Space Time Block Codes: An Overview

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***Abstract* — The demand for mobile communication systems with high data rates has dramatically increased in recent years. New methods are necessary in order to satisfy this huge communications demand, exploiting the limited resources such as bandwidth and power as efficient as possible. In wireless communication systems, multiple antennas when used with appropriate space-time coding (STC) techniques can achieve huge performance in multipath fading wireless links and to provide diversity gain and coding gain, respectively. The use of more than one antenna at transmitting end, improves the receive diversity. Data is encoded using a space–time block code, and then split into n streams which are simultaneously transmitted using n transmit antennas. Maximum likelihood (ML) decoding is achieved by a simple way through decoupling of the signals transmitted from different antennas rather than a joint detection. When the channel is not constant during the period of codeword transmission, the conventional linear maximum likelihood receiver suffers from performance degradation. The fundamentals of space-time coding were established in the context of space-time Trellis coding by Tarokh, Seshadri and Calderbank. Then Alamouti proposed a simple transmit diversity coding scheme and based on this scheme. Our main focus is upon signal detection and channel estimation for wireless communication systems. In this paper, the receiver performance for Alamouti code is analyzed. Here, we evaluate the performances of the two channel estimation schemes when employing 2 transmit and one receive antennas employing BPSK modulation methods.**

***Index Terms* —** MIMO, space–time codes, encoding and decoding, Alamouti codes.

I. INTRODUCTION

The capacity requirement in wireless LAN and cellular mobile system during last decade has grown explosively. In particular, compared to the data rates made available by today's technology, the need for wireless Internet access and multimedia applications require an increase in information throughput with order of magnitude. This increase in data rate is possible with the use of multiple antennas at the transmitters and receivers in the system. The use of multiple antennas at transmitter end can reproduce many benefits as well as substantial amounts of performance gain of receive diversity. Multiple-Input Multiple-Output (MIMO) technology [1][2] can be expected as a cornerstone of many wireless communication systems due to the potential increase in data rate and performance of wireless links offered by transmit diversity and MIMO technology.

Fig.1 represents a MIMO wireless communication system, containing multiple antennas at both the transmitter and receiver. The predominant cellular network implementation is the one which has a single antenna on mobile device and multiple antennas at the base station, resulting in reduction in the cost of

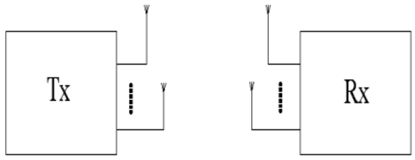

Figure 1: MIMO communication system

the mobile radio. On the other hand, when the costs for other radio frequency components in mobile devices are compromised (go down), then a second antenna in mobile device becomes more common. Today, cellular phones, laptops and other communication devices have two or more antennas.

MIMO is also planned to be used in mobile radio telephone standards such as recent 3G and 4G standards [16]. In general MIMO gives a significant increase in spectral efficiency, improves the reception and allows for a better reach and rate of transmission. All upcoming 4G systems will also employ MIMO technology.

II. SPACE TIME CODES

Use of multiple antennas can improve the reliability of data transmission, the method employed for this purpose in wireless communication is known as Space-time code (STC) [8][10][12]. STCs rely on transmitting multiple, redundant copies of a data stream to the receiver to allow reliable decoding.

Space time codes could be divided into following three types:

a) Space–time trellis codes (STTCs) [12] distribute a Trellis code over multiple antennas and multiple time-slots. It is used to provide both coding gain and diversity gain. This scheme was proposed by Tarokh [11]. Due to the combined benefits of forward error correction (FEC) coding and diversity transmission, Trellis coding is proved to be a highly effective scheme and also provides a considerable performance.

b) Space-time turbo codes (STTuC) [7] is a combination of space-time coding and turbo coding. They are originally introduced as binary error-correcting codes which are called turbo decoding algorithm.

c) Space–time block codes (STBCs), [8][10] which act on a block of data. In this coding scheme, only the diversity gain is obtained, and not the coding gain. This makes STBC less complex in implementation than STTCs and STTuC.

III. SPACE-TIME BLOCK CODES

This technique is based on the concept of diversity techniques using smart antennas and is more effective than traditional diversity techniques as it uses data coding and signal processing at both sides of transmitter and receiver [3][4][5][6]. The main shortcoming of receive diversity for mobile communications is the distance between the receive antennas so that the signals received at each antenna undergoes independent fade. This increases the cost in having multiple antennas in a small mobile unit, as they have got to be small in size and light in weight. Hence, due to allowable highly complex base station the use of transmit diversity in base stations appears to be an attractive method [9][13][14][15]. Similar to the case of MIMO channels, base stations take up the advantage of using both transmit and receive diversities when they communicate with each other. When a signal is transmitted from a base station to a mobile unit, forming a channel of Multiple Input Single Output (MISO), transmit diversity can be used.

Space-time block codes (STBC) are a generalized version of Alamouti scheme. These codes have the same key features. That is, they are orthogonal and can achieve full transmit diversity specified by the number of transmit antennas. In other words, space-time block codes are a complex version of Alamouti space-time code, where the encoding and decoding schemes are the same as in both the transmitter and receiver sides. The data are constructed as a matrix which has its rows equal to the number of the transmit antennas and its columns equal to the number of the time slots required to transmit the data. At the receiver side, when signals are received, they are first combined and then sent to the maximum likelihood detector where the decision rules are applied.

Space-time block code was designed to achieve the maximum diversity order for the given number of transmit and receive antennas subject to the constraint of having a simple decoding algorithm. In addition, space-time block coding provides full diversity advantage but is not optimized for coding gain.

A) Space-Time Block Encoding and Decoding

Figure 2 shows the structure of space-time block encoder for two transmit and one receive antenna which is the same as Alamouti encoder. As known, in general, space-time block code is defined by n_T x p transmission matrix S, where n_T represents the number of transmit antennas and p represents the number of time periods needed to transmit one block of coded symbols. The ratio between the number of symbols that space-time block encoder takes as its input (k) and the number of space-time coded symbols transmitted from each antenna defines the rate of a space-time block code. The rate of any space-time block codes with two transmit antennas is equal to one. The rate of a space-time block code can be calculated by:

$$R = \frac{k}{p} \qquad (1)$$

Although the transmission sides are the same, the receiver sides are quite different. The receiver in this case has two receive antennas instead of one, which increases the receive diversity compared with a system with one receive antenna. Figure 2 shows the Space Time Block Encoder.

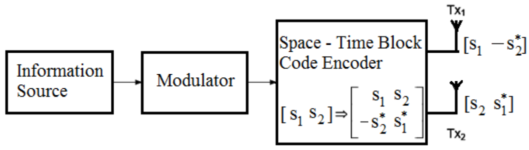

Figure 2 Space time Block Code Encoder

Table 1 shows the channel coefficients for the space-time system with two transmit and two receive antennas.

TABLE 1: CHANNEL CONFIGURATION FOR TWO TRANSMIT AND TWO RECEIVE ANTENNAS

	Rx_1	Rx_2
Tx_1	h_1	h_3
Tx_2	h_2	h_4

Here, h_1 and h_2 are channel coefficient, Tx_1 & Tx_2 are two transmit antenna and Rx_1 & Rx_2 are two receive antenna.

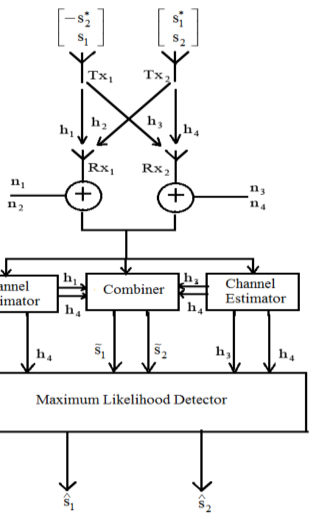

Fig 3: STBC scheme with two transmit & 2 receive antenna

The received signals at the two receive antennas denoted by r_1, r_2, r_3 and r_4 for t and t+T, respectively, can be expressed as:

$$r_1 = h_1 s_1 + h_2 s_2 + n_1 \qquad (2)$$

$$r_2 = -h_1 s_2^* + h_2 s_1^* + n_2 \quad (3)$$

$$r_3 = h_3 s_1 + h_4 s_2 + n_3 \quad (4)$$

$$r_4 = -h_3 s_2^* + h_4 s_1^* + n_4 \quad (5)$$

The combiner in the figure 3 builds the following two signals that are sent to maximum likelihood detector:

$$\tilde{s}_1 = h_1^* r_1 + h_2 r_2^* + h_3^* r_3 + h_4 r_4^* \quad (6)$$

$$\tilde{s}_2 = h_2^* r_1 - h_1 r_2^* + h_4^* r_3 - h_3 r_4^* \quad (7)$$

The maximum likelihood decoding rule for s_1 and s_2 can be derived as:

$$\hat{s}_1 = \arg\min_{(\hat{s}_1,\hat{s}_2)\epsilon C} (|h_1|^2 + |h_2|^2 + |h_3|^2 + |h_4|^2 - 1)|\hat{s}_1|^2 +$$

$$\hat{s}_2 = \arg\min_{(\hat{s}_1,\hat{s}_2)\epsilon C} (|h_1|^2 + |h_2|^2 + |h_3|^2 + |h_4|^2 - 1)|\hat{s}_2|^2 +$$

In all simulations, the first step to consider before creating a space-time block code is the type of signal needed to be transmitted. There are two types of signals that can be created and transmitted. The first type of signals is the complex signal that can be generated by using any complex modulation schemes like M-QPSK and M-QAM. The second type of signals is the real signal that can be generated using any real modulation schemes like BPSK or PAM. In this paper we have discussed the performance of BPSK signals.

The creation of any space-time block code matrix depends on the type of the transmitted signal. If the transmitted signal is real, then the transmission matrix would be G_2, G_4, or any other real square matrix of rate of one.

$$G_2 = \begin{pmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{pmatrix}$$

$$G_4 = \begin{pmatrix} x_1 & x_2 & x_3 & x_4 \\ -x_2 & x_1 & -x_4 & x_3 \\ -x_3 & x_4 & x_1 & -x_2 \\ -x_4 & -x_3 & x_2 & x_1 \\ x_1^* & x_2^* & x_3^* & x_4^* \\ -x_2^* & x_1^* & -x_4^* & x_3^* \\ -x_3^* & x_4^* & x_1^* & -x_2^* \\ -x_4^* & -x_3^* & x_2^* & x_1^* \end{pmatrix}$$

However, if the transmitted signal is complex, then a square G_2 is used, if the number of transmit antennas is equal to two but if the number of transmit antennas is greater than two, and then a complex, non-square matrix is used. The rate of such matrices could be either 1/2 or 3/4. These matrices can never achieve the full rate of one. The only exception is the Alamouti cases for two transmit antennas using a G_2 matrix.

B) Performance of Space-time Block Codes

The performance of space-time block codes depends on the type of modulation and the number of transmit and receive antennas used. On the other hand, space-time block codes with larger number of transmit antennas always give better performance than space-time block codes with lower number of transmit antennas. This is true because larger number of transmit antennas means larger transmission matrices which means transmitting more data. This would give the receiver the ability to recover the transmitted data. Moreover, with larger number of receive antennas, the same transmitted data would be received by more than one receive antenna. This is an advantage because if one receive antenna did not recover the transmitted data correctly, the second receive antenna could. The chance that at least one out of two receive antennas would receive the transmitted data uncorrupted is always higher than if there is only one receive antenna.

Figures 4 show the performance using BPSK modulation. All BPSK modulated symbols can take only one bit at a time.

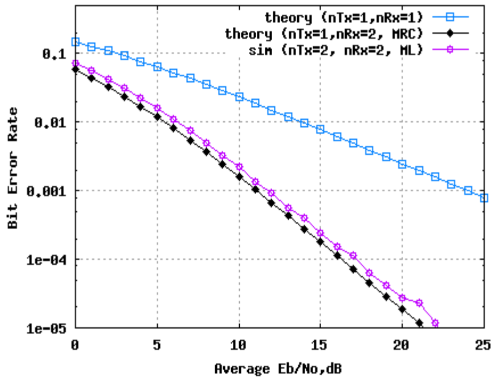

Figure 4: Space time block code scheme with M-transmit and 2-receive antennas using BPSK modulation

IV ALAMOUTI CODES

The very first space-time block code scheme, providing full transmit diversity for systems with two transmit and one receive antennas is Alamouti scheme. It is a unique scheme which uses space-time block code with an n_T x n_T complex transmission matrix to achieve the full rate of one.

A) Alamouti Encoding

At the transmitter side, a block of two symbols are taken from the source data and sent to the modulator. After that, Alamouti space-time encoder takes the two modulated symbols, in this case called s_1 and s_2 at a time and creates encoding matrix where the symbols and are mapped to two transmit antennas in two transmit times as defined in the following:

$$S = \begin{bmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{bmatrix} \quad (8)$$

where the symbol * represents the complex conjugate. Therefore, s_1^* is the complex conjugate of s_1. The encoders outputs are transmitted in two consecutive transmission periods from the two transmit antennas. In the first transmission period, the signal s_1 is transmitted from antenna one and the signal s_2 is transmitted from antenna two, simultaneously. In the second transmission period, the signal $-s_2^*$ is transmitted from antenna one and the signal s_1^* is transmitted from antenna two. The block diagram of the transmitter side using Alamouti space-time encoder is shown in Figure 5.

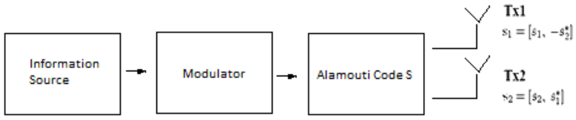

Figure 5: A block diagram of Alamouti Space time Encoder

The encoding and mapping of this scheme can be summarized as in Table 2:

	R_{x1}
T_{x1}	h_1
T_{x2}	h_2

TABLE 2: ENCODING AND MAPPING FOR TWO TRANSMIT ANTENNAS USING COMPLEX SIGNALS.

	T_{x1}	T_{x2}
t	s_1	s_2
t+T	$-s^*_2$	s^*_1

where t represents the transmission symbol period, T_{X1} and Tx_2 are the first and second transmit antennas. The transmit sequence from antennas one and two denoted by s^1 and s^2 are encoded in both space and time domains.

$$s^1 = [s_1 \quad -s_2^*] \qquad (9)$$

$$s^2 = [s_2 \quad s_1^*] \qquad (10)$$

The inner product of s^1 and s^2 is equal to zero. This confirms the orthogonality of the Alamouti space-time scheme. The encoding is performed in both time (two transmission intervals) and space domain (across two transmit antennas). The two rows and columns of S are orthogonal to each other and the code matrix is orthogonal:

$$SS^H = \begin{bmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{bmatrix} \begin{bmatrix} s_1^* & -s_2 \\ s_2^* & s_1 \end{bmatrix}$$

$$= \begin{bmatrix} |s_1|^2 + |s_2|^2 & 0 \\ 0 & |s_1|^2 + |s_2|^2 \end{bmatrix} = (|s_1|^2 + |s_2|^2) I_2 \qquad (11)$$

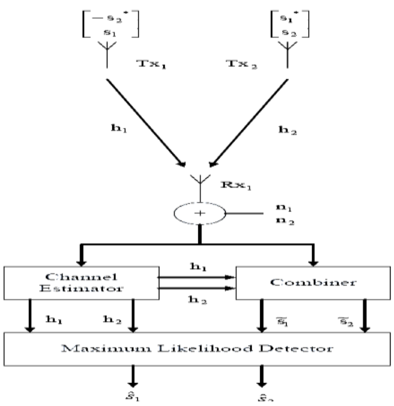

Fig 6: Alamouti Space Time System with one receive system

The block diagram of the Alamouti space-time system is shown in Figure 6. The fading coefficients denoted by h_1 (t) and h_2 (t) are assumed constant across the two consecutive symbol transmission periods and they can be defined as:

$$h_1(t) = h_1(t+T) = h_1 = |h_1| e^{j\theta_1} \qquad (12)$$

$$h_2(t) = h_2(t+T) = h_2 = |h_2| e^{j\theta_2} \qquad (13)$$

where h_1(t) and h_2(t) are the fading coefficients from the first and the second transmit antennas to the receive antenna at time. Where $|h_i|$ and θ_i, i= 1, 2, are the amplitude gain and the phase shift, respectively. T is the symbol period.

TABLE 3: TWO TRANSMIT AND ONE RECEIVE ANTENNA CHANNEL COEFFICIENTS.

The channel coefficients for two transmit antennas and one receive antenna for Alamouti space-time code can be expressed as in Table 3. Where Tx_1 & Tx_2 are the first and second transmit antennas and Rx_1 is the receive antenna

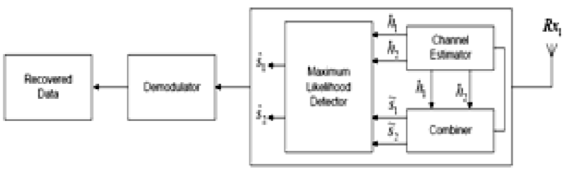

Fig 7: Alamouti Space Time Decoder

The block diagram of the receiver side using Alamouti space-time decoder is shown in Figure 7.

The receiver receives r_1 and r_2 denoting the two received signals over the two consecutive symbol periods for time t and t+T. The received signals can be expressed by:

$$r_1 = s_1 h_1 + s_2 h_2 + n_1 \qquad (14)$$

$$r_2 = -s_2^* h_1 + s_1^* h_2 + n_2 \qquad (15)$$

Or,

$$\begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{bmatrix} \begin{matrix} h_1 \\ h_2 \end{matrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} = \begin{bmatrix} h_1 s_1 + h_2 s_1 + n_1 \\ -h_1 s_2^* + h_2 s_1^* + n_2 \end{bmatrix}$$

where the additive white Gaussian noise samples at time t and t+T are represented by the independent complex variables and with zero mean and power spectral density $N_o/2$ per dimension.

B) Combining and ML Detection

In most space-time codes, it is always assumed that the receiver has a perfect knowledge of the channel coefficients. In this case, they are h_1 and h_2. Then the Alamouti space-time decoder will use them as the channel state information (CSI). The maximum likelihood (ML) decoder chooses a pair of signals $(\hat{s}_1, \hat{s}_2)$ from the signal constellation to minimize the distance metric over all possible values of $\hat{s}_1$ and $\hat{s}_2$.

$$d^2(r_1, h_1\hat{s}_1 + h_2\hat{s}_2) + d^2(r_2, -h_1\hat{s}_2^* + h_2\hat{s}_1^*)$$
$$= |r_1 - h_1\hat{s}_1 - h_2\hat{s}_2|^2 + |r_2 + h_1\hat{s}_2^* - h_2\hat{s}_1^*|^2 \qquad (16)$$

where $d(x_1,x_2) = |x_1 - x_2|$

The combiner shown in Figure 6 builds the following two combined signals that are sent to the maximum likelihood detector.

$$\begin{bmatrix}\hat{s}_1\\ \hat{s}_2\end{bmatrix} = \begin{bmatrix} h_1^* & h_2 \\ h_2^* & -h_1\end{bmatrix}\begin{bmatrix} r_1 \\ r_2^*\end{bmatrix} = \begin{bmatrix} h_1^* r_1 + h_2 r_2^* \\ h_2^* r_1 - h_1 r_2^*\end{bmatrix} \qquad (17)$$

Substituting r_1 and r_2 from (14) & (15), into (17), then the decision statistic can be written as:

$$\hat{s}_1 = (|h_1|^2 + |h_2|^2)s_1 + h_1^* n_1 + h_2 n_2^* \qquad (18)$$

$$\hat{s}_2 = (|h_1|^2 + |h_2|^2)s_2 - h_1 n_2^* + h_2^* n_1 \qquad (19)$$

The maximum likelihood decoding rule in (16) can be separated using (18) & (19) into two independent decoding rules for s_1 and s_2:

$$\hat{s}_1 = \arg \min_{(\hat{s}_1,\hat{s}_2)\in C} (|h_1|^2 + |h_2|^2 - 1)|\check{s}_1|^2 + d^2(\check{s}_1,\hat{s}_1)$$

$$\hat{s}_2 = \arg \min_{(\hat{s}_1,\hat{s}_2)\in C} (|h_1|^2 + |h_2|^2 - 1)|\check{s}_2|^2 + d^2(\check{s}_2,\hat{s}_2)$$

where C is the set of all possible modulated symbol pairs

C) Performance of Alamouti Scheme

Alamouti space-time code is an orthogonal scheme that can achieve the full transmit diversity of $n_T = 2$. The codeword difference matrix between the transmitted symbols (s_1, s_2) and the detected symbols $(\hat{s}_1, \hat{s}_2)$ is given by:

$$B(S,\hat{S}) = \begin{bmatrix} s_1 - \hat{s}_1 & s_2 - \hat{s}_2 \\ -s_2^* + \hat{s}_2^* & s_1^* - \hat{s}_1^* \end{bmatrix}$$

Because, the rows of the code matrix are orthogonal, the rows of the codeword difference have to be orthogonal. Also, because of the fact that the minimum distance between any two transmitted codes remains the same, since the codeword distance matrix for the Alamouti has two identical eigen values and the minimum squared Euclidean distance in a single constellation is equal to the minimum eigen value, no coding gain is provided by Alamouti scheme.

The bit-error-rate (BER) verses signal-to-noise-ratio (E_b/No (dB)) performance for Alamouti transmit diversity scheme on Rayleigh fading channels is evaluated by simulation. In the simulation, it is assumed that the receiver has the perfect knowledge of the channel coefficient. It is also assumed that the fading is mutually independent from each transmit antenna to each receive antenna and the total transmit power is the same for all cases. Figure 8 shows the Alamouti scheme BER versus Eb/No performance with coherent BPSK modulation. From the simulation result, it is very clear to see that Alamouti scheme has the same diversity as the two-branch maximal ratio combining

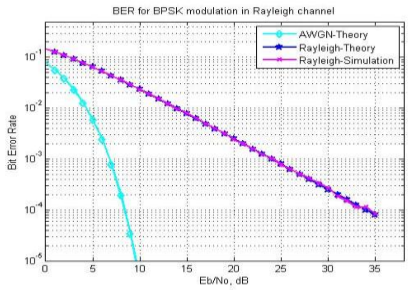

Figure 8: The performance of Alamouti scheme using BPSK modulation

(MRC). However, from Figure 4 & 8, we can see that Alamouti scheme performance is worse than the two-branch MRC by 3 dB and that is because the energy radiated from the single antenna in the MRC is the double of what radiates from each transmit antenna in the Alamouti scheme. To reach the same results, the total transmit power from each transmit antenna in the Alamouti scheme has to be equal to the transmit power of the MRC.

V CONCLUSION

Using this paper, we have covered Alamouti scheme and space-time block codes encoding, decoding and performances.

In this paper, we have introduced the concept of transmit diversity from first principles. It also covers Alamouti scheme with perfect channel knowledge at the receiver along with detailed exploration of encoding, decoding, and maximum likelihood of detection scheme for the Alamouti code. We have also discussed on analyzing the performance of the different implementations of space-time block codes that use higher number of transmit and receive antennas. Finally, we have shown the bit-error-rate performance of the space-time block code implementations and Alamouti codes. Alamouti first proposed a two-by-one orthogonal space time block code. For the same antenna configuration, the Alamouti code is less complex than a space-time trellis code and thus is the preferred choice.

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