A New Hybrid Method for Image Approximation Using the Easy Path Wavelet Transform

[1]Sunil Ahirwar,
M Tech (Computer Science & Engg.)
LNCT Bhopal, Madhya Pradesh, India
Lakecity85@gmail.com

[2]Deepika Dubey
M Tech (Computer Science & Engg.)
LNCT Bhopal, Madhya Pradesh, India
deepika1087@yahoo.in

***Abstract*—The easy path wavelet transform (EPWT) has recently been proposed by one of the authors as a tool for sparse representations of bivariate functions from discrete data, in particular from image data. The EPWT is a locally adaptive wavelet transform. It works along pathways through the array of function values and exploits the local correlations of the given data in a simple appropriate manner. However, the EPWT suffers from its adaptivity costs that arise from the storage of path vectors. In this paper, we propose a new hybrid method for image approximation that exploits the advantages of the usual tensor product wavelet transform for the representation of smooth images and uses the EPWT for an efficient representation of edges and texture. Numerical results show the efficiency of this procedure.**

***Index Terms*—Adaptive wavelet bases, easy path wavelet transform (EPWT), linear smoothing filters, -term approximation, sparse data representation, tensor product wavelet transform.**

I. INTRODUCTION

OVER the last years, wavelets have had a growing impact on signal and image processing. In the 1-D case, wavelets provide optimal representations of piecewise smooth functions. Unfortunately, in 2-D, tensor product wavelet bases are suboptimal for representing geometric structures as edges and texture, since their support is not adapted to directional geometric properties. Only in case of globally smooth images, they provide optimally sparse representations. Many different approaches have been developed to design approximation schemes that aim at a more efficient representation of 2-D data. However, while theoretical results show their good performance for sparse representation of piecewise smooth images with discontinuities along smooth curves these frames cannot be applied for image compression. On the one hand, the known curvelet/shearlet algorithms do not get completely rid of the redundancy of the underlying function systems, has so far only been proven for images with edges along C^2-curves.Instead of choosing *a priori* a basis or a frame to approximate an image *u*, one can rather adapt the approximation scheme to the image geometry. Different approaches have been developed in this direction. In this, we will exploit the advantages of the well known tensor-product wavelet transform for representation of smooth images and the ability of the adaptive EPWT to represent edges and texture in images. For that purpose, we propose a new hybrid method for image approximation that (roughly) consists of the following steps.

For a given digital image $\mathbf{u^0=(u^0(i,j))_{i=1,j=1}^{N1,N2}}$, we first try to find a suitable separation $\mathbf{u^0=(u^{sm}+u^r)}$, where $\mathbf{u^{sm}}$ is globally smooth, and the difference image contains the remaining part of the image (i.e., edges and texture). The separation will be done by a simple smoothing of $\mathbf{u^0}$ based upon local smoothing filters. Then the usual tensor product wavelet transform is applied to the smooth image $\mathbf{u^{sm}}$. Here we exploit the fact that smooth functions can be optimally represented by an M–term wavelet expansion $\mathbf{u_M^{sm}}$. In the next step, the EPWT is applied to the (shrunken) difference image $\mathbf{u^0- u_M^{sm}}$. Assuming that the original image $\mathbf{u^0}$ is piecewise smooth, the difference image $\mathbf{u^0- u_M^{sm}}$ contains a high number of components with very small absolute value. Therefore, we consider a shrunken version $\mathbf{\tilde{u}^r=S(u^0- u_M^{sm})}$ possessing a smaller number of nonzero values. In our numerical experiments, we shrink the difference, such that $\mathbf{\tilde{u}^r}$ contains only $N_1N_2/4$ nonzero values. The EPWT is now applied only to the nonzero values of $\mathbf{\tilde{u}^r}$, and the adaptivity costs can be strongly reduced compared with the EPWT for a full image.

Finally, we obtain a very good image approximation as a sum of the M-term wavelet expansion $\mathbf{u_M^{sm}}$ of the smooth image part and the N-term EPWT wavelet expansion $\mathbf{u^r_N}$ of the difference image.

II. HYBRID-MODEL FOR IMAGE APPROXIMATION

As already mentioned in the introduction, the basic idea of the new hybrid model is to find a suitable partition of a given image into a smooth part and a remainder and to apply different wavelet transforms to these two image parts. While the smooth image is known to be optimally representable by a suitable tensor product wavelet transform, we will use the new EPWT for representation of the remainder that contains textures and edges. *A. Separation of Images* We are interested in a segmentation of our image into a “smooth” part and a remainder that contains information about edges and textures. Note that this separation issue is different from image separation problems usually considered for image denoising, where one aims to separate an image into a cartoon part, i.e., a piecewise smooth function (smooth part together with edges of finite length), and a texture part.

B. Tensor-Product Wavelet Transform

Tensor-product wavelet bases are particularly efficient to approximate smooth images. For given 1-D biorthogonal wavelet bases of generated by the dual pairs , and , of scaling functions andwavelets, we consider the 2-D wavelets and the dual wavelets , , 2, 3, being defined analogously. We use the notation and analogously for . Then and are biorthogonal Riesz bases of (e.g., [17]). The fast wavelet transform is based upon filter bank algorithms. Let a function be Hölder-smooth of order in and let the -term separable wavelet approximation be obtained by keeping only the wavelet coefficients with the largest absolute value in a wavelet basis representation of Then for a sufficiently smooth wavelet basis we have where denotes the -norm, see [17]. The decay exponent is optimal, i.e., tensor product wavelet bases are optimal for sparse representation of smooth images. Therefore, we apply the tensor product wavelet transform to the obtained smoothed digital image (or to a slightly different image , see Section II-D). Using only a fixed number of most significant wavelet coefficients in the wavelet representation, we obtain an approximation of after wavelet reconstruction. In our numerical experiments, we use the well-known 9/7 biorthogonal filter bank and orthonormal Daubechies wavelets. As usual, the image is then obtained by a decomposition algorithm, a shrinkage procedure and a wavelet reconstruction.

C. EPWT for Sparse Edge Representation

Let be the original digital image and the – term wavelet approximation of the smoothed image obtained by a linear smoothing process (and a slight modification based upon shrinkage, see Section II-D). Now we consider the difference image that mostly contains edges and texture. We want to apply a new locally adaptive wavelet transform to this difference image, the EPWT. While the EPWT has been shown to be very efficient for sparse image representation we have to keep in mind its adaptivity costs for the storage of path vectors. In order to exploit the ability of the EPWT to sparsely represent edges and texture and, at the same time, to keep adaptivity costs small, we suggest to apply the EPWT not to the complete image , but only to the part with essential image information. Supposing that the original image mainly contains piecewise regular regions, which will be hardly changed by the smoothing process, the difference image possesses many very small image values. Therefore, we apply first a shrinkage procedure to and obtain with , where if if (1) The shrinkage parameter should be chosen dependently upon the image at hand in such a way that contains exactly nonzero image values, where . In our numerical experiments, we have taken such that has only nonzero values; these values are situated along the edges/texture of . Now we apply the EPWT only along the nonzero values of while the vanishing values remain untouched. More precisely, we only consider the partial image containing the image values corresponding to the index set

D. Algorithm

Let us summarize the procedure of the new hybrid algorithm for image approximation. For illustration of the previous algorithm, we present an example, where the partial results after each step of the algorithm are displayed. The original image in Fig. 1(a) shows a 256 256-part of the image “sails.” After the first step of our algorithm, we get a smoothed version , see Fig. 1(b). In this example, we have used the smoothing filter in Section II-A with and . Now we apply the second step, i.e., we calculate a difference image, keep the 16384 components with largest absolute values, and add the other values to . In this way we obtain a slightly changed smooth image , see Fig. 1(c). Compared with , it contains slightly more details; the numbers on the sails are a bit less blurry now. According to step 3 of the algorithm, we apply a wavelet shrinkage procedure with a hard threshold to , and keep only 1200 coefficients; here we use five levels of the biorthogonal 9/7-wavelet filter bank. We obtain , see Fig. 1(d). The difference image is presented in Fig. 1(e). (The image shown here contains the absolute values of the difference and is inverted, i.e., white stands for 0 and black for 255). We apply again a shrinkage to this difference image keeping only nonzero coefficients according to step 4 of the algorithm. Fig. 1(f) shows an inverted version of the obtained difference . We apply the EPWT, and a hard threshold to keep only 800 EPWT wavelet coefficients of . The reconstruction is shown in Fig. 1(g), again we present here the absolute values of its components, where white stands for zero and black for 255. Finally, we add the results of wavelet shrinkage in Fig. 1(d) and the result of the EPWT shrinkage in Fig. 1(g) and obtain the result in Fig. 1(h). For comparison, we show in Fig. 1(i) the wavelet approximation of the original image by the 9/7-transform using 2000 nonzero-coefficients.

NEW HYBRID ALGORITHM

Input : digital image $u^0=(u^0(I,j)^{N1,N2}_{i=1,j=1})$.

1) Aplly an iterative local smoothing filter for image separation:
Fix $\tau > 0$ and K Є N.
For k=1,….,K **do**
$U^k(I,j):=u^{k-1}(i,j)+\tau(u^{k-1}(i+1,j)+u^{k-1}(i-1,j)+u^{k-1}(I,j-1)+u^{k-1}(I,j+1)-4u^{k-1}(I,j))$
Using Neumann boundry conditions.
End
Put $u^{sm} :=(u^k(I,j))_{i=1,j=1}{}^{N1,N2}$.

2) Apply a shrinkage procedure to the difference image $d=u^0-u^{sm}$ by a hard threshold procedure. Choose a θ such that $d(i,j):=S\ \theta\ d(i,j)$ possesses exactly 2^J nonzero image values, where $2^J< N_1N_2$. Now compute a(slightly changed) smooth part of the original image u^0, namely $\tilde{u}^{sm}:=u^0-d=u^0-S_{\theta}d$.

3) Apply a usual wavelet shrinkage procedure to the smoothed image $\tilde{u}^{sm}$ using an orthogonal or biorthogonal two dimension wavelet transform. Let $\tilde{u}^{sm}_M$ be the approximation of $\tilde{u}^{sm}$ that is reconstructed using only the M most significant wavelet coefficient.

4) Consider the difference image $u^r := u^0 - \tilde{u}^{sm}_M$ that contain edges and texture. Apply again a shrinkage procedure to u^r obtaining $\tilde{u}^r=S_{\theta}-u^r$, where $\tilde{u}^r$ possesses exactly 2^J nonzero image values.

5) Apply the EPWT with shrinkage to the detail image $\tilde{u}^r$, where only the nonzero coefficient of $\tilde{u}^r$ are used. Let $\tilde{u}^r_N$ be the approximation of $\tilde{u}^r$ using only the N most significance EPWT wavelet coefficients.

Output: Then $\tilde{u}^0:= - \tilde{u}^{sm}_M+ \tilde{u}^r_N$ is an approximate of u^0 where we have used only M+N wavelet coefficients.

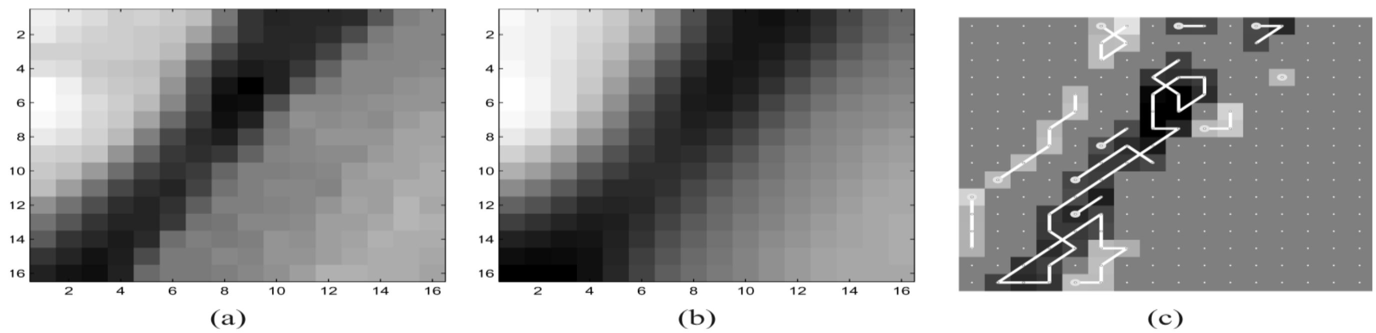

Fig. 2. (a) Original image 16*16. (b) Smoothed image. (c) Illustration of the shrunken difference with 64 nonzero values and of the first path of the EPWT.

Fig. 3. (a) Numbered pixels belong to the shrunken difference image that we consider, the first path is indicated. (b) Index sets after applying the EPWT once. (c) Second path. (d) New index sets.

V. CONCLUSION

In this paper, we have introduced a first hybrid method that uses the tensor-product wavelet transform for smooth images on the one hand and the EPWT for a sparse representation of the edges and textures of the image on the other hand. Similarly as most known adaptive transforms for image approximation, the EPWT provides very good approximation results but produces a non negligible amount of extra costs due to the adaptivity of the method. Incorporating these "adaptivity costs," adaptive methods only slightly outperform the non adaptive methods but with essentially higher computational costs. One way to obtain a real improvement for image approximation may be to study hybrid methods as we did in the paper. Also here, the remaining adaptivity costs are not negligible but considerably smaller than for the "pure" EPWT for image approximation. In particular, a further improvement of pathway determination and path encoding may lead to a compression algorithm that is truly interesting for practical purposes.

REFERENCES

[1] F. Arandiga, A. Cohen, R. Donat, N. Dyn, and B. Matei, "Approximation of piecewise smooth functions and images by edge-adapted (ENO-EA) nonlinear multiresolution techniques," *Appl. Comput. Harmon. Anal.*, vol. 24, pp. 225–250, 2008.
[2] X. Bresson and T. F. Chan, "Non-local unsupervised variational image

segmentation models, CAM Report 07-25, 2007, to be published.

[3] E. J. Candès and D. L. Donoho, "New tight frames of curvelets and optimal representations of objects with piecewise singularities," *Comm. Pure Appl. Math.*, vol. 57, pp. 219–266, 2004.

[4] E. J. Candès, L. Demanet, D. L. Donoho, and L. Ying, "Fast discrete curvelet transforms," *Multiscale Model. Sim.*, vol. 5, pp. 861–899, 2006.

[5] T. F. Chan, S. Esedoglu, and M. Nikolova, "Algorithms for finding global minimizers of denoising and segmentation models," *SIAM J. Appl. Math.*, vol. 66, pp. 1632–1648, 2006.

[6] A. Cohen and B. Matei, "Compact representation of images by edge adapted multiscale transforms," in *Proc. IEEE Int. Conf. Image Process.*, Thessaloniki, Greece, 2001, pp. 8–11.