CAPACITY ANALYSIS OF MIMO SYSTEMS

(USING WATER FILLING ALGORITHM)

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Abstract

Multiple-Input Multiple-Output (MIMO) Systems are used in wireless communication for enhancement of capacity. The main aim of this paper is to carry out investigations of MIMO system capacity and to observe the performance of MIMO system by using Waterfilling Algorithm. In which Diversity gain and spatial multiplexing gain are the two main advantages of MIMO systems that are used to study the effect of increase in bit rate with increasing the number of transmitter and receiver antennas. These advantages are due to the increase in order of diversity in diversity gain and the other is direct proportionality of transmit and receive antennas each other in spatial multiplexing gain. In this thesis the problem of maximizing the capacity of MIMO system with water filling algorithm is implemented by the singular value decomposition (SVD) of the received signal which is composed of a set of parallel sub channels, where MATLAB is used to simulate the effect of MIMO system capacity with Rayleigh fading channel. The simulation results shows that the performance of the MIMO system improves with the number of transmit and receive antennas in terms of capacity and bit error rate (BER). In this paper we have observed the variation in the capacity of MIMO system with the number of transmit and receive antennas and also observed the variations in the statistical parameters of the diagonal matrix obtained by singular value decomposition of the MIMO system. The variation of bit error rate with signal to noise ratio (SNR) for different cases of transmit and receive antennas are also observed.

Keywords: MIMO, Watwerfilling, SVD

I. INTRODUCTION

Multiple-Input-Multiple-Output (MIMO) wireless systems have recently emerged as one of the most significant techniques to improve the performances of wireless communications. MIMO wireless systems, characterized by multiple antenna elements at the transmitter and receiver, have shown astonishing increase in spectral efficiency and significant improvement in link reliability in rich multipath environments [5,11]. The MIMO technology figures prominently on the list of recent technical advances with a chance of resolving the bottleneck of traffic capacity in future Internet-intensive wireless networks. Perhaps even more surprising is that just a few years after its invention the technology seems poised to penetrate large-scale standards-driven commercial wireless products and networks such as broadband wireless access systems, Wireless Local Area Networks (WLAN), 3G networks and beyond. In general MIMO systems can be defined simply. Given an arbitrary wireless communication system, we consider a link for which the transmitting end as well as the receiving end is equipped with multiple antenna elements. The idea behind MIMO is that the signals on the transmit antennas at one end and the receive antennas at the other end are "combined" in such a way that the quality (Bit Error Rate or BER) or the data rate (bits/sec) of the communication for each MIMO user will be improved [11]. Such an advantage can be used to increase both the network's quality of service and the operator's revenues significantly. A core idea in MIMO systems is space time signal processing in which time (the natural dimension of digital communication data) is complemented with the spatial dimension inherent in the use of multiple spatially distributed antennas. As such MIMO systems can be viewed as an extension of the so-called smart antennas, a popular technology using antenna arrays for improving wireless transmission dating back to several decades. A key feature of MIMO systems is the ability to turn multipath propagation, traditionally a pitfall of wireless transmission, into a benefit for the user. MIMO effectively takes advantage of random fading and, when available, multipath delay spread for multiplying transfer rates [3,6].

This paper organized in to several sections. Section II gives the brief explanation of MIMO system, advantages of MIMO system and system model. Section III gives the mathematical expression for channel capacity and MIMO channel capacity. and also gives the theory about MIMO system using SVD. Section IV describe the waterfilling algorithm for MIMO system and

also describe the simulation results and analysis. And the section V gives the conclusion.

II. MIMO SYSTEM

Digital communication systems based on MIMO systems have recently emerged as one of the most improvement technical breakthroughs for wireless communications [11]. For arbitrary wireless communication systems, a communication link for which a transmitter and a receiver are equipped with multiple antenna elements is considered as MIMO system. The idea behind MIMO systems is to use space-time signal processing in which the natural dimension of digital communication data is complemented with the dimension by using the multiple spatially distributed antennas. MIMO systems are capable of turning multipath propagation into a benefit for the user. This is because the MIMO systems are able to provide spatial diversity, time diversity and frequency diversity by coherently combining the use of the transmitter antennas at one end and the receiver antennas at the other end. Thereby, enhancing wireless transmission over the MIMO channel improves the channel capacity and the quality of bit error rate (BER).

ADVANTAGES OF MIMO SYSTEM

Spatial Diversity Gain

In radio communications diversity gain is reoffered as the ratio of the signal field strength obtained by diversity combining to the signal strength obtained by a single path. Diversity gain is usually expressed in dB [4]. In wireless system the signal level at a receiver fluctuates (fades). Spatial diversity gain mitigates fading and is realized by providing the receiver with multiple copies of transmitted signal in space, frequency or time. With an increasing number of independent copies that at least one of the copies is not experiencing deep fade increases, thereby improving the quality and reliability of reception. A MIMO channel with M_T transmit antennas and M_R receive antennas potentially offers M_T M_R independently fading links and hence a spatial diversity.

Spatial Multiplexing Gain

Spatial multiplexing is a special type of multiplexing where different signals or data bits are transmitted through several independent (spatial) communication channels by multiple antennas and at the same time the receiving side also use multiple antennas for receiving signals-this way increase the data transmission rate which is in direct proportion to the number of antennas used for both transmission and receiving purpose. Higher the number of antennas, the higher the number of data transmission rate. It is a proprietary multiplexing techniques developed by Stanford university [4]. Benefits of spatial multiplexing are it does not require any additional power and no additional bandwidth requirement.

System model

Consider a narrow-band single user MIMO system with '*t*' transmit and '*r*' receive antennas. These antennas are assumed to be Omni directional which means that the antennas transmit and receive equally well in all directions. The data stream from a single user is demultiplexed into '*t*' lower rate data streams and each stream is fed into one of the '*t*' transmitting antennas all of which radiate at the same time in the same frequency band. By sharing the same frequency band the spectral efficiency becomes very high [5,11]. The receiver is assumed to have ideal channel estimates so it can separate and decode the symbols transmitted from each antenna. All the detected lower rate symbols from different streams are then multiplexed together to get the original high rate bit stream. The ability to separate out the symbols is due to the fact that in a scattering environment, the signals received at each receiving antenna from each transmitting antenna appear to be uncorrelated. The linear link model between the transmit and receive antennas can be represented in the vector notation as

$$y = Hx + n$$

Where x is the (t-by-1) transmit vector, y is the (r-by-1) receive vector, H is the (r-by-t) channel matrix and n is the (r-by-1) additive white Gaussian noise vector. The channel is assumed to be flat (narrow bandwidth) and slow fading so it does not change during a burst of transmission. Each entry of H, h_{ij} represents the path gain between the jth transmit antenna and i^{th} receive antenna. The h_{ij} is Rayleigh distributed [9]. In a rich scattering environment the columns of H are assumed to be independent.

III. CHANNEL CAPACITY

Shannon defined the capacity of a channel as the maximum data rate at which data transmitted from a transmitter, when passed through the channel, can be received at some receiver with negligible chance of error [5,9,11]. Information theory provides means to explore the ultimate limits of reliable data transmission. The channel capacity is a convenient measure to analyze the

potential gain of MIMO systems compared to SISO systems. If the data source and received data are viewed as random variables, then the channel capacity refers to the maximum mutual information between them. Then the capacity C is [5] $C = \max_{f(x)} I(X;Y)$. Where the maximization is taken over all possible probability distributions $f(x)$ of X. For a band limited channel with noise being Gaussian and white, Shannon derived the normalized capacity (capacity per unit bandwidth) to be [5]

$$C = log_2 (1 + \rho) \; bps/Hz.$$

MIMO channel capacity

Capacity of a random MIMO channel with power constraint P_T can be expressed as [9]

$$C = E_H \left(\max_{p(x)tr(\phi)p \leq p_T} I(X;Y) \right)$$

Where $\Phi = E\{xx^\dagger\}^2$ is the covariance matrix of the transmit signal vector x. The total transmit power is limited to P_T irrespective of the number of transmit antennas. By using channel model and the relationship between mutual information and entropy can be expanded as follows for a given H

$$I(X;Y) = H(Y) - H(Y/X)$$
$$I(X;Y) = H(Y) - H((HX + n)/X)$$
$$I(X;Y) = H(Y) - H(n/X)$$
$$I(X;Y) = H(Y) - H((n)$$

Where H is the symbol used for entropy. The transmit vector X and noise vector n are assumed independent of each other. The third equation in the above set of equations holds because H is constant (zero entropy) during the transmission of a whole block of X. The $I(X; Y)$ is maximized when 'Y' has the maximum entropy of $log_2 det[\prod eK]$ which requires Y to be with covariance matrix $E[yy^\dagger]=k$. The † denotes complex conjugate transpose. If X is a complex Gaussian vector with $E[xx^\dagger] = Q$, then 'k' can be found by

$$K = E[(HX + n)(HX + n)^\dagger]$$

$$= E[HXX^\dagger H^\dagger] + E[n\, n^\dagger]$$

$$= HQH^\dagger + (K)^n$$

$$= (K)^s + (k)^n \text{ since } X \text{ and } n \text{ are}$$
independent.

Here the $(K)^s + (k)^n$ are the signal and noise parts of the covariance matrix of the vector Y. The maximum mutual information which is also the capacity is

$$C = H(Y) - H(n)$$

$$C = log_2 [det (\Pi e ((K)^s)) + (K)^n))] -$$

$$log_2 [det (\Pi e(K)^n)]$$

$$C = log_2 [det[((K)^s*((K)^n)^{-1}) + I_r]]$$

Where I_r is the (r-by- r) identity matrix. The noise received at each receiving antenna is assumed to be uncorrelated so, $Kn = (\sigma)^2 * Ir$, $(\sigma)^2$ being the noise power on each receiving antenna. Also, when the transmitter has no knowledge about the channel, it is optimum to use equal power on each antenna; that means $Q = (P_T/t)* I_t$, P_T being the total signal power available at the transmitter. The MIMO channel capacity is then becomes

$$C = log_2 [det [HQH^| * ((K)^n)^{-1} + I_r]$$

Where $\rho = Pt / (\sigma)^2$ is the average SNR at each receiving antenna and $H^\dagger$ is the complex conjugate transpose of H. In order to investigate the characteristics of H, we perform the Singular Value Decomposition on H to diagonalize H and find the eigen values.

Singular Value Decomposition based MIMO system

The performance of a system using Singular Value Decomposition over Multiple Input-Multiple-Output channel is dependent on the accuracy of the channel state information at the transmitter and the receiver [8]. In time division duplex (TDD) systems the channel is reciprocal; hence the CSI can be retrieved through estimation of pilot symbols and applied for transmission. The SVD leads to a straight forward architecture where the MIMO channel matrix is decomposed into parallel SISO sub channels with unequal gains. Assuming that the receiver noise is known at the transmitter and the total transmitter power is fixed, the optimum power allocation for these sub channels is the water filling solution. The Singular Value Decomposition (SVD) of the matrix H is $H = UDV^\dagger$ where U and V are unitary and D is a diagonal matrix with positive real elements. The matrix is not required to be square to have an SVD. The elements of D are said to be the singular values of the matrix H. These elements are also the positive square roots of the eigenvalues of the matrix $HH^\dagger$ or $H^\dagger H$. In a

time division duplex channel, the channel state information is available at both the ends of the communication link with a reasonable accuracy. SVD systems use this channel information to compute the SVD. The transmitter filters vector x through *V* before sending it over the channel and the vector *Y* is filtered using $U^\dagger$. The overall transmission equation then becomes

$$Y = U^{\dagger} (H x + n)$$

$$Y = U^{\dagger} ((U D V) x + n)$$

$$Y = U^{\dagger} ((U D V) V^{\dagger} x + n) \qquad x = V^{\dagger} x$$

$$Y = (U^{\dagger}U) D (VV^{\dagger}) x + U^{\dagger} n$$

$$Y = D x + n \qquad\qquad n = U^{\dagger} n$$

Where *U* is a (r-by-r) and *V* is a (t-by-t) unitary matrix and *D* (r-by-t) is a diagonal matrix of singular values $\{\sigma_i\}$ of *H*. These singular values have the property that $\sigma_i = \sqrt{\lambda_i}$ where λ_i is the ith eigen value of $(HH^\dagger)$. Component wise it can be written as $Y_i = D_{ii} x_i + n_i$.

The parallel decomposition of the channel is obtained by defining a transformation on the channel input and output *X* and *Y* through transmit precoding and receive shaping. In transmit pre-coding the input to the antennas, *X* is generated through a linear transformation on the input vector *X* as $X = V^{\dagger} X$. Receiver shaping performs a similar operation at the receiver by multiplying the channel output, *Y* with $U^{\dagger}$.

IV. WATERFILLING ALGORITHM

Many engineering problems that can be formulated as constrained optimization problems result in solutions given by water filling structure the classical example of which is the capacity achieving solution for the MIMO channel [7]. The problem of jointly designing the transmitter and the receiver for communication through MIMO channel also results in a water filling solution. The well-known classical waterfilling solution solves the problem of maximizing the mutual information between the input and the output of a channel composed of several subchannels (such as a frequency-selective channel, a time-varying channel, or a set of parallel subchannels arising from the use of multiple antennas at both sides of the link) with a global power constraint at the transmitter. This capacity-achieving solution has the visual interpretation of pouring water over a surface given by the inverse of the subchannel gains hence the name waterfilling or waterpouring [6].

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Waterfilling capacity of MIMO Channel

When the channel knowledge is absent at the transmitter, the individual sub channels are not accessible. So the equal power allocation in all the sub channels is logical under this scenario. When the transmitter has perfect knowledge of the channel, the waterfilling method is used to optimize the transmitted signal power. The principle of the waterfilling theorem sees the division of total power in such a way that a greater portion goes to the sub channels with higher gain and less or evens none to the channels with small gains [7,9,11]. The sub channels with lower gain i.e. those with higher noise for which no power is allocated at all refer to those sub channels which are not used for transmitting any signal during the transmission. The objective of this algorithm is to allocate power across the channel so as to maximize the total capacity. This power allocation is subject to the constraint that the sum of the power poured into all subchannel is equal to P_T, the total power available to the transmitter. The relative channel strengths and the amount of power to allocate to each channel is determined by knowledge of the channel matrix, *H*. We use the eigen decomposition of *H* to obtain *H*(r-by-t) = *U D V*†; Where $UU^{\dagger} = I_r = V V^{\dagger} = I_t$ and *D* = diagonal $\lambda_1, \lambda_2, \lambda_3 \ldots \lambda_n$ with λ_i as the positive square root of ith eigen value and *i= 1* to *n* non zero λ values and *n= min {r,t}*. Figure 4.1 illustrates the water filling procedure into the sub channels of the composite (7-by-7) MIMO channel [7]. The first step is to determine the parameter μ. The parameter μ, is a mathematical parameter, used to determine the power assigned to each of the sub channels of the composite MIMO channel. After determining the μ, the square of the inverse of eigen values are compared with μ.

If the square of the inverse of i th eigen value is greater than μ , i.e. if $1/\lambda^{-2} \geq \mu$,then that i^{th} eigen channel is too weak to be used for the communication process. The last two sub channels in the above illustrated example of a (7-by-7) MIMO channel are such eigen channels which are not used for transmitting any signal at that point of time. Such channels are said to be switched off and they are put away from the communication process which means that those particular sub channels are not allocated with any transmitting power.
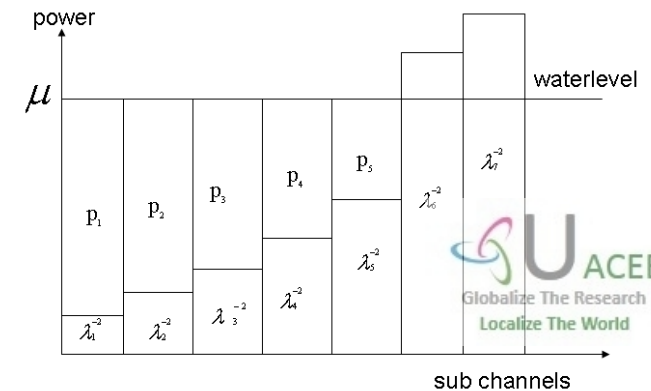
Fig: 4.1. Water filling in parallel sub channels

Once the total available power, P_T and the gains of the parallel sub channels are known, the optimum power allocated to the ith sub channel is

$$p_i = \left(\mu - \frac{1}{\lambda_i^2} \right) + \cdot$$

If this quantity $p_i = \left(\mu - \frac{1}{\lambda_i^2} \right)$ is positive then the power is allocated to the ith sub channel otherwise, the sub channel is left unused. The water filling parameter 'µ' is determined iteratively by the total power Pt, such that µ satisfies the following equation.

$$P_T = \sum_{i=1}^{m} \left[\mu - \frac{1}{\lambda^2} \right]$$

i = 1,2………..m; where m is the number of sub channels that have survived after checking the above conditions and are to be used for transmission of the signal. Now the capacity of MIMO channel with water filling can be expressed as [7,9]

$$C = \sum_{1}^{m} \log_2 \left[1 + \left(p_i / \sigma^2 \right) * \lambda_i^2 \right] bps/Hz$$

Above equation enables the visualization of the MIMO channel as a number of parallel SISO pipes with gain equal to the respective eigen values and it enables us to understand that the waterfilling capacity for MIMO channels is the sum of the capacities of the SISO equivalent parallel sub channels, obtained from performing SVD on MIMO channel matrix. If the channel is known at the transmitter, the capacity can be enhanced by using the good channels i.e. those with the highest gain by applying an unequal power distribution.

Summary of Steps involved in the Waterfilling Power Allocation to the MIMO Subchannels.

1. The first step is to determine the water filling parameter or threshold, μ, which is also shown as water level in the figure 4.1 used for illustrating water filling. Note that the μ is just a Mathematical parameter used to determine the power allocated to each of the eigen channels.

2. After determining μ, the inverse of eigen values of the matrix H is compared with the threshold.

3. Now if $\frac{1}{\lambda^{-2}} \geq \mu$ then, the gain of the ith eigen channel is too small and this eigen channel will not be considered for communication, like the last two eigen channels shown in Fig: 4.1

4. Assume the case of a square dimension of MIMO channel, i.e $r = t$ and also $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \cdots \lambda_t$. And also consider that m eigen values have survived after the above described procedure.

5. Once the total available power, P_T and the gains of the parallel sub channels are known, the optimum power allocated to the ith sub channel is

$$p_i = \left[\mu - \frac{1}{\lambda_i^2} \right]$$

And the power allocated to each of these eigen channels, p_i is determined by the waterfilling rule such that the following equations are satisfied.

$$\frac{1}{\lambda_1^2} + P_1 = \frac{1}{\lambda_2^2} + P_2 = \text{\small\dotfill} = \frac{1}{\lambda_m^2} + P_m = \mu \quad \text{for } i = 1,2, \dots ., m.$$

$$P_T = \sum_{i=1}^{m} \left[\mu - \frac{1}{\lambda_i^2} \right] \quad \text{and}$$

$$P_T = P_1 + P_2 + \dots \dots P_m$$

If this quantity $p_i = \left[\mu - \frac{1}{\lambda_i^2} \right]$, is positive then the power p_i is allocated to the ith sub channel otherwise, the sub channel is left unused. The water filling parameter 'µ' is determined next part.

The parameter µ is determined as shown below.

$$\frac{1}{\lambda_1^2} + P_1 = \frac{1}{\lambda_2^2} + P_2 = \text{\small\dotfill} = \frac{1}{\lambda_m^2} + P_m = \mu$$

$$P_T = P_1 + P_2 + \dots \dots P_m$$

$$m_\mu = \sum_{i=1}^{m} \frac{1}{\lambda_i^2} + \sum_{i=1}^{m} P_i = \sum_{i=1}^{m} \frac{1}{\lambda_i^2} + P_T$$

$$\mu = \frac{1}{m} \left[\sum_{i=1}^{m} \frac{1}{\lambda_i^2} + P_T \right]$$

Results and discussion

MATLAB Simulation

The capacity of the MIMO channel has been simulated for various number of transmitter and receiver antennas using the water-filling algorithm for allocation of optimum power to the parallel sub channels, represented by the diagonal elements of the diagonal matrix which was obtained by performing the Singular Value Decomposition on the MIMO channel matrix. For each case of (t-by-r) 10,000 MIMO channel matrices have been generated and the mean capacity or ergodic capacity of the MIMO channel is plotted against SNR in

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dB. The elements of the MIMO channel matrix generated in the MATLAB simulation are Rayleigh distributed and each element in the matrix represents the gain that exists between a pair of Transmitting and Receiving antennas. Fig 4.2 shows the comparison of capacities of MIMO channel with and without using water filling algorithm for the various cases of (r-by-t) MIMO pairs. For producing the graphs, 10,000 versions of the random Rayleigh channel matrices. The result obtained shows that there is an improvement in capacity of MIMO channel when the water filling solution to achieve capacity maximization is used to allocate different powers to the sub channels.
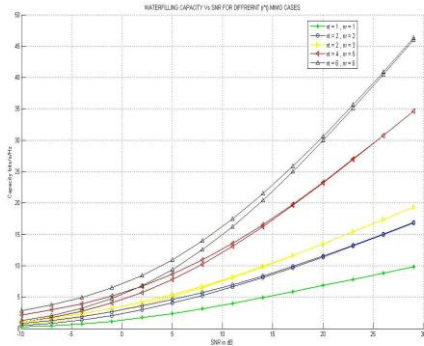

Fig: 4.2 plot of Comparison of waterfilling and non-waterfilling capacities with (t-by-r) pairs

In addition to improving the spectral efficiency by making use of the spatial multiplexing gain which is obtained by sending independent sub streams of data on the parallel sub channels, the MIMO system also offers a diversity gain by using the transmit and receive diversity which comes due to the use of multiple antennas at both the transmitter and receiver. For plotting the BER versus SNR curves shown in Figure.4.3. Random Rayleigh distributed MIMO channel matrices were generated and SVD was performed on it. Then blocks of 64 data symbols was transmitted from each antenna by using 4 QAM modulations. The data transmission and reception of the precoding and the receiver shaping described earlier. The results obtained from simulation show that the MIMO system results in a lower bit error rate or in other words, makes the channel more robust to multipath fading effect to support a better communication link quality.
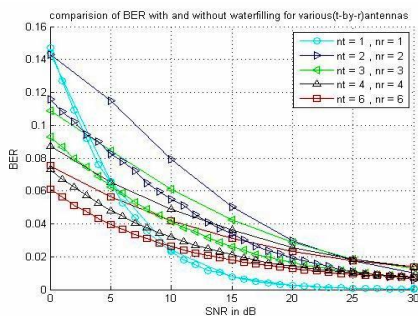

Fig. Plot of comparison of BER Vs SNR for various number of transmit and receive antennas

V. CONCLUSIONS

This paper mainly gives investigations on capacity of MIMO channel. The waterfilling algorithm was implemented in this work to carry out simulations on the MIMO channel capacity. In the section 3 the results obtained from MATLAB simulations are all summarized and each graph is interpreted. In the literature, as in [2], capacity bounds for uncorrelated Ricean fading MIMO channel for several antenna configuration was studied. But in this paper study of channel capacity for Rayleigh faded MIMO channel is considered with implementation of waterfilling. The graphs of capacity Vs SNR show that the capacity of the MIMO channel increases as the number of antennas used at both the transmitter and the receiver increases. The graph of BER Vs SNR shows that the bit error rate decreases when the number of antennas at both the transmitter and the receiver are increased. The idea of single user power optimization in a single user MIMO system can be extended to a Multi- user MIMO system.

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