Performance Analysis of FIR filter in frequency Domain and Time Domain for Wireless Communication System

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Abstract—**This Paper gives design the methodology for implementation of FIR filter in the frequency domain and time domain. The filter coefficients for these two implementation are taken by sampling the time representation and frequency representation of RRC filter. The RRC filter transfer function is used to obtain Raised Cosine filter which is a pulse shaping filter. Then the performance evaluation is made by comparing the simulation results of the two filter implementation on the basis of computational complexity, inter symbol interference rejection depending on average gain in immunity against ISI, Error Vector Magnitude (EVM), Peak distortion and AWGN rejection. Then it is shown that by choosing frequency domain filter coefficients as the samples of the ideal frequency transfer function can boost the performances of such a filters mainly in terms of immunity against inter symbol interference.**

Keywords—FIR, RRC, WCDMA, IIR, UMTS, ISI

I. INTRODUCTION

In order to evaluate the performance of proposed FIR implementation method in the frequency domain, especially in terms of immunity against inter symbol interference and noise rejection, two FIR filter implementations are considered:

- A time domain FIR filter of which the coefficients are obtained by sampling the RRC impulse response with an oversampling factor of 4 and which processes the I and Q paths successively;
- The frequency domain FIR filter with coefficients extracted as will be discussed in section 5.4 and which combines the I and Q paths into one complex signal and processes the two paths simultaneously.

In designing the filter, the sampling rate also needs to be considered[10]. The higher the sampling rate the easier it is after a digital to analog converter to remove high frequency components generated in the sampling process. The higher the sampling rate, the faster the converter must operate, which in general will lead to greater cost and greater power consumption. To keep the approach realistic, it is chosen an oversampling factor of 4 (thus a sampling rate f_s = 4 × chip rate = 15.36 MHz) as a compromise between high performance on one hand and cost and complexity on the other hand [8]. Then it is created two real random input signals of length 100 points with values between −1 and 1 to simulate inputs on the I and Q paths, and included 3 null samples between each adjacent values to simulate an oversampled signal by a factor of 4. These signals where passed through a classical time domain filter to get the transmitted signal. The two filter implementations (the time domain and proposed frequency domain) are then applied to the transmitted signal.

A. TIME DOMAIN IMPLEMENTATION OF FIR FILTER

Third Generation (3G) communication systems use WCDMA technique to transmit and receive data. The spectrum of a signal of the type used in WCDMA has adjacent channel side lobes. These side lobes introduce interference. Number of considerations discussed in [7] proposed to the use of a FIR filter in the WCDMA transmitter, together with a matched filter in the receiver in a way to get a Nyquist filter as the combined filter response. In the time domain, the FIR operation is achieved by convolution.

$$FIR[x(t)] = x(t) \otimes h(t) \qquad (1)$$

Where h (t), is filter coefficients, (⊗) is the convolution product, x(t) is discrete input, FIR[x(t)] implies the FIR filtering in frequency domain. FIR filters in the transmission and reception chains are used to implement root raised cosine (RRC) filters of which the combined transfer function results in a raised cosine filter (a type of Nyquist filter).In Equation (2), the time domain representation for the RRC filter is given.

$$h(t) = (\sin[(1-a)p * t/T] + 4a(t/T)\cos[(1+a)]/((p * t/T)[1-(\frac{4at}{T})^2]) \square$$

(2)

Where T=1/chip rate=1/ 3.84 Mhz=.260us for WCDMA.

So the filter coefficients for time domain implementation of FIR filter are extracted by sampling the impulse response given by Eq. 2 shown in Figure 1. These samples are taken as the coefficients of the time domain implementation filter.

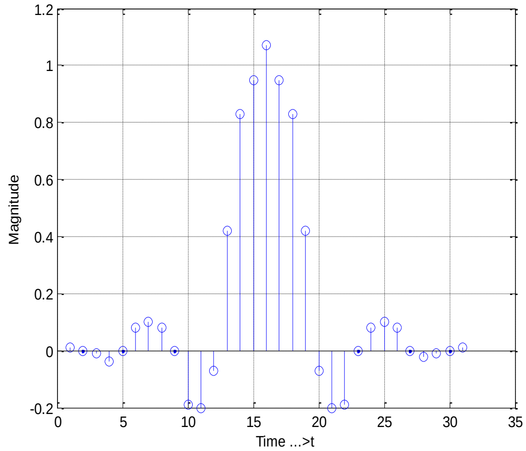

Figure 1 Samples of time domain impulse response of RRC filter

B FREQUENCY DOMAIN IMPLEMENTATION OF FIR FILTER

Implementing FIR filters by FFT is not an innovative idea. Many researchers have proposed FIR filters in the frequency domain in the past two decades [2, 3, 4]. But no previous work is proposed for a frequency-domain FIR filter for the UMTS standard, mainly because of computational complexity problems. Instead of performing the FFT of the filter's coefficients extracted by sampling the RRC time representation like shown above in Eq.3, take the ideal square root of the raised cosine filter transfer function shown in Eq. 5.3 and sample it to get the coefficients that should be used in the frequency domain FIR filter. The resultant coefficients are then used in the frequency domain implementation of the FIR filter.

$$H(f)=T \qquad 0<=\mathrm{mod}(f)<=m$$
$$H(f) = T/2(1+\cos[p * t/a(\mathrm{mod}(f) - m)]) \qquad m <= \mathrm{mod}(f) <= M$$
$$H(f)=0 \qquad \mathrm{mod}(f)>M \qquad (3)$$

where $m = 1 - \frac{a}{2T}$ $\quad M = 1 + \frac{a}{2T}$

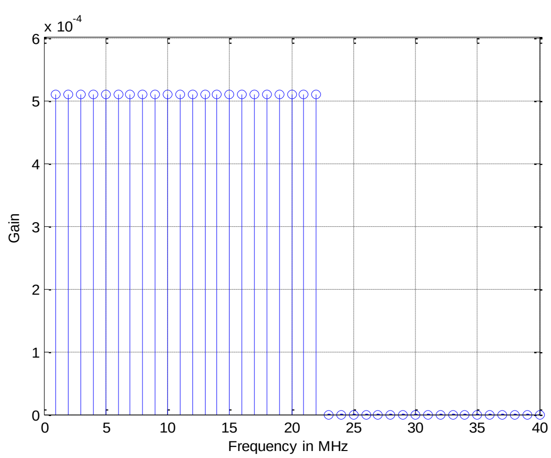

Figure 2 Samples of root raised cosine frequency response.

The FIR filtering in frequency domain is implemented as follows

$$FIR_f = F^{-1}[H.F(x)] \qquad (4)$$

Where FIR_f denotes the FIR filtering in frequency domain H is the Fourier transform of filter coefficients h (t), (⊗) is the convolution product, (.) is the dot product and F(x) is the Fast Fourier transform of discrete input x (t). The frequency domain filter proposed here directly processes the I and Q paths. This is done by combining the paths before entering the filter into complex samples that will be treated by the direct and inverse FFT then breaking the output again into two paths. In fact, from the linearity property of the Fourier transform, it can be stated that

$$\begin{aligned} FIR_f(X_I + jX_Q) &= F^{-1}[F(X_I + jX_Q)H] \\ &= F^{-1}[F(X_I)H + jF(X_Q)H] \\ &= FIR_f(X_I) + jFIR_f(X_Q) \end{aligned} \qquad (5)$$

where FIR_f denotes the FIR filtering in frequency domain and X_I and X_Q are the inputs on the I and Q paths respectively.

II SIMULATION RESULTS AND PERFORMANCE EVALUATION

A COMPUTATIONAL COMPLEXITY IN TIME DOMAIN FIR FILTER

To process one sample in the time domain, a symmetric FIR filter needs $\frac{N}{2}$ multiplications and $N - 1$ additions. This means that a FIR filter executes 0.5(3N−2) operations per sample. This processing should be executed for both I and Q paths, which means that the actual number of operations per sample (a couple of an I-sample and a Q-sample) doubles and becomes 3N − 2[11]. Consequently, the computational complexity (the number of operations per second without distinguishing between addition and multiplication) of a FIR filter operating at frequency F is given a:

$$CC_{FIRt} = (3N - 2)\,F \qquad (6)$$

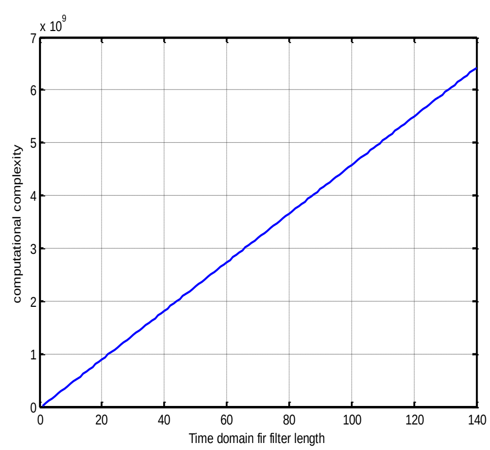

Figure 3 Computational complexity in time domain FIR filter.

B COMPUTATIONAL COMPLEXITY IN FREQUENCY DOMAIN FIR FILTER

The frequency domain FIR implementation, uses the overlap-add method [2] which consists of decomposing the signal into simple components, processing each of the components, and recombining the processed components into the final signal[6]. If the input signal is segmented into sections of length L and a FIR filter of length N is to be implemented, a FFT of length L+N −1 or more should be performed to avoid time aliasing. In this study, the input signal is segmented into sections of length N and thus FFT of length P where P is the minimal power of 2 greater than or equal to 2N can be used. Each FFT of length P requires $5P\log_2(P)$ operations and processes N samples at a frequency F to which it has to be to added 6 operations per sample to perform the complex multiplication by the filter coefficients H[1]. Hence, the computational complexity of the frequency domain FIR filters of length N as:

$$CC_{FIRf} = 2(5P/N \log_2 P + 3)\, F \tag{7}$$

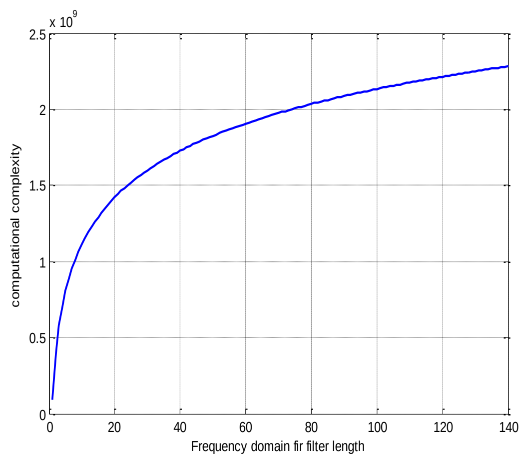

Figure 4 Computational complexities in frequency domain FIR filter.

Comparing the computational complexities of the 2 different implementations of the FIR filter for different filter lengths observing Figure 5. From this Figure, it can be found that it is more advantageous to implement FIR filters in the frequency domain for higher lengths which is generally the case of the UMTS standard because Table 5.1 shows that the computational complexity of time domain filter increases linearly whereas in frequency domain FIR filter first it increases sharply for lower filter length and afterwards it attains almost a constant value [4]. So frequency domain implemented FIR filters have less computational complexity than time domain FIR filters for higher filter length but classic frequency domain implementation is normally more computationally complex than a time domain implementation for small filter lengths.

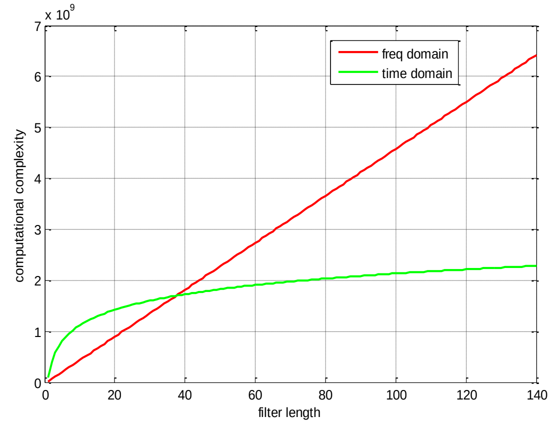

Figure 5 Comparison of computational complexity of two filters in time domain and frequency domain.

Table 1 Comparison of computational complexity of two filters in Mega Operations per Second (MOPS)

Sr. No	Filter length	Time domain filter	Frequency domain filter
1	20	800 MOPS	1400 MOPS
2	40	1800 MOPS	1750 MOPS
3	60	2700 MOPS	1900 MOPS
4	80	3600 MOPS	2100 MOPS
5	100	4600 MOPS	2300 MOPS
6	120	5500 MOPS	2400 MOPS
7	140	6400 MOPS	2500 MOPS

C GAIN IN IMMUNITY AGAINST INTERSYMBOL INTERFERENCE IN TIME DOMAIN FIR FILTER

The main property of Nyquist filters is that they result in zero inter symbol interference (ISI) at the optimum sampling point for filtered data [7]. The ISI is measured as the variance of the error between the samples of the input signal and those of the received signals at the optimum sampling points. The gain in immunity against ISI is 51% in the case the case of time domain FIR filter as obtained in Figure 6.

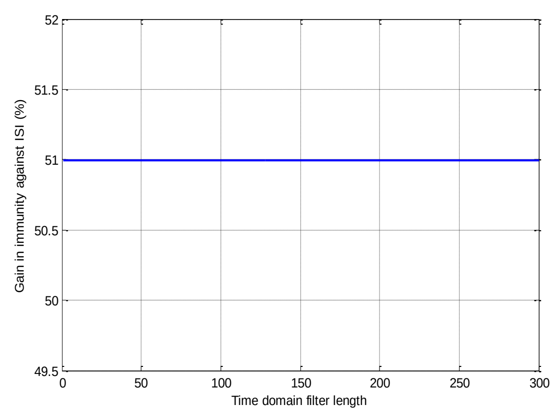

Figure 6 Gain in immunity against ISI in time domain FIR filter.

D GAIN IN IMMUNITY AGAINST INTERSYMBOL INTERFERENCE IN FREQUENCY DOMAIN FIR FILTER

The ISI is measured as the variance of the error between the samples of the input signal and those of the received signals at the optimum sampling points. From Figure 7 the gain in immunity against ISI is 74.5% in the case of frequency domainFIRfilter.

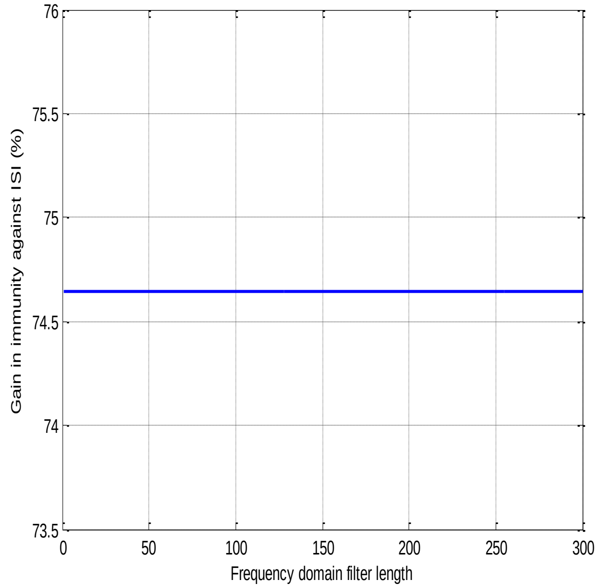

Figure 7 Gain in immunity against ISI in frequency domain FIR filter.

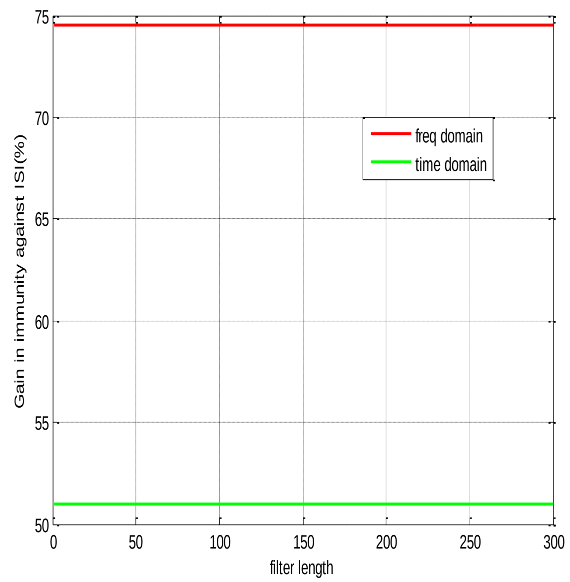

Figure 8 Comparison of gain in immunity against ISI of two filters in time domain and frequency domain.

Table 2 Comparison of gain in immunity against ISI of two filters.

Sr. No	Filter length	Time domain filter	Frequency domain filter
1	50	50.9%	74.5%
2	100	50.9%	74.5%
3	150	50.9%	74.5%
4	200	50.9%	74.5%
5	250	50.9%	74.5%
6	300	50.9%	74.5%

As it is clear from the Figure 8 and Table 2 that gain in immunity against inter symbol interference is greater in the case of frequency domain filtering .So they are more immune to ISI than time domain FIR filters.

E. ERROR VECTOR MAGNITUDE IN TIME DOMAIN FIR FILTER

It is another parameter to indicate ISI. In literature discussing UMTS radio receivers, a parameter called EVM is commonly used to indicate the amount of ISI [7]. The UMTS standard provide the following definition of EVM: “The Error Vector Magnitude is a measure of the difference between the reference waveform and the measured waveform. This difference is called the error vector. The EVM result is defined as the square root of the ratio of the mean error vector power to the mean reference power expressed as a percentage”. These technical specifications define also a minimum requirement for the EVM: the EVM shall not exceed 17.5% in the user equipment radio transmission and reception in the FDD mode [5]. The formula used to calculate the EVM is presented in (Eq. 8) where P_{error} is the root mean square power of the error vector is and $P_{Reference}$ is the root mean square power of the ideal transmitted signal. Figure 9 gives the Error vector magnitude in time domain FIR filter.

$$\text{EVM } (\%) = \sqrt{\frac{P_{error}}{P_{Reference}}} * 100 \quad (8)$$

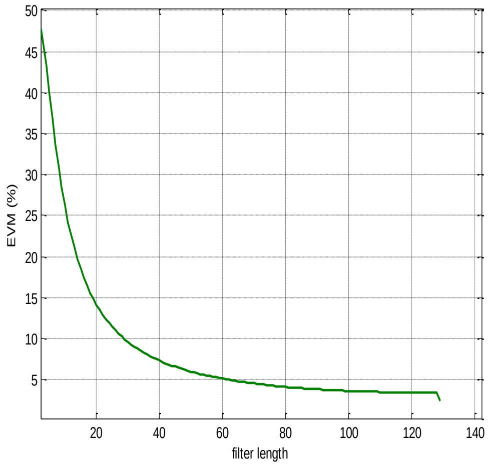

Figure 9 Error vector magnitude in time domain FIR filter.

F ERROR VECTOR MAGNITUDE IN FREQUENCY DOMAIN FIR FILTER

The Error Vector Magnitude is a measure of the difference between the reference waveform and the measured waveform. This difference is called the error vector. The EVM result is defined as the square root of the ratio of the mean error vector power to the mean reference power expressed as a percentage”. The formula used to calculate the EVM is presented in Eq. 9 where P_{error} the root mean square power of the error vector is is and $P_{reference}$ is the root mean square power of the ideal transmitted signal. Figure 10 gives the Error Vector Magnitude in frequency domain FIR filter [5].

$$\text{EVM } (\%) = \sqrt{\frac{P_{error}}{P_{Reference}}} *100 \qquad (9)$$

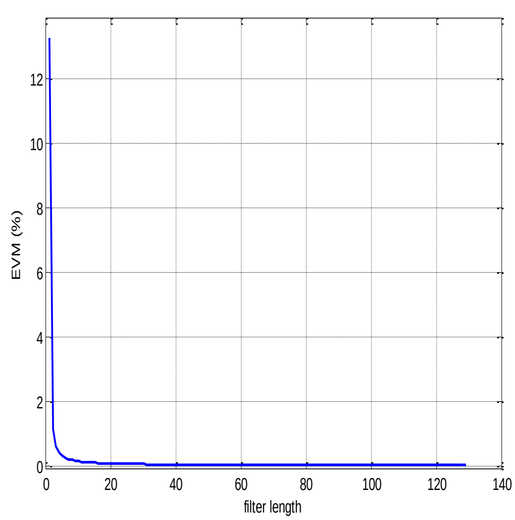

Figure 10 Error vector magnitude in frequency domain FIR filter.

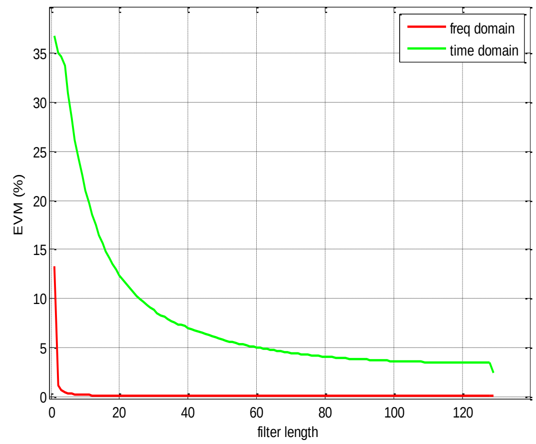

Figure 11 Comparison of EVM of two filters in time domain and frequency domain.

Table 3 Comparison of EVM of two filters.

Sr.No	Filter length	Time domain filter	Frequency domain filter
1	10	25%	14%
2	20	14%	.1%
3	40	7.5%	.1%
4	60	5%	.1%
5	80	4.5%	.1%
6	100	4.5%	.1%
7	120	3.5%	.1%

Observing the Figure11 it is found that EVM obtained in frequency domain FIR filters 14% which satisfies the condition that the EVM shall not exceed 17.5% in the user equipment radio transmission and reception in the FDD mode. Simulations with different filter lengths and for the two different FIR implementations discussed in this study shows that the frequency domain implementation using the sampled ideal filter transfer function is always better than the time domain implementation [13]. From Figure 11, it is seen that the frequency domain implementation ensures that the minimum requirement of EVM is met even with a FIR filter of length 8. Table 3 gives the comparison of two filter implementation

G. PEAK DISTORTION IN TIME DOMAIN

Another parameter commonly used to measure the ISI of transmit-receive filter combination is the peak distortion given by Equation 10.

$$D_p(\text{dB})=20\log_{10}\left[\left(2\sum_{k=1}^{\infty} h\left(\frac{N}{2}+kM\right)\right)/\left(h\left(\frac{N}{2}\right)\right)\right] \qquad (10)$$

h is the impulse response of the transmit-receive filter combination, N is the length of this response and M is the oversample factor. Furthermore it has been assumed that h is symmetric. In order to extract h, a pulse signal is passed into two sets of transmit-receive filters. Here a transmit and a receive filters implemented in the time domain. The oversample factor is always M = 4. Figure 12 gives the peak distortion obtained in frequency domain implementation of FIR filter.

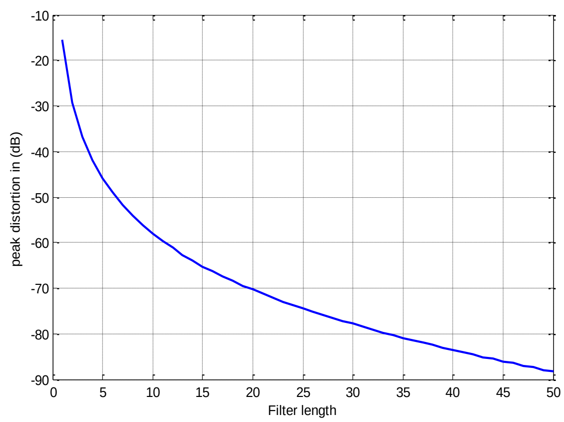

Figure 12 Peak distortion in time domain FIR filter**.**

H PEAK DISTORTION IN FREQUENCY DOMAIN

Figure13 gives the peak distortion obtained in frequency domain implementation of FIR filter. This is obtained from the same Eq.10.

$$D_p(\text{dB})=20\log_{10}\left[\left(2\sum_{k=1}^{\infty} h\left(\frac{N}{2}+kM\right)\right)/\left(h\left(\frac{N}{2}\right)\right)\right] \qquad (11)$$

Here a transmit and a receive filters are implemented in the frequency domain. h is the impulse response of the transmit-receive filter combination, N is the length of this response and M is the oversample factor.

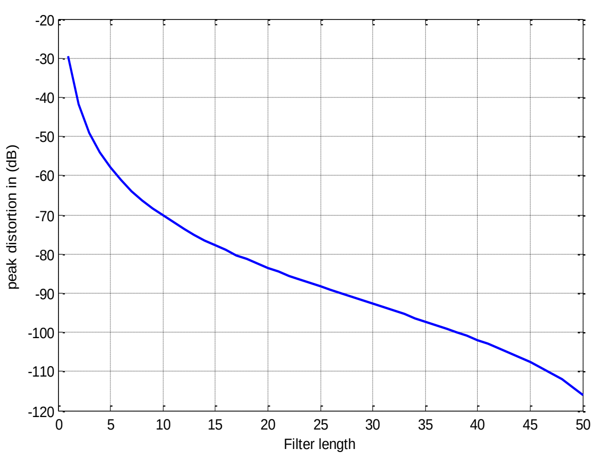

Figure 13 Peak distortion in frequency domain FIR filter.

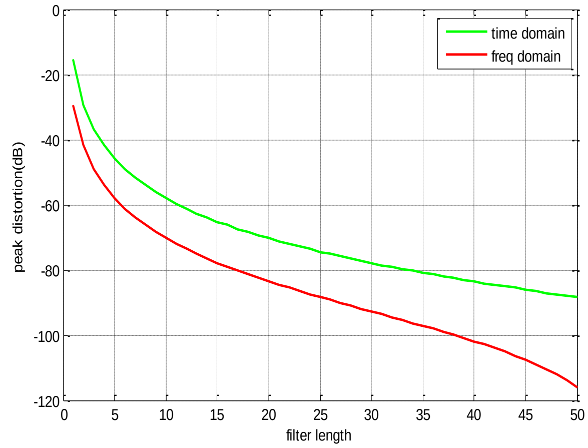

Figure 14 Comparison of peak distortion of two filters in time domain and frequency domain.

Table 4 Comparison of peak distortion of two filters

Sr. no	Filter length	Time domain filter	Frequency domain filter
1	1	-15 dB	-30 dB
2	5	-45 dB	-58 dB
3	10	-58 dB	-70 dB
4	15	-65 dB	-78 dB
5	20	-70 dB	-84 dB
6	25	-74 dB	-88 dB
7	30	-78 dB	-94 dB
8	35	-81 dB	-98 dB
9	40	-82 dB	-104 dB

Observing the Figure 14 it is found that peak distortion obtained in frequency domain FIR Filter is less than the time domain filter. Table 4 clearly shows that peak distortion decreases in both the cases with increase in filter length but it is lesser in case of frequency domain FIR filter. So it confirms that frequency domain FIR filter presents less peak distortion than the time domain filter [15].

I. ADDITIVE WHITE GAUSSIAN NOISE REJECTION IN TIME DOMAIN.

The essential reason for implementing the raised cosine filter as two RRC filters, one in the transmitter and the other in the receiver, is that filtering in the receiver is essential to reject the noise and interference present in the receiver [7]. To eliminate the Additive White Gaussian Noise (AWGN) caused by the channel, the transmit and receive filters transfer functions should be identical. Here in Figure 15 the Additive White Gaussian Noise (AWGN) is added to the transmitted signal which has signal to noise ratio SNR of 16dB.This noisy signal is then passed through FIR filter implemented in time domain this will reject the added (AWGN) noise as it can be seen in Figure 16.

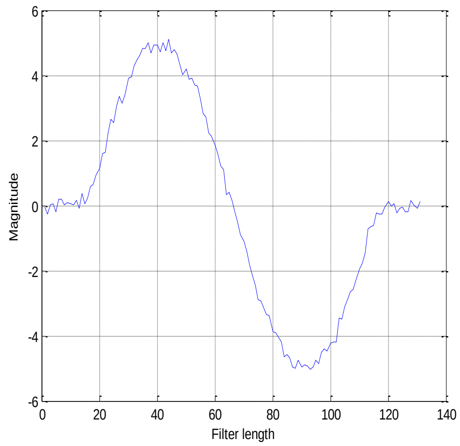

Figure 15 AWGN added to transmitted signal with SNR 16 dB in time domain.

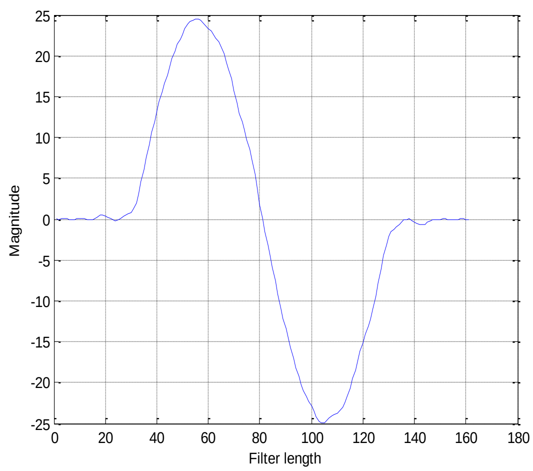

Figure 16 AWGN rejected by the FIR filter implemented in time domain.

J. ADDITIVE WHITE GAUSSIAN NOISE REJECTION IN FREQUENCY DOMAIN.

Here again the same procedure is followed to reject the AWGN. Two frequency domain FIR filters are required one at the transmitter and other at the receiver. These two filters are designed by the RRC filter transfer function, the combined effect results the raised cosine filter, which acts as a pulse shaping filter. This raised cosine filter helps in rejecting the AWGN that is caused by the channel [13]. Figure 17 shows the transmitted signal in the frequency domain and to which AWGN is added. The SNR of the signal is 16dB.This noisy signal is then passed through FIR filter implemented in frequency domain which will reject the AWGN in the signal. AWGN rejected signal is shown in figure 18.

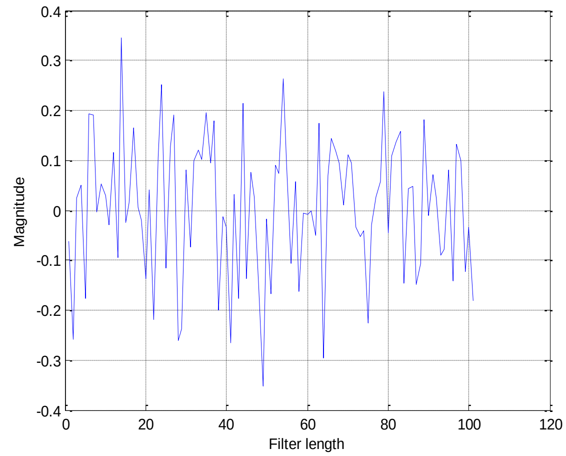

Figure 17 AWGN added to transmitted signal with SNR 16 dB in frequency domain.

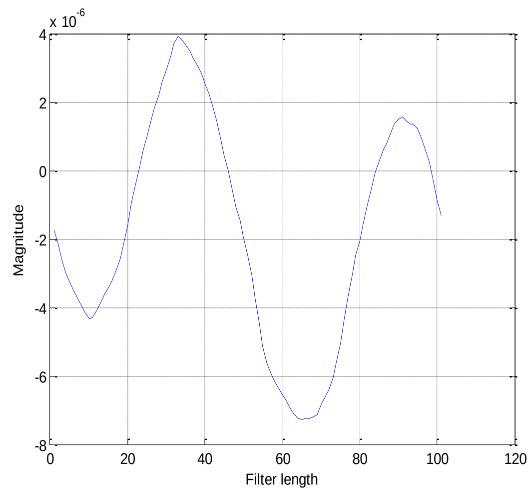

Figure 18 AWGN rejected by the FIR filter implemented in frequency domain.

III CONCLUSION

So observing Figure 5 and Table 1 it is estimated that the computational complexity of time domain filter increases linearly whereas in frequency domain FIR filter first it increases sharply and afterwards it attains almost a constant value. So frequency domain implemented FIR filters have less computational complexity than time domain FIR filters for higher filter length but classic frequency domain implementation is normally more computationally complex than a time domain FIR filters for small filter lengths. From Figure11 and Table 3 it is found that EVM obtained in frequency domain FIR filters is 14% which satisfies the condition the EVM shall not exceed 17.5% in the user equipment radio transmission and reception in the FDD mode. Figure 8 and Table 2 depicts that gain in immunity against ISI is 50.9% for time domain FIR filter whereas it is 74.5% in case of frequency domain FIR filter. Figure 14 and Table 4 gives the comparison between two filters in terms of peak distortion. It is found that peak distortion decreases in both the cases with increase in filter length but it is lesser in case of frequency domain FIR filter. So it confirms that frequency domain FIR filter presents less peak distortion than the time domain filter. Then Figure 15 shows that AWGN noise added to the signal is rejected by time domain FIR filter.

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