Edge Detection Based on Mathematical Morphology and Wavelet Transform using Type-1 and Type-2 Fuzzy Rules

Jalil Shirazi and Masrour Dowlatabadi

Abstract— ***In this paper, we proposed an improved edge detection algorithm based on fuzzy combination of mathematical morphology and wavelet transform. We used type-1 and type-2 fuzzy rules. The proposed method overcomes the limitation of wavelet based edge detection and mathematical morphology based edge detection in noisy images. Experimental results show superiority of the proposed method, as compared to the traditional Prewitt, wavelet based and morphology based edge detection methods. The proposed method is an effective edge detection method especially for noisy images and keeps clear and continuous edges***.

Keywords— Edge detection, Wavelet transform, Mathematical morphology, Type-1 fuzzy logic, Type-2 fuzzy logic.

I. INTRODUCTION

One of the basic topics in image processing is edge detection. Edge is the most basic feature of images. If the edges in an image are identified accurately, some basic properties such as area, perimeter and shape can be measured. Edge detection can play a signification application in different fields such as computer vision, pattern recognition, image segmentation, remote sensing and medical image analysis.

Many classical edge detectors have been developed over time. Some of the well known edge detection operators based on the first derivative of the image are Roberts, Prewitt, Sobel [1]. The Laplacian methods like Marrs-ildreth do it finding the zeros of second derivative from the image [1]. However, classical edge detectors usually fail to handle images with strong noise.

For improving the edge detection, fuzzy logic were used [2, 3, 4]. For example, the threshold value in edge detection was calculated based on fuzzy rules [2]. In order to overcome the effects of noises, different methods of edge detection based on wavelet transform was proposed [5]. Mallat has given the wavelet multi-resolution edge detection method based on singularity detection [6]. Xu *et al*. [7] proposed a spatially selected filtering technique by multiplying the adjacent DWT (Discrete Wavelet Transform) scales to enhance the significant structures. Zhang *et al*. [8] proposed edge detection based on multiplying the wavelet coefficients at two adjacent scales to magnify significant structures and suppress noise.

In addition, the morphological image processing has developed into an important research field of image processing like edge detection. Morphological filters were used in edge detection [9, 10] and combination of the morphological method and wavelet was used [11].

While wavelet based edge detection can suppresses most noises in noisy images, but detected edges are discontinuities. On the other hand, mathematical morphology based edge detection methods can obtains more completely continuous edges, but it is sensitive to noise.

In this paper, as a novel research, we proposed an edge detection method based on the fusion of the wavelet transform and the mathematical morphology using type-1 and type-2 fuzzy rules. In order to overcome some limitation of type-1 fuzzy system, type-2 fuzzy system is used and compared with type-1 fuzzy system results. The experimental results show that the proposed method can suppress noises effectively and detect more edge details.

The paper is structured as follows. Wavelet transform, mathematical morphology and fuzzy logic are described in section 2. The proposed method and experimental results are presented in section 3. The results are compared in section 4. The paper is concluded in section 5.

II. WAVELET TRANSFORM, MATHEMATICAL MORPHOLOGY AND FUZZY LOGIC

In this section, a brief overview of wavelet transform in edge detection, mathematical morphology, type-1 and type-2 fuzzy logic is given.

A. Edege Detection Based on Wavelet Transform

One dimensional binary wavelet function is expressed as follows:

Suppose θ (x) is a differentiable smooth function whose integral is 1 and converges to 0 at infinity, the ψ (x) is defined as the first-order derivative of θ (x):

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$$\psi(x) = \frac{d\theta(x)}{dx} \tag{1}$$

Set:

$$\theta_{2^j}(x) = \frac{1}{2^j}\theta(\frac{x}{2^j}) \tag{2}$$

So, the wavelet transformation of function f (x) in the scale 2 is:

$$W_{2^j} f(x) = f * \psi_{2^j}(x) = f * (2^j \frac{d\theta_{2^j}}{dx})(x) \tag{3}$$

Where * denotes convolution operation. The wavelet used in this paper is the Mallat wavelet [6], whose θ (x) is a cubic spline.

For image processing, one dimensional wavelet transform can be extended to two dimensional wavelet transform by using the tensor product of two one dimensional analyses, one in the horizontal and the other in the vertical direction.

Set two dimension smoothing function θ(x, y), its first-order partial derivative in horizontal and vertical directions will act as two basic wavelets, respectively. The convolution of scale transformed wavelets of these two basic wavelets and the image function f(x, y) are defined as the horizontal and vertical component of wavelet transformation. The two dimension wavelet function is defined as:

$$\psi_s^1(x,y) = \frac{\partial\theta(x,y)}{\partial x} \; \psi_s^2(x,y) = \frac{\partial\theta(x,y)}{\partial y} \tag{4}$$

The wavelet transform of image function f(x, y) of wavelet $\psi_s^1(x,y)$ and $\psi_s^2(x,y)$ on scale s are defined as:

$$w_s^1 f(x,y) = f(x,y) * \psi_s^1(x,y) \tag{5}$$

$$w_s^2 f(x,y) = f(x,y) * \psi_s^2(x,y) \tag{6}$$

For $s = 2^j$, these two wavelet transform become binary wavelet transform is termed as two dimensional binary wavelet transformation of the image f (x, y).

The two dimensional binary wavelet transforms are the first order derivative of the smoothing image along horizontal and vertical direction respectively and can be viewed as two components of the gray gradient vector of smoothed image f(x, y). The modulus of gray gradient vector is defined as follows :

$$M_S f(x,y) = \sqrt{((w_s^1 f(x,y))^2 + (w_s^2 f(x,y))^2} \tag{7}$$

B. Edge Detection Based on Mathematical Morphology

Mathematical morphology theory is developed from set theory. It was introduced by Matheron as a technique for analyzing geometric structure of metallic and geologic samples [9]. It was extended to image analysis by Serra [9]. Based on set theory, mathematical morphology is established by introducing fundamental operators applied to two sets. One set is said to be processed by another which is known as structuring element. The aim of this transformation is to search the special set structure of original set.

The basic mathematical morphological operators are dilation and erosion, and the other morphological operations are derived from these two basic operations. Suppose, *F*(*x*, *y*) indicate a grey scale two dimensional image and B indicate structuring element. Dilation of a grey scale image *F* (x, y) by a grey scale structuring element *B*(s, t) is indicated by

$$(F \oplus B)(x,y) = \max\{F(x-s, y-t) + B(s,t)\} \tag{8}$$

Erosion of a grey-scale image *F*(x, y) by a grey-scale structuring element *B*(s, t) is indicated by

$$(F\Theta B)(x,y) = \min\{F(x+s, y+t) - B(s,t)\} \tag{9}$$

Opening and closing of grey scale image *F*(x, y) by grey scale structuring element *B*(s, t) are indicated respectively by

$$F \circ B = (F\Theta B) \oplus B \tag{10}$$

$$F \bullet B = (F \oplus B)\Theta B \tag{11}$$

Erosion decreases the grey scale value of the image, while dilation increases it. Both of them are sensitive to the image edge whose grey scale value changes apparently. Opening is erosion followed by dilation and closing is dilation followed by erosion. We used (10) and (11) to detect fine edges in low-frequency of the image.

C. Type-1 and Type-2 Fuzzy Logic

Fuzzy logic is an area of soft computing that enables a computer system to cause with uncertainty. A fuzzy inference system consists of a set of if-then rules defined over fuzzy sets. Fuzzy sets generalize the concept of a traditional set by allowing the membership degree to be any value between 0 and 1. The knowledge that is used to build fuzzy rules is uncertain. Such uncertainty leads to rules whose backgrounds or consequents are uncertain, which translates into uncertain antecedent or consequent membership functions.Type-1 fuzzy systems, whose membership functions are type-1 fuzzy sets, are unable to directly handle such uncertainties. Type-2 fuzzy logic is a generalization of conventional fuzzy logic (type-1), in which the predecessor or following membership functions are type-2 fuzzy sets. Such sets are fuzzy sets whose membership grades themselves are type-1 fuzzy sets; they are very useful in circumstances where it is difficult to determine an exact membership function for a fuzzy set [12].

In type-1 fuzzy system, where the output sets are type-1 fuzzy sets, the defuzzification is performed in order to get a number. In the type-2 case, the output sets are type-2. In type-

1 fuzzy system, defuzzification gives a crisp number at the output and the extended defuzzification operation in the type-2 case gives a type-1 fuzzy set at the output. Since this operation takes from the type-2 output sets of the fuzzy system to a type-1 set, it is called “type reduction”. The type-reduced fuzzy set may then be defuzzified to obtain a single crisp number.

D. Proposed method and Experiments

Fig. 1 is the diagram of our proposed method based on fusion of the wavelet transform and the mathematical morphology using fuzzy rules. At first, a Mamdani fuzzy inferences system was implemented using type-1 fuzzy logic, with three inputs, one output and 18 rules. We used Mallat wavelet transform [7].

It was shown that using the adjacent scale multiplication achieves better results than using other combination of two scales [7]. We proposed a novel method that yields a higher performance in the edge detection. We used two images from details coefficients of scales 2 and 3 wavelet transform as first and second fuzzy system inputs. As the third input, mathematical morphology on smoothed image from wavelet transform was used. At first, type-1 fuzzy system is used in the proposed method and then type-2 fuzzy system is applied.

We used relation 7 for giving the first and second fuzzy system inputs. We tested relation 7 for different wavelet scales and get the best results for scale 2 and scale3.

We used fuzzy rules on three images pixels that are fuzzy system inputs images. We applied the same membership functions for the first and the second fuzzy system input and it is shown in Fig. 2 (a). The membership functions are used for the third input and fuzzy system output are shown in Fig. 2(b) and Fig. 2 (c) respectively.

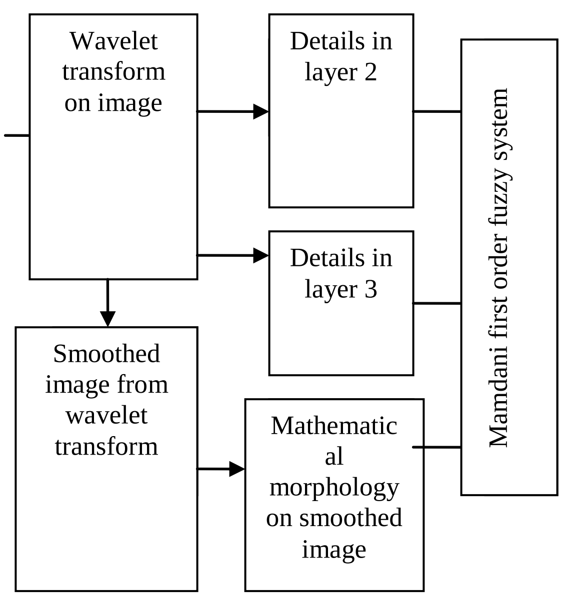


Fig. 1 Block diagram of our proposed method.

For the purpose of evaluating the performance of proposed edge detection method, we conducted the edge detection experiments using Prewitt, wavelet transform based, mathematical morphology based and proposed algorithm edge detectors respectively. Fig.3 (a) shows noisy image. We added 5% salt and pepper noise to original images. Fig. 3 (b-e) show the corresponding detected images using the Prewitt, wavelet transform based, mathematical morphology based and type-1 fuzzy proposed algorithm edge detectors respectively.

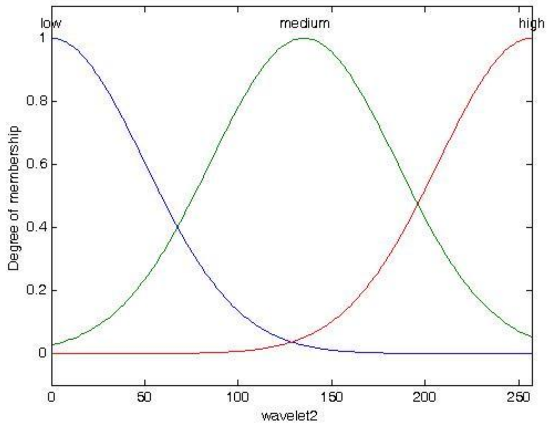
For the purpose of evaluating the performance of type-2 fuzzy in comparing type-1 fuzzy, we conducted the edge detection experiments using type-2 fuzzy rules. The structure of the type-2 fuzzy rules is the same as for the type-1 case; this is because the difference between type-2 and type-1 is related with the nature of the membership functions. Since a higher type changes the nature of the membership functions, the operations that depend on the membership functions change; however, the basic principles of fuzzy logic are independent of the nature of membership functions and hence, do not change [12].

We conducted the proposed edge detection method using type-2 fuzzy rules. Fig. 4 shows the used type-2 fuzzy system. The membership functions that are used for the first input, second input, third input and type-2 fuzzy system output are shown in Fig. 5 (a), Fig. 5 (b), Fig. 5 (c) and Fig. 5 (d) respectively. Fig. 6 show the detected edges images using type-2 fuzzy system proposed method.

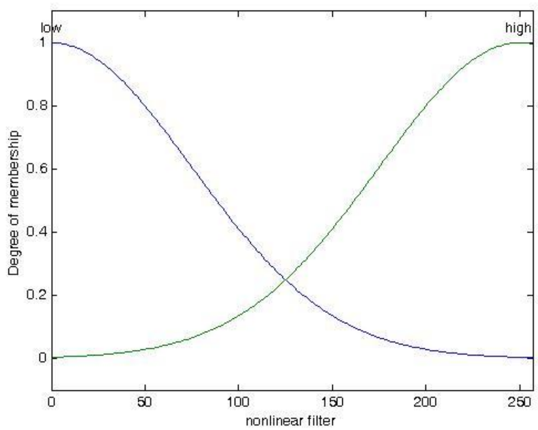
E. Results and Discussion

Fig. 3 (b) indicates that Prewitt method is noise sensitive and produces many false edges in detected image. Fig. 3 (c) shows that wavelet based method is noise resistant, but produces many discontinuous edges and loses some information. Fig. 3 (d) indicates that morphology based method is noise sensitive and produces many false edges in detected image. Fig. 3 (e) and Fig. 6 indicate that proposed method is not only noise immune but also produces more edges detail.

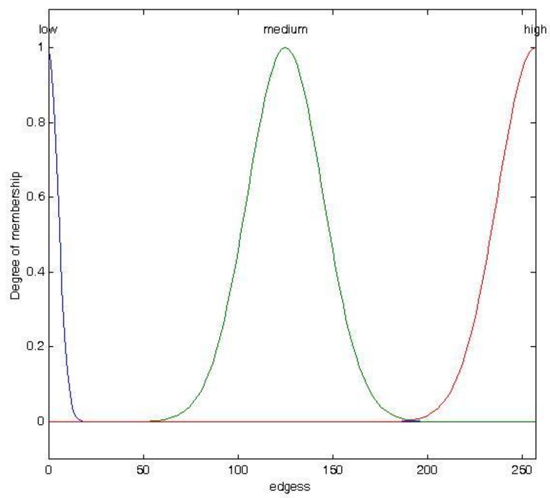
There are some pixels that the proposed methods can’t accurately decide they are edge pixels or not. Fig. 3 (e) and Fig.6 indicate that although type-1 and type-2 fuzzy proposed method have the same efficiency against the noise, but type-2 fuzzy proposed method have better performance for ambiguities pixels.



(a)

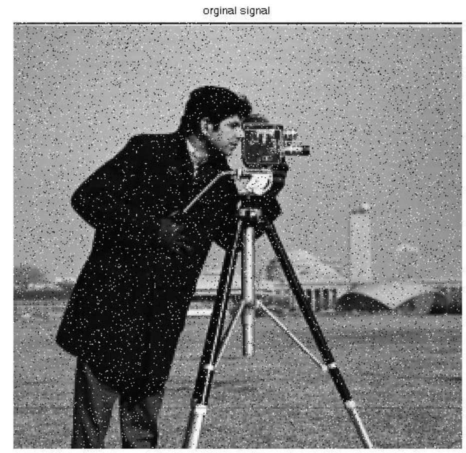


(b)



(c)

Fig. 2 (a) The membership function for the first and second fuzzy system inputs, (b) The membership function for third input, (c) The membership function for output.

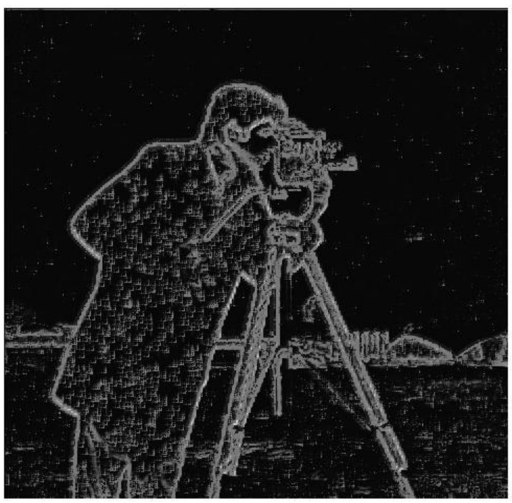


(a)

(b)

(c)

(d)



(e)

Fig.3 (a) Noisy image, (b) Edges detected based on Prewitt, (c) Edges detected based on wavelet, (d) Edges detected based on morphology, (e) Edges detected based on type-1 fuzzy proposed method.

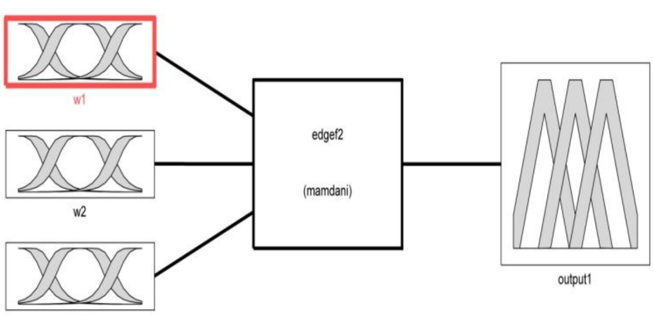
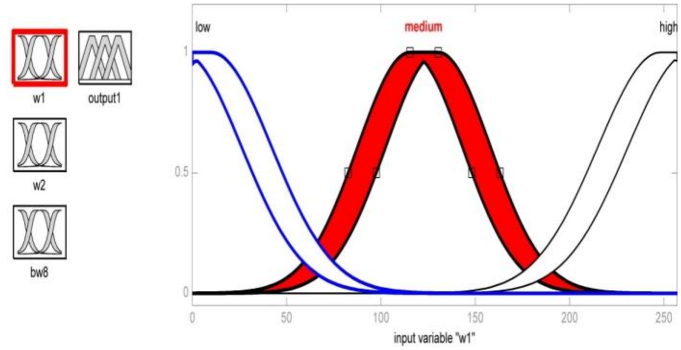
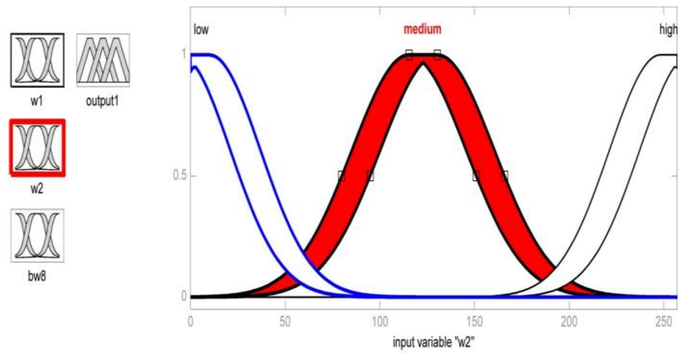


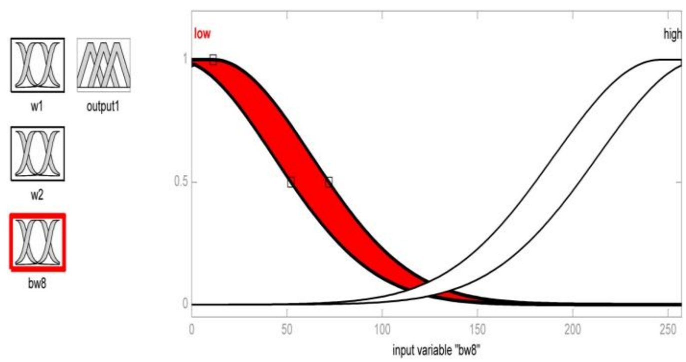
Fig. 4 The used type-2 fuzzy system.



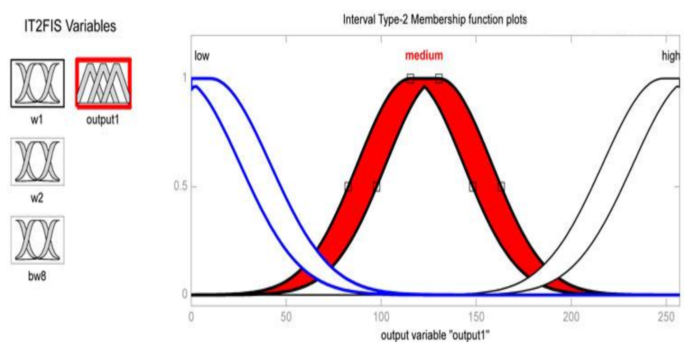
(a)



(b)



(C)



(d)

Fig. 5 (a) The membership function for the first type-2 fuzzy system input, (b) The membership function for the second input, (c) The membership function for the third input, (d) The membership function for the output.

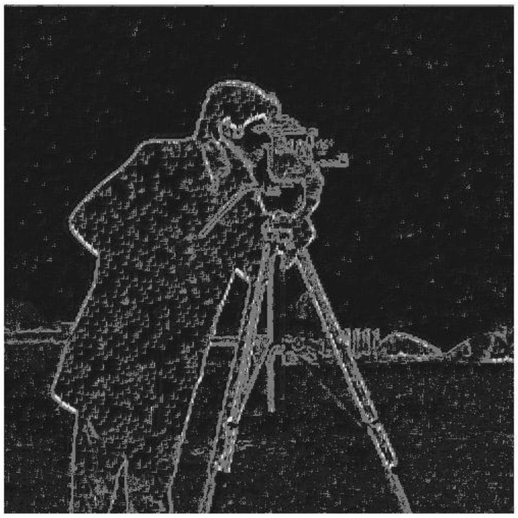


Fig. 6 Edge detected based on type-2 fuzzy proposed method.

III. Conclusion

In this paper, we have proposed an edge detection approach based on fusion of wavelet transform and mathematical morphology using type-1 and type-2 fuzzy rules. The proposed method combines the merits wavelet, fuzzy logic and morphology respectively. We compared proposed method in edge detection with Prewitt, wavelet based and mathematical morphology based edge detectors. In addition, the type-1 fuzzy proposed method and the type-2 fuzzy proposed method are compared. Experimental results show that the proposed algorithm not only suppresses noise effectively, but also keeps clear and continuous edges. We have shown that the proposed method outperform other compared edge detectors that are widely used in the edge detection. Also, it was shown the type-2 fuzzy proposed method outperforms the type-1 fuzzy proposed method. This research could be one of the first of its kind that would suggest the type-2 fuzzy proposed method as a replacement to the conventional edge detectors. An important step in most medical imaging analysis systems is to extract the boundary of an area we are interested in. An extension to this work would be to using fuzzy type-2 proposed method for medical images edge detection.

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