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Multi-resolution Discrete Cosine Transform Based Image Compression

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***Abstract*—Multi-resolution analysis has been a very popular with compression specially in video streaming because it eliminates the blocking effects of the DCT which become more prominent in videos. Wavelets are based on the multi-resolution but it is well acknowledged that wavelets are far more expensive in terms of hardware and software as compared to DCT. DCT, however, has been used to compress image but not for multi resolution image analysis. We propose a multi-resolution discrete cosine Transform based method for image compression. This paper is an attempt to explore the possibilities of using DCT for multi-resolution image analysis. Naive implementation of block DCT for multi-resolution expansion has many difficulties that lead to signal distortion. One of the main causes of distortion is the blocking artifacts that appear when reconstructing images transformed by DCT. The algorithm is based on line DCT which eliminates the need for block processing. The line DCT is one dimensional array based on cascading the image rows and columns in one transform operation. Several images have been used to test the algorithm at various resolution levels. The reconstruction mean square error rate is used as an indication to the success of the method. The proposed algorithm has also been tested against the traditional block DCT.**

***Keywords* - Discrete Cosine Transform, Image Compression, Multi-Resolution Analysis and DCT.**

I. INTRODUCTION

Discrete cosine transform (DCT) has become the most popular technique for image compression over the past several years. One of the major reasons for its popularity is its selection as the standard for JPEG. DCTs are most commonly used for non-analytical applications such as image processing and signal-processing DSP applications such as video conferencing, fax systems, video discs, and HDTV. Mapping an image space into a frequency space is the most common use of DCTs. For example, video is usually processed for compression/decompression as 8 x 8 blocks of pixels. Large and small features in a video picture are represented by low and high frequencies. An advantage of the DCT process is that image features do not normally change quickly, so many DCT coefficients are either zero or very small and require less data during compression algorithms. DCTs are fast and, like FFTs, require calculation of coefficients. The entire standards employ block based DCT coding to give a higher compression ratio. The aim here is to see if we can get the same results in compression using DCT as we can get by using the wavelets. The word multi-resolution refers to the simultaneous presence of different resolutions. The basic difference between DCT and wavelets is that in wavelets rather than creating 8 X 8 blocks to compress, wavelets decompose the original signal into sub-bands. Wavelets are basically an optimizing algorithm for representing a lot of change in the pictures. With DCT algorithm, the 8 X 8 blocks can lose their crisp edges, whereas, with wavelets the edges are very well defined. Now, if the wavelets produce much better results than DCT then why do we need to try DCT for multi-resolution? The reason is that there are certain drawbacks to WAVELETS especially in terms of computation time required. For the highest compression rates, it takes a longer time to encode. The other reason is that MPEG is already a standard using DCTs and computer hardware comes with MPEGs built in. There is hardly any hardware available in the market these days which comes with wavelets as a built in standard.

The above arguments are our motivation for this paper to come up with an algorithm of multi-resolution analysis for image processing which will have the efficiency and cost of the DCT and the compression results of the wavelets.

II. MULTI-RESOLUTION ANALYSIS

The concept of multi-resolution analysis was formally introduced by Mallat [1989] and Meyer [1993] [2]. Multi-resolution analysis provides a convenient framework for developing the analysis and synthesis filters [1]. The basic components for a multi-resolution analysis are: an infinite chain of nested linear function spaces and an inner product defined on any pair of functions. Multi-resolution has been widely used recently with great success with the wavelets. Wavelets and multi-resolution analysis have received immense attention in the recent years. There have been a lot of problems which have made use of wavelets and multi-resolution analysis and thus making it a popular scheme for compression. The basic idea behind multi-resolution analysis, as explained in [1], is to decompose a complicated function into smaller and simpler low resolution part together with wavelet coefficients. These coefficients are very important to recover the original signal when we apply the inverse.

Mallat [1989] described multi-resolution representation as a very effective method for analyzing the information content for images. The original scale and size of objects in an image depends upon the distance between the image and the camera. To compress an image to a smaller size, we ought to keep the essential information of the image. In Mallat's words a multi-

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resolution decomposition enables us to have a scale-invariant interpretation of the image [3]. Multi-resolution analysis provides a hierarchical structure. It means that in order to get to 15 % compression, the image is not compressed directly to 15% as in block DCT, instead the image is compressed in stages; reducing the image to a half at every stage. At different resolutions the details of a signal generally characterize different physical aspects of the image. It is a common observation that at coarse resolutions the details correspond to larger overall aspects of the image while at fine resolutions the distinguishing features are prominent. Some of the common applications of multi-resolution analysis are image compression, edge detection, and texture analysis. Multi-resolution analysis is not only restricted to the previously mentioned techniques but recently researchers also found some more applications of multi-resolution analysis and found good results. These applications include image restoration and noise removal. Multi-resolution analysis tries to understand the content of the image at different resolutions [4]. Based on the concept of multi-resolution, we have image pyramids. These pyramids are essentially simple structures for representing images at more than one resolution. The base of the pyramid, as we can guess, contains the image representations at the highest possible resolution, whereas the apex contains the low resolution approximation. The other technique that arises from the concept of multi-resolution analysis is sub-band coding. In this technique an image is decomposed into a set of components called the sub bands. These components are limited in their bandwidth, which means that they cannot be greater than a certain pre-defined size. This is where the importance of multi-resolution analysis lays, i.e. the reconstruction of the original image; reconstruction of the original image is done by up sampling, filtering and summing the individual sub-bands without the loss of any pertinent information [5].

III. DISCRETE COSINE TRANSFORM (DCT)

DCT has been very popular transform for many years. The fact that DCT is a near optimal transform is the main reason for its popularity. The DCT transform de-correlates the image data [3]. In DCT, an image is typically broken into 8x8 blocks. These blocks are each transformed into 64 DCT coefficients [14]. In [7], the most commonly used DCT definition of one dimensional sequence of length N is given by equation (1):

$$C(u) = \alpha(u)\sum_{x=0}^{N-1} f(x)\cos\left[\frac{\pi(2x+1)u}{2N}\right] \quad (1)$$

The above equation is defined for u = 0, 1, 2, 3…..N-1. There are two kinds of DCT coefficients; AC and DC. The DC coefficient corresponds to the value of C (u) when u = 0. In other words, DC coefficient provides the average value of the sample data [3]. The rest of the coefficients are called AC coefficients. Based on the one dimensional DCT as described above, the two dimensional DCT can be achieved. The above equation shows the two dimensional DCT. It is clear from the above equation that it is derived by multiplying the horizontal one dimensional basis function with the vertical one dimensional basis function as given in equation (2).

$$C(u,v) = \alpha(u)\alpha(v)\sum_{x=0}^{N-1}\sum_{y=0}^{N-1} f(x,y)\cos\left[\frac{\pi(2x+1)u}{2N}\right]\cos\left[\frac{\pi(2y+1)v}{2N}\right] \quad (2)$$

Both one and two equations are one dimensional DCTs work in similar fashion. One dimensional DCT is used mainly in sound signals because of its one dimensional nature, whereas, two dimensional DCT is used in images because of their two dimensional nature.

DCT has many important properties that help significantly in image processing, especially in image compression. One of these properties is energy compaction as shown in figure -1.

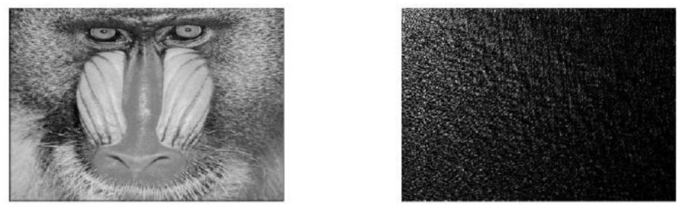

Figure 1. Images show Energy Compaction Property of DCT

Energy compaction means that most of the pertinent information of the image is stored or compacted in top left corner of the image. In DCT, lower frequencies are on the top left corner and they contain most of the information. The high frequencies don’t have much of the information needed to reconstruct the image. With this property it is easier for the quantizer and encoder to simply leave out the high frequencies as a means of compressing the image without losing the information. Since some of the data is lost or neglected, DCT results in Lossy compression.

Another important property of DCT is de-correlation. It was mentioned earlier that DCT is very good at removing the inter pixel redundancy. This property is used in reducing the amplitude of the signals [7]. Figure 2 shows the de-correlation property of DCT.

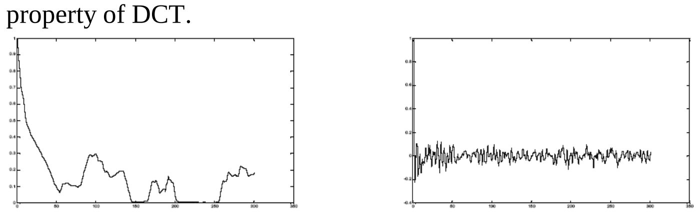

Figure 2. Decorrelation property of DCT

One of the main problems and the criticism of the DCT is the blocking effect. In DCT images are broken into blocks 8 x 8 or 16 x 16 or bigger. The problem with these blocks is that when the image is reduced to higher compression ratios, these blocks become visible. This has been termed as the blocking effect. This is evident in figure - 3. This image is compressed using 8 x 8 blocks and only 4 coefficients are retained. The blocking effect is very prominent in this image. This blocking effect creates a lot more problems in videos and it becomes

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really hard to recognize the person in the image during teleconferencing as shown in figure - 3

Figure 3. Blocking effect of DCT using 8 x 8 blocks.

IV. MULTI-RESOLUTION DCT

Multi-resolution analysis (MRA) has been very popular with compression specially in video streaming because it eliminates the blocking effects of the DCT which become more prominent in videos. In this new approach for DCT first of all we will eliminate the blocking completely. Instead of using the blocks as done in the standard DCT, we will use the full rows and columns of the image. Once we have the rows and columns of the image, we will apply the techniques of multi-resolution on it to achieve the required compression. The idea here is not to prove that this new approach is better than the block DCT or the wavelets for compression purposes. In fact, the idea here is to show that multi-resolution analysis can be applied to DCT.

The New Algorithm: The steps of the new algorithm are as follows:

i. Read the image and convert into a double using MTALAB function “im2double”.
ii. Divide the image into rows and columns [row col] = size(a).
iii. Compute the total size of the image 2(row*col).
iv. Concatenate the rows and columns to each other and make a long linear line of data.
v. Apply 1-D Dct to the data obtained from step 4.
vi. Apply multi-resolution analysis and discard the 50% of the data obtained from step 5.
vii. Repeat the process until the desired compression is reached.
viii. Apply inverse 1-D DCT
ix. Separate the rows and columns from the data received from step 8.
x. Reconstruct the original image matrix by averaging pixels from the rows and the columns. This averaging is necessary to remove any distortion or noise introduced in the process.
xi. Compute the mean square error and compare it with the block DCT.

As it has been mentioned earlier, the results obtained from the above algorithm will be compared against block DCT. The blocks used in block DCT will be at least 16 x 16 or more.

There are two main parts to this algorithm which are of immense importance. The first step deals with eliminating the blocking artifacts which has been a problem with the block DCTs. The other part deals with applying multi-resolution analysis to the coefficients obtained after applying the DCT. In steps 2 and 4 it is clear that instead of making the blocks for the DCT we are taking the whole rows and columns and making a big one dimensional array for the pixels as shown in figure – 4.

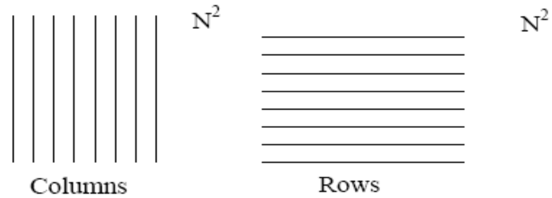


Figure 4. Rows and columns of the image separated.

The size of both columns and rows is N^2. So the total size of the image will be $2N^2$ figure 4. The rows are concatenated to each other to form a one big horizontal line of pixels. The columns are also concatenated to each other in the same way. Now we concatenate concatenated columns to the end of the concatenated rows forming a 1-D signal of size $2N^2$. All these steps have been taken to eliminate the need of making the blocks for running DCT. This procedure is termed as *“cascading rows and columns”.* This process is shown pictorially in the following figure - 5.

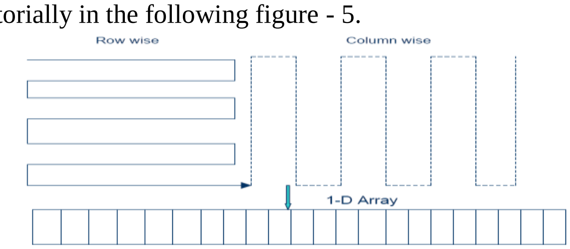


Figure 5. Cascading the rows and columns into a 1-D array

Once we have all the pixels arranged in a 1-D array then DCT is applied on it. We use the MATLAB built-in function “dct2”. This function computes a 2-D DCT but we can use it to compute the one dimensional DCT by using a matrix size of Nx1. After the DCT coefficients are obtained, we apply the multi-resolution analysis to the DCT coefficients to obtain the desired compression. Here the compaction energy property of the DCT is utilized. It has been established in the previous chapters that most of the information in DCT is retained at lower frequencies, meaning at the top left corner of the blocks. This property makes it easier and simpler to apply multi-resolution analysis on DCT coefficients. Figure - 6 shows the energy compaction property of the DCT. On the X-axis there is frequency and the amplitude is plotted on the Y-axis. It is evident from the graph that much of the image information is stored in the left corner of the graph which is the low frequency.

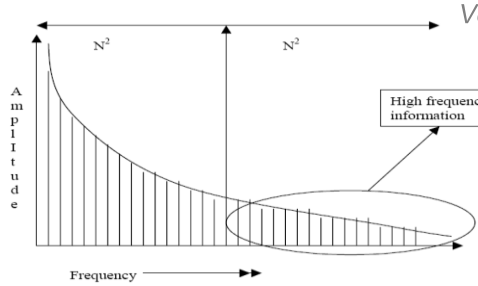


Figure 6. Shows the energy compaction property of DCT coefficients

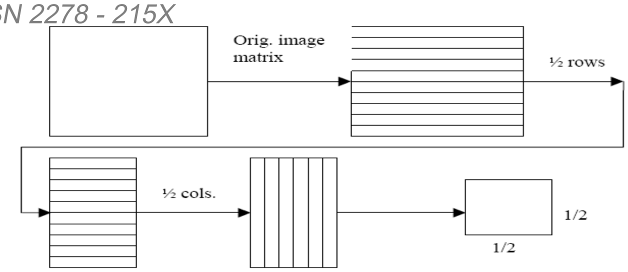


Figure 7. An alternate approach to eliminate blocks in DCT

It means that in DCT coefficients most of the information is represented in the lower frequencies. This property plays a very important role while applying the multi-resolution analysis. In multi-resolution analysis we discard the 50% of the coefficients at each level until we get to the desired compression. Based on figure - 6, this process becomes easier as we can simply discard half of the coefficients at higher frequencies without losing much of the information because we have already established that most of the information is compacted at the lower frequencies.

V. ALTERNATIVE APPROACH

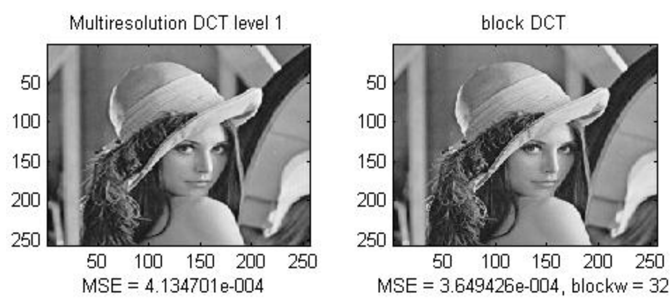
The above proposed algorithm is one way of eliminating the blocks i.e. by taking all rows and columns and concatenating them to make 1-D data. This approach has worked well and the results will be presented in the next section that consolidates our claim that the blocking artifacts can be avoided while still having acceptable errors. An alternate way of eliminating the blocks is explained as follows:

Instead of taking rows and columns of the image matrix in this approach first we take only the rows of the original image matrix. The rows are concatenated to form a 1-D array of the pixels. DCT is applied to these 1-D row-wise pixels and using the energy compaction property to the DCT, half of the pixels are discarded. Next columns are taken from this compressed matrix and the process is repeated for concatenating, applying DCT and the multi-resolution part where half of the pixels are discarded. In the next resolution, we alternate this process and this time the columns are taken first and then the rows.

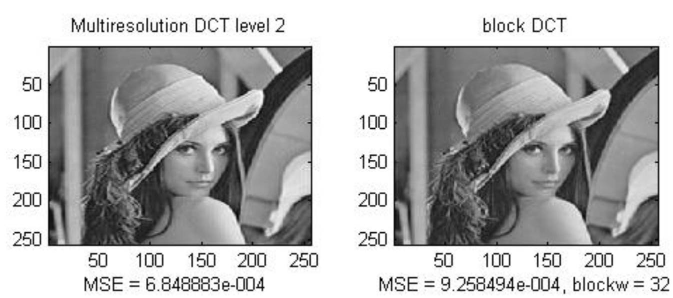
This approach achieves 50% compression in two cycles; first for the rows and then for the columns as compared to the first approach in which 50% compression is achieved in one cycle because the rows and columns are taken at the same time. It is shown in the figure - 7 that from the original image matrix the rows are taken and reduced to half and then on the reduced coefficients the columns are taken and then reduced again to half to achieve the first 50% compression or the first resolution. The fact that this approach reduces the size of the image at each level, aliasing is introduced in this process. Since this approach is based on one dimensional DCT and we have already established in the previous chapters that MDCT also uses a similar approach. In order to use this approach properly, MDCT can be used because of its special technique of time domain alias cancellation (TDAC).

VI. EXPERIMENTAL RESULTS

The main idea of this paper is to come up with a new algorithm that eliminates the blocking from the DCT as well as to show that multi-resolution analysis can be applied to the DCT. Several images have been used with our algorithm as well as the block DCT in order to compare the results. The block size used in block DCT is 32 x 32. We have not used any other block size because if a smaller block size is used then definitely the results will not be as good for the block DCT and it will not be a fair comparison. For both the images, we have run the test up to 6 resolutions. The mean square average is used to computer the errors for comparisons. Although the resolution is reduced at every level we keep the same size for the images as the original size in order to make easier comparisons. We obtain the same size as the original image in the reconstruction part by padding zeros to the reduced resolution to come up with the same size image. The results of the multi-resolution DCT as well as block DCT applied to the images can be seen in the following images as shown in figure - 8.



(a) Level 1



(b) Level 2

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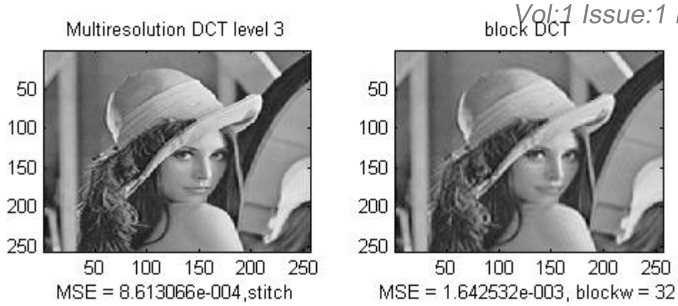
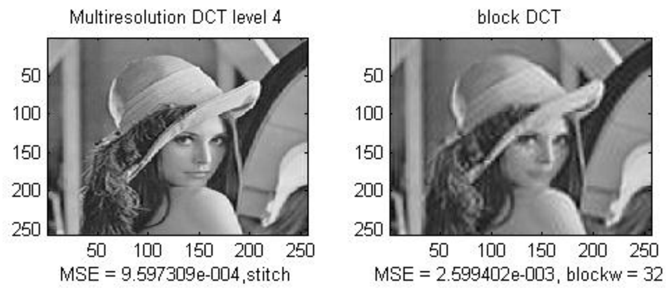
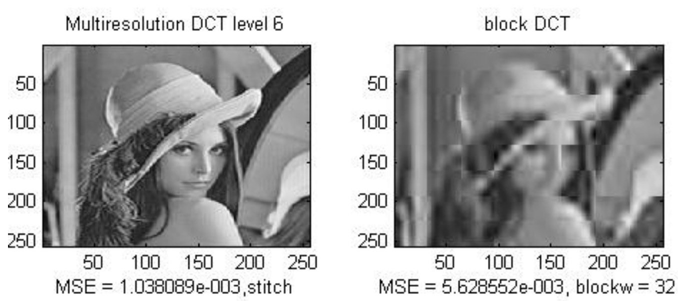


(c) Level 3



(d) Level 4

(e) Level 5



(f) Level 6

Figure 8. Image of Lena at different resolutions (a-f) with Multiresolutuin DCT and Block DCT

The above images represent the comparison between our new multi-resolution DCT algorithm and the block DCT at different levels. At each level after the DCT is taken, 50% of the high frequency DCT coefficients are discarded and replaced by zeros to keep the original size. IDCT is taken after that to reconstruct image. At the next level the DCT is taken again for the reconstructed image and the DCT coefficients are spread again across the image. The sharp edges will be in the high frequency range and the other important information in the lower frequency range. It is evident from the above images that; since we keep discarding the high frequency coefficients, meaning the edges, the images at each subsequent level get again for the reconstructed image and the DCT coefficients are spread again across the image. The sharp edges will be in the high frequency range and the other important information in the lower frequency range. It is evident from the above images that; since we keep discarding the high frequency coefficients, meaning the edges, the images at each subsequent level get blurrier.

VII. CONCLUSION

The purpose of this paper was to come with a new algorithm that would eliminate the blocking from the DCT as well as apply multi-resolution analysis to the DCT. The experimental results obtained proves our claims in the sense that the images reconstructed after compression do not have blocking artifacts and also we have shown that multi-resolution analysis can be applied to the DCT and, in fact, the results are very encouraging compared to our algorithm with the block DCT using 32 x 32 blocks. The reason for choosing this block size is that any blocks smaller would produce worse results and any blocks bigger would take quite longer time to do the processing.

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