (10000) produce the corresponding outputs of the first element. Fig. 5 and 6 shows the element wise distribution of the static amplitude and phase distribution where the maximum values of the converter output are considered as 32 and 2π respectively.

V. Conclusion

The differential evolution algorithm is used successfully for synthesizing a cosecant square pattern in the radiation pattern of time-modulated linear antenna arrays where a five bit digitally controlled values of static amplitude and phase shift are used to excite the array elements. Inclusion of five bit digital converter is combined with the additional dimension 'time' in TMLAAs greatly reduces the design complexity and hence the cost. Because, the high cost attenuator is not required to taper the static amplitude but relatively low cost digital converter can be used for the same.

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