THREE DIMENSIONAL FLOW OVER A POROUS VERTICAL PLATE WITH PERIODIC PERMEABILITY IN SLIP FLOW REGIME

Poonam Gupta
Department of Mathematics
Anand International College of Engineering,
Jaipur, INDIA
maths_lecturer@yahoo.co.in

***Abstract*—: In this paper we discuss the effects of combined convection flow of a viscous incompressible fluid past an infinite vertical porous plate embedded with highly porous medium with periodic permeability under constant heat flux in presence of slip boundary conditions for velocity. The velocity and temperature are obtained analytically and their behaviour for different variations in the governing parameters are shown graphically and discuss numerically.The skin friction and Nusselt number are also evolved analytically and tabulated numerically for variations in the said parameters .**

***Keywords*— Heat flux, Moving plate, Periodic permeability, Slip regime, Three dimensional flow.**

I. INTRODUCTION

In the recent years flow of a viscous liquid through a porous medium has attracted the attention of a number of scholars because of its possible applications in many branches of science and technology like petroleum industry, seepage of water in river beds etc. In view of these applications, a series of investigations have been made by Raptis (1983), Reptis and Perdikis (1985) have studied unsteady flow problems through a highly porous medium over infinite porous plate. In all the studies mentioned above the permeability of porous medium was assumed to be constant. Taking permeability variations into consideration Taneja and Jain (2002) have studied the heat and mass transfer flow with radiation in two dimensional flow.

Problems of constant heat flux have attracted the attention of fewer research workers. Singh and Jain (1993), Maharshi and Tak (2002) and some others have discussed the effects of constant heat flux. Recently Sarangi and Jose (2004) have also studied flow through a porous medium on a vertical plate with constant heat flux. All above investigations are for two dimensional flow problems. However, when the variations in permeability is transverse to the flow, the flow is essentially three dimensional. The problem of such a transverse effects was first considered by Gersten and Gross (1974). Acharya and Padhya (1983) investigated the free convection and mass transfer flow of a viscous fluid past a vertical porous plate with constant suction and spanwise periodic varying plate temperature. Singh and Rana (1992), Ahamed and Sharma (1997) have studied the three dimensional viscous flow and heat transfer along a porous plate when sinusoidal transversal suction at the wall is applied. Recently Harmindar S. Takhar (2007), Singh and Sharma (2002), K.D. Singh Jr. Dr., Rakesh Sharma and Khem Chand (2000) have studied three dimensional viscous flow and heat transfer along porous plate with periodic permeability of the porous medium.

The studied reported herein analyzes the effects of transverse periodic variations of the permeability under constant heat flux on three dimensional combined convective flow on a moving vertical plate in presence of slip flow regime. Solutions for velocity, temperature, skin friction and rate of heat transfer are analyzed for different variations in the governing parameters entered in the problem. It is being observed that increase in slip parameter decreases skin friction in the direction of x (τ_x).

II. FORMULATION OF THE PROBLEM

We consider a flow embedded with highly porous medium of a viscous incompressible fluid past an infinite plate with periodic permeability under constant heat flux in a slip flow regime. The plate lying vertically in x* - z* plane and y* axis is normal to it. The bounded plate is moving with velocity U_0 and moreover, free stream velocity of the flow is also assumed as U_0. The permeability of porous medium is taken as

$$K^*(z^*) = \frac{K_0^*}{(1+\varepsilon\cos\pi z^*/\lambda)}, \qquad \ldots(1)$$

where K_0^* is mean permeability of medium, λ is wave length of permeability distribution and ε is amplitude of permeability variations. Due to such permeability variations the problem is three dimensional.

Let u*, v*, w* be the velocity components in x*, y* and z* directions respectively and T* be the temperature, the equations governing the flow in dimensional form are

$$\frac{\partial v^*}{\partial y^*} + \frac{\partial w^*}{\partial z^*} = 0, \qquad \ldots(2)$$

$$v^*\frac{\partial u^*}{\partial y^*} + w^*\frac{\partial u^*}{\partial z^*} = g\beta\,(T^*-T_\infty^*) + \upsilon\left(\frac{\partial^2 u^*}{\partial y^{2*}} + \frac{\partial^2 u^*}{\partial z^{2*}}\right) - \frac{\upsilon(u^*-U_0)}{K^*}, \qquad \ldots(3)$$

$$v^*\frac{\partial v^*}{\partial y^*} + w^*\frac{\partial v^*}{\partial z^*} = -\frac{1}{\rho}\frac{\partial P^*}{\partial y^*} + \upsilon\left(\frac{\partial^2 v^*}{\partial y^{*2}} + \frac{\partial^2 v^*}{\partial z^{*2}}\right) - \upsilon\,\frac{v^*}{K^*}, \qquad \ldots(4)$$

$$v^* \frac{\partial w^*}{\partial y^*} + w^* \frac{\partial w^*}{\partial z^*} = -\frac{1}{\rho}\frac{\partial P^*}{\partial z^*} + \upsilon\left(\frac{\partial^2 w^*}{\partial y^{*2}} + \frac{\partial^2 w^*}{\partial z^{*2}}\right) - \upsilon \frac{w^*}{K^*}, \quad \ldots(5)$$

$$v^* \frac{\partial \theta^*}{\partial y^*} + w^* \frac{\partial \theta^*}{\partial z^*} = \frac{\kappa}{\rho C_p}\left(\frac{\partial^2 T^*}{\partial y^{*2}} + \frac{\partial^2 T^*}{\partial z^{*2}}\right), \quad \ldots(6)$$

with the boundary conditions as:

$$\left.\begin{array}{l} y^* = 0\ ;\ u^* = U_0 + L_1 \dfrac{\partial u^*}{\partial y^*},\ v^* = -V_0,\ w^* = 0,\ \dfrac{\partial T^*}{\partial y^*} = -\dfrac{q}{k} \\ y^* \to \infty ; u^* \to U_0,\ w^* \to 0\ ,\ P^* \to P_\infty^*\ , T^* \to T_\infty^* \end{array}\right\}. \quad \ldots(7)$$

Introducing the non dimensional quantities as

$$y = \frac{y^*}{\lambda},\ z = \frac{z^*}{\lambda},\ u = \frac{u^*}{U_0},\ v = \frac{v^*}{U_0},\ w = \frac{w^*}{U_0},$$

$$P = \frac{P^*}{\rho U_0^2},\ T = \frac{(T^* - T_\infty) U_0 k}{q \nu},\ \alpha = \frac{V_0}{U_0},\ h_1 = \frac{L_1}{\lambda},$$

using these non dimensional quantities in equations (2) to (6), equations reduce to

$$\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \quad \ldots(8)$$

$$v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = GT\,Re + \frac{1}{Re}\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) - \frac{(1 + \in \cos \pi z)(u - 1)}{Re\ K_0}, \quad \ldots(9)$$

$$v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -\frac{\partial P}{\partial y} + \frac{1}{Re}\left(\frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) - \frac{(1 + \in \cos \pi z)v}{Re\ K_0}, \quad \ldots(10)$$

$$v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = -\frac{\partial P}{\partial z} + \frac{1}{Re}\left(\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right) - \frac{(1 + \in \cos \pi z)w}{Re\ K_0}, \quad \ldots(11)$$

$$v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z} = \frac{1}{Re\,Pr}\left(\frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right), \quad \ldots(12)$$

with the boundary conditions

$$\left.\begin{array}{l} y = 0 : u = 1 + h_1 \dfrac{\partial u}{\partial y},\ v = -\alpha,\ w = 0,\ \dfrac{\partial T}{\partial y} = -Re \\ y \to \infty : u = 1,\ w = 0,\ P = P_\infty,\ T = 0 \end{array}\right\}, \quad \ldots(13)$$

where

$$Gr = \frac{g\,q\,\beta \nu^2}{\kappa U_0^4} \text{ (Grashoff number)},$$

$$Re = \frac{U_0 l}{\nu} \text{ (Reynolds number)},$$

$$Pr = \frac{\mu C_p}{\kappa} \text{ (Prandtl number)},$$

$$K_0 = \frac{K_0^*}{\lambda^2} \text{ (Permeability parameter)}.$$

III. Solution of the Problem

In order to solve the problem, we assume the solutions of the following form because the amplitude ε (<< 1) of the permeability variation is very small ,so the solution in the neighbourhood of the plate in the form

$$f(y,z) = f_0(y) + \in f_1(y,z) + \in^2 f_2(y,z) + \ldots \quad \ldots(14)$$

where f stands for u, v, w, p and θ. When $\in = 0$ equations (8) to (12) reduce to two dimensional free convection flow with constant suction at the plate, and given by

$$\frac{\partial v_0}{\partial y} = 0, \quad \ldots(15)$$

$$v_0 \frac{\partial u_0}{\partial y} + w_0 \frac{\partial u_0}{\partial z} = GT_0 Re + \frac{1}{Re}\left(\frac{\partial^2 u_0}{\partial y^2} + \frac{\partial^2 u_0}{\partial z^2}\right) - \frac{(u_0 - 1)}{Re\ K_0}, \quad \ldots(16)$$

$$v_0 \frac{\partial v_0}{\partial y} + w_0 \frac{\partial v_0}{\partial z} = -\frac{\partial P_0}{\partial y} + \frac{1}{Re}\left(\frac{\partial^2 v_0}{\partial y^2} + \frac{\partial^2 v_0}{\partial z^2}\right) - \frac{v_0}{Re\ K_0}, \quad \ldots(17)$$

$$v_0 \frac{\partial w_0}{\partial y} + w_0 \frac{\partial w_0}{\partial z} = -\frac{\partial P_0}{\partial z} + \frac{1}{Re}\left(\frac{\partial^2 w_0}{\partial y^2} + \frac{\partial^2 w_0}{\partial z^2}\right) - \frac{w_0}{Re\ K_0}, \quad \ldots(18)$$

$$v_0 \frac{\partial T_0}{\partial y} + w_0 \frac{\partial T_0}{\partial z} = \frac{1}{Re\,Pr}\left(\frac{\partial^2 T_0}{\partial y^2} + \frac{\partial^2 T_0}{\partial z^2}\right), \quad \ldots(19)$$

and the corresponding boundary conditions are

$$\left.\begin{array}{l} y = 0 : u_0 = 1 + h_1 \dfrac{\partial u_0}{\partial y},\ v_0 = -\alpha,\ w_0 = 0,\ \dfrac{\partial T_0}{\partial y} = -Re \\ y \to \infty : u_0 = 1,\ w_0 = 0,\ P_0 = P_\infty,\ T_0 = 0 \end{array}\right\}. \quad \ldots(20)$$

The solutions of the equations (15) to (19) are given as:

$$\left.\begin{array}{l} u_0 = C_2 e^{-m_1 y} - A_1 e^{-\alpha Pr Re\, y} + 1, \\ T_0 = \dfrac{1}{\alpha . Pr} e^{-\alpha\, Pr Re\, y}, \\ v_0 = -\alpha, \quad w_0 = 0, \quad P_0 = P_\infty . \end{array}\right\} \quad \ldots(21)$$

When $\in \neq 0$, substituting equation (14) into the equations (8) to (12) and comparing the like powers of ε and neglecting higher powers of ε, we get the following equations

$$\frac{\partial v_1}{\partial y} + \frac{\partial w_1}{\partial z} = 0, \quad \ldots(22)$$

$$v_1 \frac{\partial u_0}{\partial y} - \alpha \frac{\partial u_1}{\partial y} = GT_1 Re + \frac{1}{Re}\left(\frac{\partial^2 u_1}{\partial y^2} + \frac{\partial^2 u_1}{\partial z^2}\right) - \frac{[(u_0 - 1)\cos \pi z + u_1]}{Re\ K_0}, \quad \ldots(23)$$

$$-\alpha \frac{\partial v_1}{\partial y} = -\frac{\partial p_1}{\partial y} + \frac{1}{Re}\left(\frac{\partial^2 v_1}{\partial y^2} + \frac{\partial^2 v_1}{\partial z^2}\right) - \frac{(-\alpha \cos \pi z + v_1)}{Re\ K_0}, \quad \ldots(24)$$

$$-\alpha \frac{\partial w_1}{\partial y} = -\frac{\partial p_1}{\partial z} + \frac{1}{Re}\left(\frac{\partial^2 w_1}{\partial y^2} + \frac{\partial^2 w_1}{\partial z^2}\right) - \frac{w_1}{Re\ K_0}, \quad \ldots(25)$$

$$v_1 \frac{\partial T_0}{\partial y} - \alpha \frac{\partial T_1}{\partial z} = \frac{1}{Re\,Pr}\left(\frac{\partial^2 T_1}{\partial y^2} + \frac{\partial^2 T_1}{\partial z^2}\right), \quad \ldots(26)$$

and the corresponding boundary conditions are

$$\left.\begin{array}{l} y = 0 : u_1 = h_1 \dfrac{\partial u_1}{\partial y},\ v_1 = 0,\ w_1 = 0,\ \dfrac{\partial T_1}{\partial y} = 0 \\ y \to \infty : u_1 = 0,\ w_1 = 0,\ p_1 = 0,\ T_1 = 0 \end{array}\right\}. \quad \ldots(27)$$

These are the partial differential equations which describe the three-dimensional flow through a porous medium. For solution, we shall first consider equations (22), (24) and (25) being independent of the main flow and the temperature field.

Let us assume v_1, w_1 and p_1 as of the form

$$v_1(y,z) = -v_{11}(y)\cos\pi z \;, \qquad \ldots(28)$$

$$w_1(y,z) = \frac{1}{\pi}\frac{\partial v_{11}}{\partial y}\sin\pi z \;, \qquad \ldots(29)$$

$$p_1(y,z) = p_{11}(y)\cos\pi z\,. \qquad \ldots(30)$$

Equations (28) and (29) are chosen so that the equations of continuity (22) is satisfied. Substituting (28) and (29) in equations (24) and (25), we get the following equations

$$\frac{d^2 v_{11}}{dy^2} + \mathrm{Re}\,\alpha\frac{dv_{11}}{dy} - \left(\pi^2 + \frac{1}{K_0}\right)v_{11} = -\mathrm{Re}\frac{dp_{11}}{dy} - \frac{\alpha}{K_0}, \qquad \ldots(31)$$

$$\frac{d^3 v_{11}}{dy^3} + \alpha\,\mathrm{Re}\frac{d^2 v_{11}}{dy^2} - \left(\pi^2 + \frac{1}{K_0}\right)\frac{\partial v_{11}}{\partial y} = -\pi^2 p_{11}\,\mathrm{Re}\,, \qquad \ldots(32)$$

with the boundary conditions

$$\left.\begin{array}{l} y = 0 : v_{11} = 0,\; \dfrac{\partial v_{11}}{\partial y} = 0 \\ y \to \infty : p_{11} = 0\,,\; \dfrac{\partial v_{11}}{\partial y} = 0 \end{array}\right\} \qquad \ldots(33)$$

On solving equations (31) and (32) under the boundary conditions (33), we get

$$v_1 = \frac{\alpha(\pi e^{-m_2 y} - m_2 e^{-\pi y} - \pi + m_2)}{(m_2-\pi)(\pi^2 K_0 + 1)}\cos\pi z\,, \qquad \ldots(34)$$

$$w_1 = \frac{\alpha m_2 (e^{-\pi y} - e^{-m_2 y})}{(m_2-\pi)(\pi^2 K_0 + 1)}\sin\pi z\;, \qquad \ldots(35)$$

$$p_1 = \frac{\alpha m_2(\mathrm{Re}\,\alpha + 1/K_0)e^{-\pi y}}{\mathrm{Re}\,\pi(m_2-\pi)(\pi^2 K_0+1)}\cos\pi z\;, \qquad \ldots(36)$$

For the main flow and the temperature field solution, we assume u_1 and T_1 as

$$u_1(y,z) = u_{11}(y)\cos\pi z\,, \qquad \ldots(37)$$

$$T_1(y,z) = T_{11}(y)\cos\pi z\,. \qquad \ldots(38)$$

By substituting equations (37) and (38) in (23) and (26), we get

$$\frac{d^2 u_{11}}{dy^2} + \mathrm{Re}\,\alpha\frac{du_{11}}{dy} - \left(\pi^2 + \frac{1}{K_0}\right)u_{11} - G\,T_{11}\,\mathrm{Re}^2 - \frac{(u_0 - 1)}{K_0} - \mathrm{Re}\,v_{11}\frac{du_0}{dy}\,, \qquad \ldots(39)$$

$$\frac{d^2 T_{11}}{dy^2} + \mathrm{Re}\,\mathrm{Pr}\,\alpha\frac{dT_{11}}{dy} - \pi^2 T_{11} = -v_{11}\,\mathrm{Pr}\frac{dT_0}{dy}\,, \qquad \ldots(40)$$

and the boundary conditions are

$$\left.\begin{array}{l} y = 0 : u_{11} = h_1\dfrac{du_{11}}{dy},\; \dfrac{dT_{11}}{dy} = 0 \\ y \to \infty : u_{11} = 0\,,\; T_{11} = 0 \end{array}\right\}. \qquad \ldots(41)$$

On solving equations (39) and (40) using boundary conditions (41), we get the following results

$$T_{11} = C_5 e^{-m_3 y} - A_2\left\{B_1 e^{-(m_2+\alpha.\mathrm{Pr.Re})y} - B_2 e^{-(\pi+\alpha.\mathrm{Pr.Re})y} - B_3 e^{-\alpha.\mathrm{Pr.Re}.y}\right\},$$

$$\begin{aligned} u_{11} = C_6 e^{-m_4 y} - G\,\mathrm{Re}^2 \{ & D_1 e^{-m_3 y} - A_2 (D_2 e^{-(m_2+\alpha.\mathrm{Pr.Re})y} \\ & - D_3 e^{-(\pi+\alpha.\mathrm{Pr.Re})y} - D_4 e^{-\alpha.\mathrm{Pr.Re}\,y} \} - A_3 \{ - D_5 e^{-(\pi+m_2)y} \\ & + D_6 e^{-(\alpha\,\mathrm{Pr.Re}+\pi)y} + D_7 e^{-(m_1+m_2)y} - D_8 e^{-(m_2+\alpha.\mathrm{Pr.Re})y} \\ & - D_9 e^{-m_1 y} + D_{10} e^{-\alpha.\mathrm{Pr.Re}\,y} \} - D_{11} e^{-m_1 y} + D_{12} e^{-\alpha.\mathrm{Pr\,Re}.y}. \end{aligned}$$

IV. Skin Friction and Nusselt Number

After obtaining velocity field and temperature field, we now discuss the x and z components of skin friction at the wall and the Nusselt number as:

τ_x (skin friction in x - direction)

$$= \frac{\tau_x^*}{\rho UV} = \frac{1}{\mathrm{Re}}\left[\frac{\partial u_0}{\partial y} + \varepsilon\frac{\partial u_{11}}{\partial y}\cos\pi z\right]_{y=0},$$

τ_z (skin friction in z - direction)

$$= \frac{\tau_z^*}{\rho UV} = \frac{1}{\mathrm{Re}}\left[\varepsilon\frac{\partial w_{11}}{\partial y}\sin\pi z\right]_{y=0},$$

and the Nusselt number is

$$\mathrm{Nu} = \frac{1}{T(0)} = \frac{1}{\{T_0(0) + \varepsilon T_{11}(0)\cos\pi z\}}$$

where

$$\left(\frac{\partial u_0}{\partial y}\right)_{y=0} = -C_2 m_1 + A_1\alpha\,\mathrm{Pr\,Re}\,,$$

$$\begin{aligned} \left(\frac{\partial u_{11}}{\partial y}\right)_{y=0} = -m_4 C_6 - G\,\mathrm{Re}^2 \Big[& -D_{11} m_3 - A_2 \{ - D_2(m_2 + \alpha.\mathrm{Pr.Re}) \\ & + D_3(\pi + \alpha.\mathrm{Pr.Re}) + D_4\alpha.\mathrm{Pr.Re} \} - A_3 \{ D_5(\pi + m_1) \\ & - D_6(\alpha\,\mathrm{Pr.Re} + \pi) - D_7(m_1 + m_2) + D_8(m_2 + \alpha.\mathrm{Pr.Re}) \\ & + D_9 m_1 - D_{10}\alpha.\mathrm{Pr.Re} \} + D_{11} m_1 - D_{12}.\alpha.\mathrm{Pr.Re} \Big], \end{aligned}$$

$$T_0(0) = -\,\mathrm{Re}\,,$$

$$T_{11}(0) = -C_5 m_3 - A_2\left[\begin{array}{l} -B_1(m_2 + \alpha.\mathrm{Pr.Re}) \\ \quad + B_2(\pi + \alpha.\mathrm{Pr.Re}) + B_3\,.\alpha.\mathrm{Pr\,Re} \end{array}\right],$$

where values of constants are given in Appendix.

V. Discussion and Conclusion

In order to understand the solution physically, numerical calculations have been made for the velocity distribution, temperature distribution, x and z components of skin friction and the Nusselt in presence of different parameters viz. Permeability parameters (K_0), slip flow parameter (h_1), suction parameter (α), Grashoff number (Gr), Reynolds number (Re), we have chosen air as a fluid (Pr = 0.71), fixing $\in$ = 0.2 and z = 1/4.

In figure 1, velocity distribution is plotted against y on different values of Re, α, h_1, K_0 and Gr. From the figure we observe that increase in Re, h_1, Gr and K_0 increases the velocity of the fluid while increase in suction parameter α decreases the velocity of the fluid. It is further seen that increase in K_0 increases the pore space in the medium and hence increases the flow velocity. In the way increases in slip velocity at the plate adds some additional velocity to the flow.

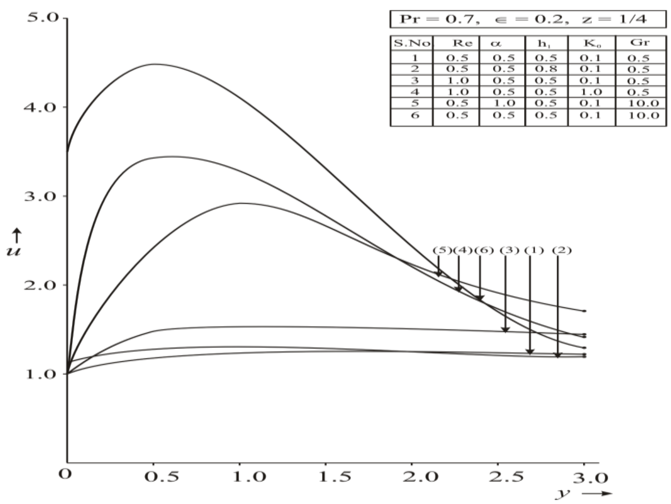

Figure 1. **Velocity distribution u plotted against y for different values of Re, α, h_1, Ko and Gr.**

Temperature distribution is plotted against y in figure 2 for fixed values of Pr, ∈ and z. We observe that temperature of the fluid increases with the increase of Re, and decreases with the increase of K_0 and α.

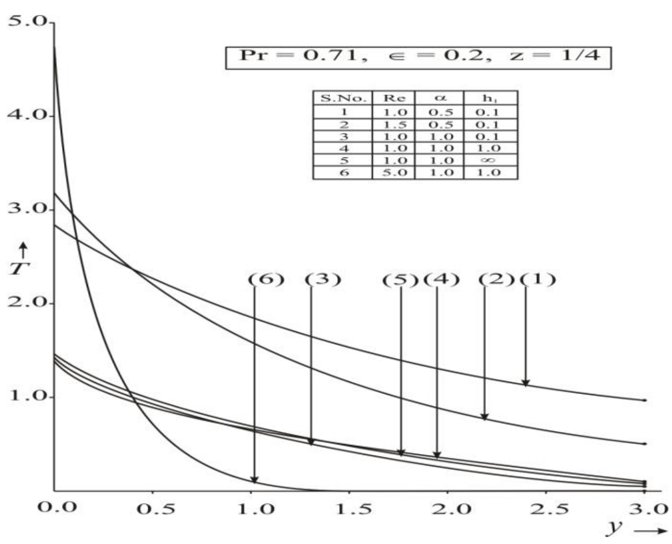

Figure 2. **Temperature distribution T plotted against y for different values of Re, α and h_1.**

In figures 3 skin friction in the directions of x (τ_x) and figures 4 skin friction in the directions of z (τ_z) are plotted against Re respectively. From figure 3 it is observed that increase in h_1, α and K_0 decreases τ_z, while increase in Gr increases τ_x. Moreover from figure 4, we noted that increase in α and K_0 increases τ_z.

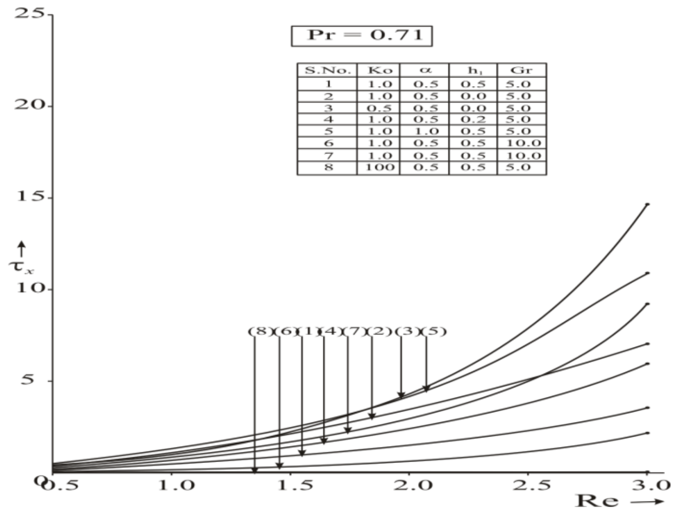

Figure 3. τ_x **plotted against Re for different values of α, h_1, Ko and Gr.**

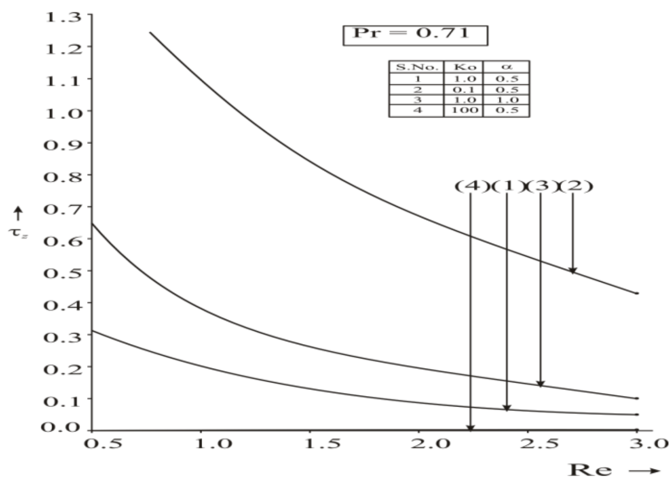

Figure 4. τ_z **plotted against Re for different values of Ko and α.**

Nusselt number (Nu) is plotted against Re on different values of α and K0 in figure 5. It is interesting to observed that for the case of constant heat flux at the plate, Nusselt number increases with both the parameters α and K0.

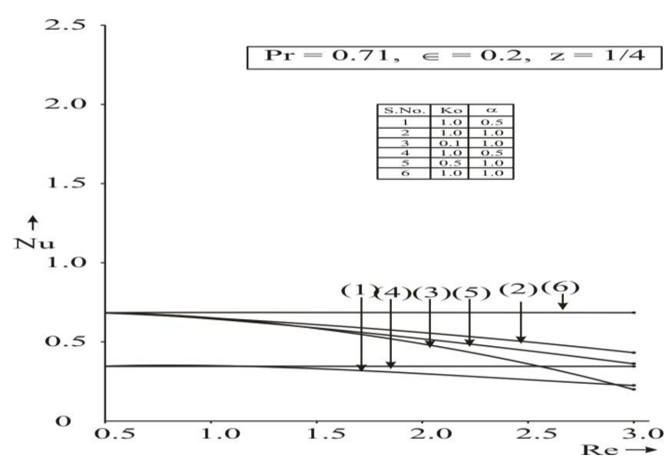

Figure 5. **Nussult number (Nu) plotted against Re for different values of Ko and α.**

VI. APPENDIX

$$C_1 = \frac{1}{\alpha . \Pr}, \qquad m_1 = \frac{\mathrm{Re}\,\alpha}{2} + \sqrt{\frac{\mathrm{Re}^2\alpha^2}{4} + \frac{1}{K_0}},$$

$$C_2 = \frac{A_1(1+h_1\alpha \Pr \mathrm{Re})}{(1+h_1 m_1)}, \quad m_1' = -\frac{\mathrm{Re}\,\alpha}{2} + \sqrt{\frac{\mathrm{Re}^2\alpha^2}{4} + \frac{1}{K_0}},$$

$$A_1 = \frac{\mathrm{Gr}\,\mathrm{Re}^2 C_1}{\left\{\alpha^2 \Pr^2 \mathrm{Re}^2 - \alpha^2 \mathrm{Re}^2 \Pr - \frac{1}{K_0}\right\}}, m_2 = \frac{\alpha \mathrm{Re}}{2} + \sqrt{\frac{\alpha^2 \mathrm{Re}^2}{4} + \left(\pi^2 + \frac{1}{K_0}\right)},$$

$$m_3 = \frac{\alpha \Pr \mathrm{Re}}{2} + \sqrt{\frac{\alpha^2 \Pr^2 \mathrm{Re}^2}{4} + \pi^2}, \ m_2' = -\frac{\mathrm{Re}\,\alpha}{2} + \sqrt{\frac{\mathrm{Re}^2\alpha^2}{4} + \left(\pi^2 + \frac{1}{K_0}\right)},$$

$$m_3' = -\frac{\alpha \Pr \mathrm{Re}}{2} + \sqrt{\frac{\alpha^2 \Pr^2 \mathrm{Re}^2}{4} + \pi^2}, B_1 = \frac{\pi}{(m_2^2 + \alpha \Pr \mathrm{Re}\, m_2 - \pi^2)},$$

$$B_2 = \frac{m_2}{\alpha . \Pr \mathrm{Re}\,\pi}, \ B_3 = \frac{(m_2 - \pi)}{\pi^2}, \ A_2 = -\frac{\alpha \Pr \mathrm{Re}^2}{(m_2 - \pi)(\pi^2 K_0 + 1)},$$

$$m_4 = \frac{\alpha \mathrm{Re}}{2} + \sqrt{\frac{\mathrm{Re}^2\alpha^2}{4} + \left(\pi^2 + \frac{1}{K_0}\right)}, m_4' = -\frac{\mathrm{Re}\,\alpha}{2} + \sqrt{\frac{\mathrm{Re}^2\alpha^2}{4} + \left(\pi^2 + \frac{1}{K_0}\right)},$$

$$C_5 = \frac{A_2}{m_3}[B_1 (m_2 + \alpha \Pr \mathrm{Re}) - B_2 (\pi + \alpha \Pr \mathrm{Re}) - B_3\, \alpha \Pr \mathrm{Re}],$$

$$C_3 = \frac{m_2\, \alpha (\mathrm{Re}\,\pi\,\alpha + 1/K_0)}{(m_2 - \pi)(\pi^2 K_0 + 1) \mathrm{Re}\,\pi}, \ C_4 = \frac{m_2\,\alpha}{(\pi^2 K_0 + 1)(m_2 - \pi)} - \frac{\alpha}{(\pi^2 K_0 + 1)},$$

$$A_3 = \frac{\mathrm{Re}}{(m_2 - \pi)(\pi^2 K_0 + 1)}, \ D_1 = \frac{C_5}{\left\{m_3^2 - \mathrm{Re}\,\alpha\, m_3 - \left(\pi^2 + \frac{1}{K_0}\right)\right\}},$$

$$D_2 = \frac{B_1}{\left\{(m_2 + \alpha \Pr \mathrm{Re})^2 - \alpha \mathrm{Re}\,(m_2 + \alpha \Pr \mathrm{Re}) - \left(\pi^2 + \frac{1}{K_0}\right)\right\}},$$

$$D_3 = \frac{B_2}{\left\{(\pi + \alpha \Pr \mathrm{Re})^2 - \mathrm{Re}\,\alpha\,(\pi + \alpha \Pr \mathrm{Re}) - \left(\pi^2 + \frac{1}{K_0}\right)\right\}},$$

$$D_4 = \frac{B_3}{\left\{\alpha^2 \Pr^2 \mathrm{Re}^2 - \alpha^2 \Pr \mathrm{Re}^2 - \left(\pi^2 + \frac{1}{K_0}\right)\right\}},$$

$$D_5 = \frac{m_1 m_2 C_2}{\left\{(\pi + m_1)^2 - \mathrm{Re}\,\alpha\,(\pi + m_1) - \left(\pi^2 + \frac{1}{K_0}\right)\right\}},$$

$$D_6 = \frac{A_1 m_2\, \alpha \Pr \mathrm{Re}}{\left\{(\alpha\, \mathrm{Pe}\, \mathrm{Re} + \pi)^2 - \mathrm{Re}\,\alpha\,(\alpha \Pr \mathrm{Re} + \pi) - \left(\pi^2 + \frac{1}{K_0}\right)\right\}},$$

$$D_7 = \frac{m_1 c_2 \pi}{\left\{(m_1 + m_2)^2 - \mathrm{Re}\,\alpha\,(m_1 + m_2) - \left(\pi^2 + \frac{1}{K_0}\right)\right\}},$$

$$D_8 = \frac{A_1\, \pi\, \alpha \Pr \mathrm{Re}}{\left\{(m_2 + \alpha \Pr \mathrm{Re})^2 - \mathrm{Re}\,\alpha\,(m_2 + \alpha \Pr \mathrm{Re}) - \left(\pi^2 + \frac{1}{K_0}\right)\right\}},$$

$$D_9 = \frac{m_1 C_2 (\pi - m_2)}{\left\{m_1^2 - \mathrm{Re}\,\alpha\, m_1 - \left(\pi^2 + \frac{1}{K_0}\right)\right\}},$$

$$D_{10} = \frac{(\pi - m_2) A_1\, \alpha \Pr \mathrm{Re}}{\left\{\alpha^2 \Pr^2 \mathrm{Re}^2 - \alpha^2 \Pr \mathrm{Re}^2 - \left(\pi^2 + \frac{1}{K_0}\right)\right\}},$$

$$D_{11} = \frac{C_2}{K_0\left\{m_1^2 - \mathrm{Re}\,\alpha\, m_1 - \left(\pi^2 + \frac{1}{K_0}\right)\right\}}, D_{12} = \frac{A_1}{K_0\left\{\alpha^2 \Pr^2 \mathrm{Re}^2 - \alpha^2 \Pr \mathrm{Re}^2 - \left(\pi^2 + \frac{1}{K_0}\right)\right\}},$$

$$\begin{aligned} C_6 = \{&G\,\mathrm{Re}^2 [D_1(1+h_1 m_3) - A_2 D_2 (1 + h_1 m_2 + h_1 \alpha \Pr \mathrm{Re}) \\ &+ A_2 D_3 (1 + h_1\pi + h_1 \alpha \Pr \mathrm{Re}) + A_2 D_4 (1 + h_1 \alpha \Pr \mathrm{Re})] - A_3 D_5 (1 + h_1 \pi + h_1 m_1) \\ &+ A_3 D_6 (1 + h_1 \pi + h_1 \alpha \Pr \mathrm{Re}) + A_3 D_7 (1 + h_1 m_1 + h_1 m_3) \\ &- A_3 D_8 (1 + h_1 m_2 + h_1 \alpha \Pr \mathrm{Re}) - A_3 D_9 (1 + h_1 m_1) + A_3 D_{10} (1 + h_1 \alpha \Pr \mathrm{Re})] \\ &+ D_{11}(1 + h_1 m_1) - D_{12}(1 + \alpha \Pr \mathrm{Re}\, h_1)\} / (1 + h_1 m_4) \end{aligned}$$

REFERENCES

[1] Acharya, B.P., Padhya, S. 1983. Free convection viscous flow past a hot vertical plate with periodic temperature. Ind. J. pure Appl. Math., 14(7): .838-849.

[2] Ahamed, N., Sharma, D. 1997. Three dimensional free convective flow and heat transfer through a porous medium. Ind. J. Pure Appl. Math., 28:1345-1353

[3] Gerston, K., Gross, J.F. 1974. Flow and heat transfer along a plane wall with periodic suction. Journal of Applied Mathematics and Physics (ZAMP)., 25(3):399-408.

[4] Harmindar S. Takhar, 2007. Three-Dimensional Heat and Mass Transfer Flow of a Viscous Fluid with Periodic Suction Velocity. Inter J Fluid Mech Res. v34.i3.60: 267-286

[5] K.D. Singh Jr. Dr., Rakesh Sharma and Khem Chand, (2000). Three Dimensional Fluctuating Flow and Heat Transfer through a Porous Medium with Variable Permeability. ZAMM - Journal of Applied Mathematics and Mechanics / Zeitschrift für Angewandte Mathematik und Mechanik. Volume 80, Issue 7: 473–480

[6] Maharshi, A., Tak, S.S. 2002. Fluctuating free convection through porous medium due to infinite vertical plate with constant heat flux. J. Ind. Acad. Math.,22(2):293-302

[7] Raptis, A. 1983. Unsteady free convection flow through a porous medium. Int. J. Eng. Sci., 21(4): 345-349

[8] Raptis, A., Perdikis, C.P. 1985. Oscillatory flow through a porous medium by the presence of free convective flow. Int. J. Engin. Sci., 23:51-55

[9] Sarangi, K.C., Jose, C.B. 2004.: Unsteady MHD free convective flow and mass transfer through porous medium with constant suction and constant heat flux. J. Ind. Acad. Math., 26:115-126

[10] Singh, K.D., Rana, S.K. 1992. Three dimensional flow and heat transfer through a porous medium. Ind. J. pure Appl. Math., 23(7):905-914

[11] Singh, K.D., Sharma, R.K . 2002. Three dimensional free convective flow and heat transfer through a porous medium with periodic permeability. Ind. J. pure Appl. Math., 33(6):941-949

[12] Singh, P., Jain, N.C. 1993. MHD temperature distribution for an unsteady stratified laminar boundary layer oscillatory flow along an infinite flat plate with variable suction and constant heat flux. Modelling, Measurement and Control B, AMSE Pub., 50(2):53-64.

[13] Taneja, Rajeev, Jain, N.C. 2002. Effects of magnetic field on free convection mass transfer flow through a porous medium with radiation and variable permeability in slip flow regime. Jnanabha, 31/32:69-81.