Design of Optimal Robust Controller for Third Order System using Particle Swarm Optimization

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Abstract: In this paper particle swarm optimization(PSO)has been applied for the design of the optimal robust controllers for third order systems . The controller design problem is posed as constrained nonlinear optimization problem. The parameters of the chosen controller are obtained solving the nonlinear constrained optimization problem.The performance index which has been used in the design is integral square error(ISE).The constraints are frequency domain performances related with robust stability .

keywords—PSO, ISE, nonlinear optimization, optimal robust controller.

I. INTRODUCTION

There are model uncertainties present in a dynamical system or plant. Due to these model uncertainties there is a need to design robust controller. Robust controller provides robustness in the face of uncertainties [1]. Recently, H_∞-control techniques have found extensive applications for the design of robust controllers [2]-[3]. These techniques make use of H_{∞}-norm and robustness of the system is achieved in terms of stability and performance. The main disadvantage of the design techniques based on H_{∞}- theory is that the order of the controller is high. The parameter optimization techniques help in order reduction. Parameter optimization methods start with controller structures that are motivated by the ideas from classical, modern or other techniques. What is meant by the controller structure is a system model with one or more parameter values that can be adjusted. The next step in a parameter optimization method is to select an objective function or performance index that gives the quality of performance.

After a controller structure, an objective function and some constraints have been specified, the problem can be posed as non-linear optimization problem which can be solved to get the parameters of the controller [4]. The objective function may consist of time domain and/or frequency domain performances expected from the system.

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In general, this objective function is non-linear, non-differentiable, discontinuous and non-convex in nature. The optimization methods based on calculus will not work. Only search methods can be used. The classical methods use nominal model of plant.The robustness of the control loop is indicated by phase margin and gain margin. Evolutionary Algorithms guarantee to provide global or near global optimal solution [5]-[9].

In this paper, the objective function which has been used in the optimization is indicative of the time domain performance of the system, namely, integral square (ISE) error .The constraints which have been imposed in the optimization are related with the robust stability.

2. OPTIMAL ROBUST CONTROL

While designing the robust controller, the model uncertainty of the plant is explicitly considered, two kinds of model uncertainties: structured and non-structured. Structured model uncertainty or parametric model uncertainty is caused by the parametric modifications of the plant and can be described by the approaches, such as, interval methods [10]-[11]. The causes of non-structured model uncertainty are, usually, non-linearities of the plant or modifications of the operating point. This type of the model uncertainties can be represented using H_∞-theory. The classical methods of the controller design use a nominal model of the plant. The classical measures of the robustness of the system are gain and phase margins. In the robust controller design methods based on the H_{∞}-theory, a family of the models of the plant is used. A nominal model of the plant and model uncertainty are considered. It is necessary to guarantee the stability of the feedback control system taking into account the model uncertainty. The conditions of the robust stability described using H_{∞}-norm.

A. Condition for Robust Stability

Consider the control system shown in the Fig.1. The controller is described by means of a transfer function with fixed structure *C(s,k).* The vector of the controller parameters; k, is

$$k = [k_1, k_2 \ldots\ldots, k_m]^T \tag{1}$$

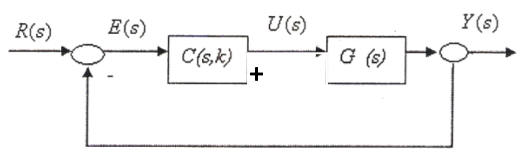

Fig.1. Control system composed of a controller with fixed structure and a plant with model uncertainty

The plant is described by a multiplicative model according to equation (2). By using the multiplicative model, the transfer function of the real (perturbed) plant *G(s)* is described by the following [3]:

$$G(s)=G_o(s)\,(1+\Delta(s)W_m(s)) \tag{2}$$

Where
$G_0(s)$ is nominal transfer function,
$\Delta(s)$ is perturbation in the plant and
$W_m(s)$ is weighting function that represents an upper bound of the multiplicative uncertainty.
It is assumed that the model uncertainty, $W_m(s)$, is stable and bounded, and that no unstable poles of $G_o(s)$ are canceled in forming G(s).

The condition for robust stability is stated as follows [3]:
If the nominal control system ($\Delta(s)=0$) is stable with the controller C(s,k), it guarantees robust stability of the control system, if and only if the following condition is satisfied:

$$\left\| \frac{C(s,k)Go(s)Wm(s)}{1+C(s,k)Go(s)} \right\|_\infty < 1 \tag{3}$$

This condition for robust stability represents only a sufficient condition. So, the robust stability of a control system can be evaluated by means of the H_∞ - norm.

Generally, the multiplicative model is used. If the plant is described by an additive model, it can be easily' converted into a multiplicative model. This paper will consider the multiplicative model.

3. OPTIMAL ROBUST CONTROLLER DESIGN

In Fig.1, for the nominal case, the tracking error signal is given by

$$e(s)=\frac{r(s)}{1+G_o(s)C(s,k)} \tag{4}$$

The performance index ,J, is given by

$$J=\min_c \int_0^\infty e^2(t)dt \tag{5}$$

It can be described in the frequency domain of the Parseval theorem]:

$$J=-\frac{1}{2\pi j}\int_{-j\infty}^{+j\infty} E(s)E(-s)ds \tag{6}$$

The reference signal (set point) is an unit step function given by:

$$R(s)=\frac{1}{s} \tag{7}$$

The error E(s) can be expressed then as a rational function:

$$E(s)=\frac{D(s)}{A(s)}=\frac{\sum_{j=0}^{m} d_j s^{m-j}}{\sum_{i=0}^{n} a_i s^{n-i}} \tag{8}$$

In this case, the degree m of the polynomial D(s) must be smaller than the degree n of the polynominal A(s), so that the squared error J in equation (6) has a finite value. Introducing the error E(s) from equation (8) into equation (6) results in the following

$$J=-\frac{1}{2\pi j}\int_{-j\infty}^{+j\infty} \frac{[\sum_{j=0}^{m} d_j s^{m-j}][\sum_{j=0}^{m} d_j (-s)^{m-j}]}{[\sum_{i=0}^{n} a_i s^{n-i}][\sum_{i=0}^{n} a_i (-s)^{n-i}]} ds$$

In design of optimal robust controller, both the tracking performance and robust stability are considered. The controller design is formulated as constrained optimization problem as follows:

$$\min_k J_n(k) \text{ subject to } \max_\omega(\alpha(\omega,k))^{0.5}\langle 1$$

The objective of the minimization is to find out the vector of controller parameters k so that the value of the performance index $J_n(k)$ is minimum and the condition of robust stability $\max_{\omega\in[0,\infty)}(\alpha(\omega,k))^{0.5}\langle 1$ is satisfied.

4. PARTICLE SWARM OPTIMIZATION

It is a population based stochastic optimization technique developed in 1995 [12],from the simulation of social behavior of bird flocking or fish schooling. PSO has been found to be simple, effective and robust in solving problems with nonlinearity, non-differentiability and multidimensional optimization [13]. In PSO, each particle represents a candidate solution to the optimization problem. At the beginning, each particle spans randomly through the problem space and updates its velocity and position with the two best values. The first best value, called pbest is the best solution achieved so for. Another value, called gbest is the Global best solution obtained so far by any particle in the swarm. At each interaction, each particle moves to pbest and gbest locations. The cost function evaluates the performance of particles to determine whether the best solution is achieved. In the present thesis work, the PSO is used to solve the constrained optimization problem.

In PSO algorithms each particle moves with an adaptable velocity within the regions of decision space and retains a memory of the best position it ever encountered. The best position ever attained by each particle of the swarm is communicated to all other particles. The updating equations of the velocity and position are given as follows:-
A particle position is given by $x_i(k)$
A particle velocity is given by $v_i(k)$
A best "remembered" individual particle position is given by $p_i(k)$
A best "remembered" swarm position is given by $p_g(k)$
Cognitive and social parameters referred to as acceleration constants are given by c_1 and c_2.Random numbers between 0 and 1 are r_1 and r_2.A inertia weight is given by w.P_i refers who best position found by particles. Velocity of Individual particle is updated as follows:
$v_i(k+1) = wv_i(k) + r_1c_1[p_i(k) - x_i(k)] + r_2c_2[p_g(k) - x_i(k)]$
Position of individual particle is updated as follows:
$x_i(k+1) = x_i(k) + v_i(k+1)$
The details of the PSO algorithm are given in flowchart.

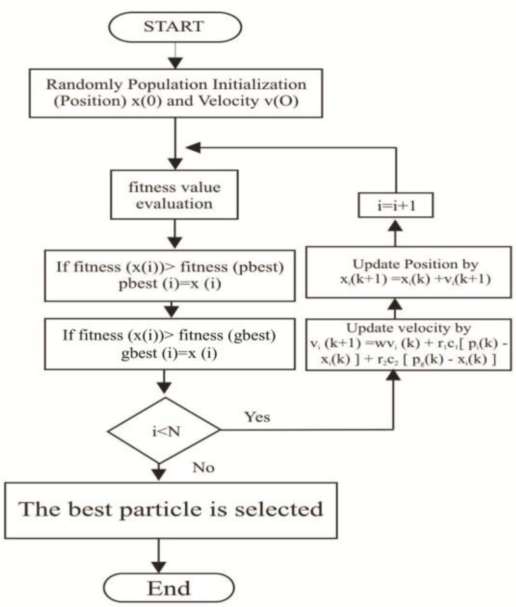

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Fig.2. Flowchart of the PSO algorithm

5. DESIGN EXAMPLE

To illustrate the method, a detailed design example is presented. Consider the control system shown in the Fig. 4.1The model of plant taken from [14] is described by the following transfer function:

$$G_0(s) = \frac{1.8}{s^3+2s^2} \tag{9}$$

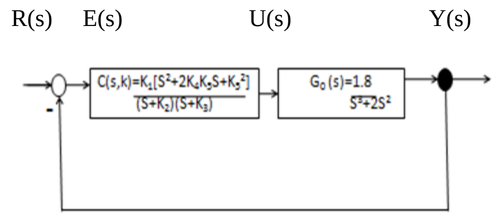

Fig.3. Control system with uncertain plant

The controller structure *C(s,k)* is chosen in the following form [14]

$$C(s,k) = k_1 \frac{s^2 + 2k_4k_5s + k_5^2}{(s+k_2)(s+k_3)} \tag{10}$$

The vector k of controller parameters is given *by k* = $[k_1, k_2, k_3, k_4, k_5]^T$ which is to be obtained solving the optimization problem.
The multiplicative uncertainty $W_m(s)$ is taken as [14]:

$$W_m(s) = \frac{0.1}{s^2 + 0.1s + 10} \tag{11}$$

The error signal *E(s)*, assuming the input signal is a unit step, is evaluated as follows

$$E(s) = \frac{1}{1 + C(s,k)Go(s)} R(s)$$
: (12)

The squared error $J_5(k) = E' E$ is obtained by calculating error *E* due to step input at each instant from 1 to 10 seconds in the interval of .05 sec. This squared error is to be minimized under the robust stability constraint given by the equation (3). The H_∞ norm in equation (3) is calculated using MATLAB function normhinf. Bode plots of system without controller are shown in the Fig.4.2. The gain and phase margins are infinity and -24.4 deg respectively.

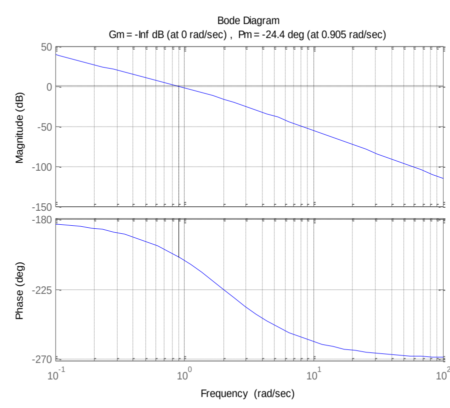

Fig.4. Bode plots of the system without controller

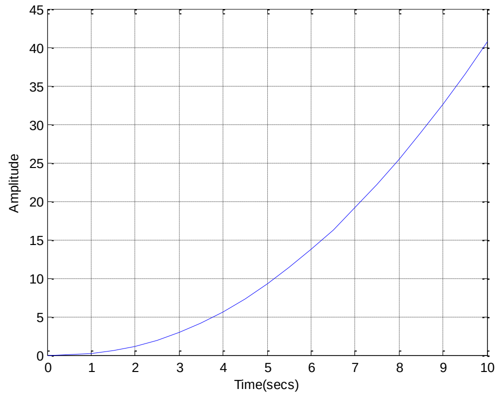

Fig.5. Step response of the plant without controller

Performance index J_5 *(k)* is to be minimized is taken as ISE. In writing m-file the integral has been taken as summation over 201 points. Time is vector consisting of time instants from 0 to 10 sec in the interval of 0.05 second. $J_5(k)$ has been minimized under the constraint of robust stability given by equation (3). The following PSO parameters were used in running the PSO

TABLE 1 OPTIONS SETTINGS REQUIRED AS INPUT FOR PSO

Sr.No.	Options	Values
1	Number of particles in swarm for each variable to be optimized	25
2	Cognitive acceleration coefficient	2.8
3	Social acceleration coefficient	1.3
4	Maximum number of iterations	2500
5	Maximum duration of optimization	2500
6	Maximum number of function evaluations	2500
7	Maximum difference between best and worst function	1e-6

The controller parameter vector was searched in following bounds:
k_1= [10,1000]; k_2 = [1,100]; k_3 [1,100]; k_4 = [0,1]; k_5 = [0.1,10].

The PSO algorithm converged with minimum value of $J_5(k^*)$=3.5131 and optimal solution vector k* = $[999.99, 14.5065, 14.5146, 1, .5387]^T$
Bode plots of system with designed controller are shown in the Fig.6. The gain and phase margins are 9.14 dB and 49.6 deg respectively.

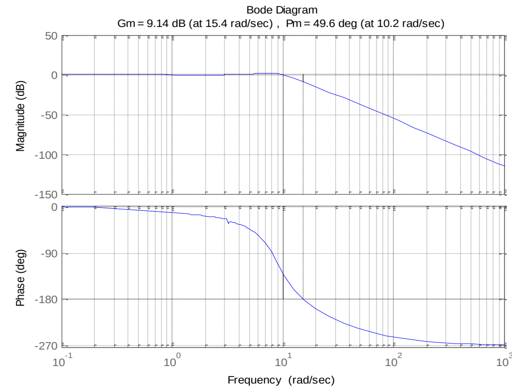

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Fig.6. Bode plots of system with the designed controller

The closed loop step response of the feedback control system shown in the Fig. 3 with designed controller is shown in Fig. 7.

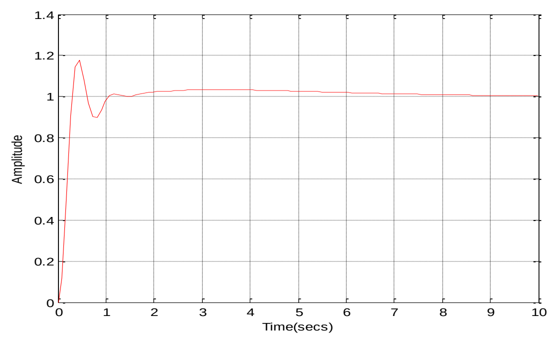

Fig.7. Step response of the controlled plant without uncertainty

The closed loop step response of the feedback control system shown in the Fig.3 with uncertainty given in equation (11) with designed controller is shown in Fig. 8.

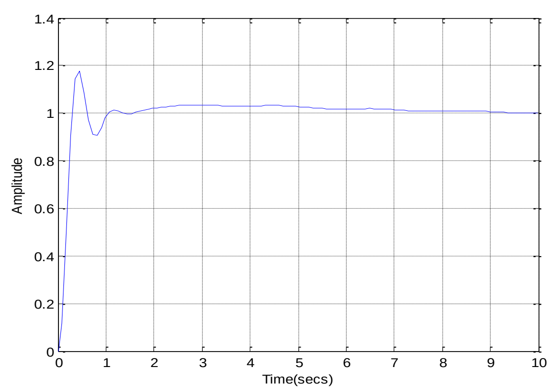

Fig.8. Step response of the controlled plant with uncertainty

The performance of the control system shown in Fig.3 with designed controller is compared with respect to closed-loop step response with and without uncertainty. The tracking behavior of the control system with and without uncertainty is shown in Fig. 9.

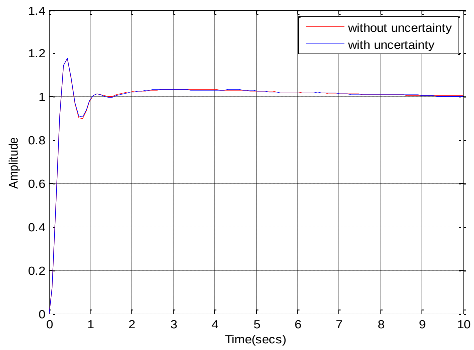

Fig.9. Step response of the controlled plant with and without uncertainty

There is no difference between the two responses. The designed controller gives satisfactory response in the face of plant uncertainty.

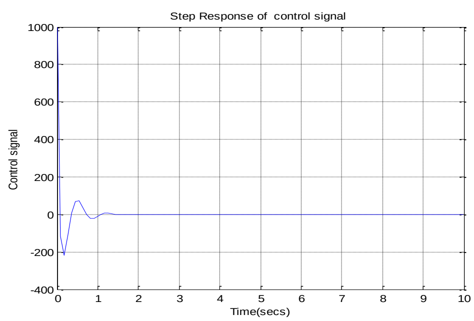

Fig.10. Control signal

The following time domain performances have been achieved:

6. CONCLUSIONS

In this paper a method is presented to design an optimal robust controller with fixed structure, Known in the literature as the mixed H_2/H_∞ problem. The design problem is formulated as an optimization problem with constraint of type H_∞ norm. The tracking performance of the closed loop system with proposed method has been found. Therefore the proposed control algorithms are shown to be effective. In the future, this control method can be further extended and applied to multivariable system.

REFERENCES

[1] K. Mueller , Entwurf Robuster Regelungen, Teubner Verlag, Struttgart, 1996.

[2] T.E. Djaferis, Robust Control Design: A Polynomial Approach ,Kluwer Academic, Dprdrecht, 1995.

[3] J.C. Doyle et al., "Mixed H_2 and H_∞ Performance Objective II: Optimal Control", IEEE Trans. on . Automatic Control, 39, 8, pp. 1575-1587, 1994

[4] M. Jamshidi, L.D.S. Coelho, R.A. Krohling and P.J. Fleming, Robust control system with genetic algorithm, CRC Press, 2003.

[5] B.S. Chen, Y.M. Cheng and C.H. Lee,"A genetic approach to mixed H_2/H_∞ optimal PID control", IEEE Control Systems Magazines, pp. 51-59 ,1995.

[6] P.P. khargonekar and M.A. Rotea, "Mixed H_2/ H_∞ control: A convex optimization approach'', IEEE Transactions on Automatic Control, vol. 36,no. 7,pp. 824-837,1992.

[7] W.R.Esposito,C.A.Floudas"Deterministic global optimization in nonlinear optimal control problems", Journal of Global Optimization,vol. 17, pp. 97-126,2000.

[8] A.B. Singer and P.I. Barton, "Global optimization with nonlinear ordinary differential equations", Journal of Global optimization,vol. 34, pp. 159-160,2006.

[9] M.M. Ali, C. Khompatraporn and Z. Zabinsky "A numerical Evaluation of several Stochastic Algorithms on selected Continuous Global Optimization Test Problems", Journal of Global Optimization, vol. 31,pp. 635-672,2005.

[10] S.P. Bhattacharyya, H. Chapellat and L.H. Keel, Robust Control: The Parametric Approach, Prentice Hall, Englewood Cliffs, New Jersey, 1995.

[11] J.Ackermann, Robust Control: System with Uncertain Physical Parameters, Springer- Verlag, Berlin,1993.

[12] J.Kennedy and R.C. Eberhark, Swarm intelligence, Morgan Kaufmann, San Francisco, CA, 2001.

[13] L.H. Lin, F.C. Wang and J.Y. Yen, "Robust PID controller design using particle swarm optimization", Proc. of the 7th Asian Control Conference, Hong Kong, China, pp.1673 -1678, Aug. 2009.

[14] C.G.Lo Bianco and A Piazzi, "Mixed H_2 /H_∞ fixed structure control via semi-infinite optimization", Proc. of the 7th IFAC International Symposium on CACSD, Gent, pp. 329-334,1997.

[15] K.J. Astrom and C.C.Hang, W.K. Ho, "Towards Intelligent PID control'', Automatica,vol. 28, no. 1, pp.1-9,1992.

[16] C.C. Hang, K.J. Astrom and W.K. Ho,"Refinements of the Ziegler-Nichols tuning formula", IEEE Proc.-D,vol. 138,no. 2, pp. 111-118, 1991.

[17] S. Tengchen, B.S. Chen and Y.P. Lin, "A mixed H_2/ H_∞ control design for linear time-varying systems: dynamic programming approach", Journal of Control system and technology,vol. 1, no. 1, pp.43-59,1993.

[18] D.S. Bernstein and D.S. Haddad, "LQG control with an H_∞ performance bound", A Riccati Equation approach, IEEE Transactions on Automatic Control,vol. 34, no. 3, pp. 293-305,1993.

[19] P.Trivedi and B.Bandyopadhyay, "An algorithm for third order sliding mode control by keeping a 2-sliding constraint",12th International Workshop on Variable Structure Systems,pp. 463-466,2012.