

# Rrecursive wideband minimum phase digital integrators

Dharmendra K. Upadhyay  
Division of ECE, NSIT

Sector-3, Dwarka, New Delhi-110078  
Delhi, India  
upadhyay\_d@rediffmail.com

Prof. R. K. Singh

Department of ECE, Uttarakhand Technical University  
Dehradun, India  
rksinghkec12@rediffmail.com

**Abstract**— New designs of recursive wideband digital integrators are obtained by applying the zero-reflection approach over the existing recursive wideband digital integrators. The designed recursive digital integrators have all the poles and zeros inside the unit circle. These minimum phase integrator designs have highly improved linearity in phase responses as compared to the existing integrator designs. Therefore, the designed minimum phase recursive wideband digital integrators are more suitable for real-time control and signal processing applications.

**Keywords**—digital integrator, linear phase, minimum phase, recursive, wideband

## I. INTRODUCTION

Digital integrators are used extensively in the areas of digital signal processing, bio-medical engineering, sonar engineering, digital control systems etc. The frequency response of an ideal digital integrator is given by

$$H_i(e^{j\omega}) = \frac{1}{\omega} e^{-j(0.5\pi)} \quad (1)$$

Where  $j = \sqrt{-1}$  and  $\omega$  is the angular frequency in radians. Many techniques have been developed to design digital integrators using recursive systems in digital signal processing (DSP) [1-12]. Initially, the recursive digital integrators have been obtained by performing a simple linear interpolation between the magnitude responses of the classical rectangular, trapezoidal and the Simpson recursive digital integrators [1-4]. In [5], a linear programming optimization approach is also proposed to design recursive digital integrators. Later, Ngo has proposed recursive wideband digital integrator of third-order system based on Newton-Cotes integration rule [6]. Ngo design approximates the ideal integrator over the whole Nyquist band with nearly 4.5% maximum percentage relative error (MPRE) in magnitude response over the full Nyquist band. In [7-9], Gupta-Jain-Kumar (GJK) have also proposed recursive wideband digital integrators for maximum percentage relative errors of nearly 3% in magnitude responses over wideband using linear interpolation between three different integration techniques.

Further, Al-Alaoui has proposed 2-segment, optimized 3-segment and optimized 4-segment recursive digital integrators for lower relative errors over wideband [10]. Recently, Upadhyay-Singh (US) have proposed a recursive wideband digital integrator of only 0.48% MPRE in magnitude response over the full Nyquist band except near to  $\omega = \pi$  [11]. It is noticed that the magnitude responses of existing integrator designs are highly accurate, but have the large amount of non-linearity in phase responses. Therefore, the design of recursive wideband linear phase digital integrators is the main issue in current research environment. In [12], Upadhyay-Singh have also proposed the recursive wideband digital integrator of first-order system for lower relative error over wideband.

In this paper, novel designs of recursive wideband minimum phase digital integrators are presented, which have maximum phase deviations of not more than  $12^\circ$  from the ideal linear phase responses over wideband. All the figures are obtained by using MATLAB.

This paper is organized as follows: Novel designs of recursive wideband minimum phase digital integrators are obtained in Section II. The comparisons of proposed integrator designs with the original designs are given in Section III. In last, the conclusions are given in Section IV.

## II. DESIGN OF RECURSIVE MINIMUM PHASE DIGITAL INTEGRATORS

Figure 1 shows the phase responses of existing Gupta-Jain-Kumar (GJK) recursive wideband digital integrators [8], [9]. The transfer functions of GJK recursive digital integrators  $H_{lmi1-2}(z)$  of third-order systems are given in (2) and (3) respectively.

It is noticed that the existing GJK recursive digital integrators have more non-linearity in phase responses over wideband. These designs have nearly  $35^\circ$  maximum phase deviations from the ideal linear phase responses over wideband. Therefore, there is the need to improve the linearity in phase responses of GJK recursive digital integrators over wideband.

$$H_{1mi1}(z) = 0.329T \frac{(z+2.663)(z^2 - 0.2079z + 0.03864)}{z^2(z-1)} \quad (2)$$

$$H_{1mi2}(z) = 0.34T \frac{(z+2.541)(z^2 - 0.2081z + 0.03858)}{z^2(z-1)} \quad (3)$$

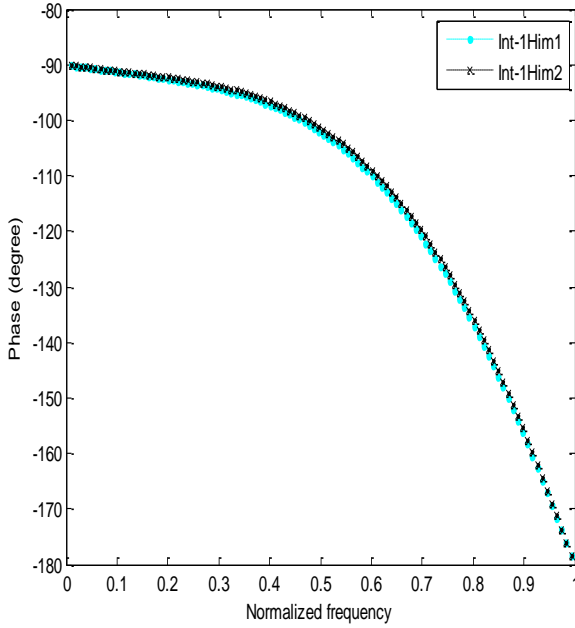


Figure 1. Phase responses of  $H_{1mi1-2}(z)$  (GJK) recursive wideband digital integrators

From (2) and (3), it is noticed that the existing GJK recursive digital integrators  $H_{1mi1-2}(z)$  have one zero outside the unit circle. Therefore, the minimum phase designs can be obtained by applying the zero-reflection approach over the existing GJK non-minimum phase digital integrators.

Now, the zero-reflection approach is applied over the GJK digital integrators  $H_{1mi1-2}(z)$  to reflect the non-unity zeros from outside the unit circle to inside the unit circle. The zero-reflection approach is similar to the pole-reflection approach as in [8-12]. The transfer functions of recursive wideband minimum phase digital integrators thus obtained are given in (4) and (5) respectively.

$$H_{2mi1}(z) = 0.876T \frac{(z+0.3755)(z^2 - 0.2079z + 0.03864)}{z^2(z-1)} \quad (4)$$

$$H_{2mi2}(z) = 0.8639T \frac{(z+0.3935)(z^2 - 0.2081z + 0.03858)}{z^2(z-1)} \quad (5)$$

### III. COMPARISONS

Figure 2 shows the percentage relative errors (PREs) of proposed  $H_{2mi1-2}(z)$  and the existing  $H_{1mi1-2}(z)$  (GJK) recursive wideband digital integrators in magnitude responses with the ideal integrator. Further, Figures 3 and 4 shows the phase responses of proposed  $H_{2mi1-2}(z)$  and the existing  $H_{1mi1-2}(z)$  (GJK) recursive wideband digital integrators.

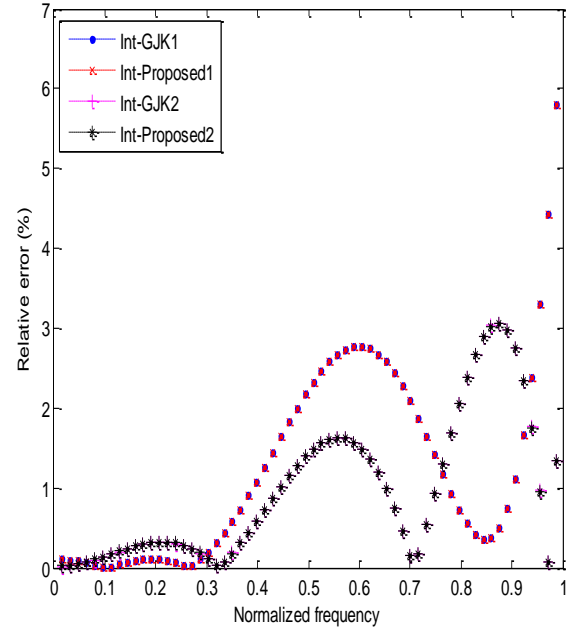


Figure 2. PREs of proposed  $H_{2mi1-2}(z)$  and the GJK  $H_{1mi1-2}(z)$  recursive digital integrators with the ideal integrator

From Figure 2, it is observed that the magnitude responses or percentage relative errors of designed recursive digital integrators  $H_{2mi1-2}(z)$  are exactly same as of the original GJK recursive digital integrators  $H_{1mi1-2}(z)$ . It is also observed that all the recursive digital integrator designs  $H_{1-2mi1-2}(z)$  have not more than 3% relative errors over wideband. However, from Figures 3 and 4, it is clear that the linearity in phase responses of designed minimum phase recursive digital integrators  $H_{2mi1-2}(z)$  is highly improved as compared to the original or existing GJK recursive wideband digital integrators  $H_{1mi1-2}(z)$ . The absolute phase errors (APEs) of

proposed  $H_{2mi-2}(z)$  and the existing  $H_{1mi-2}(z)$  (GJK) recursive digital integrators are also shown in Figure 5.

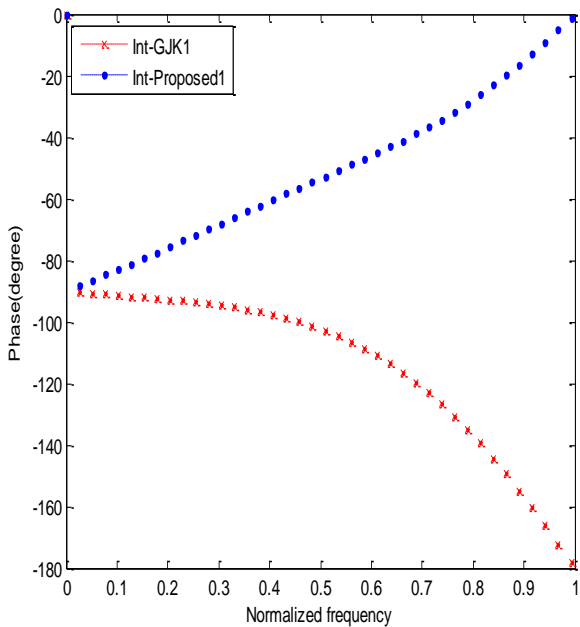


Figure 3. Phase responses of proposed  $H_{1mi1}(z)$  and the existing GJK  $H_{2mi1}(z)$  recursive digital integrators

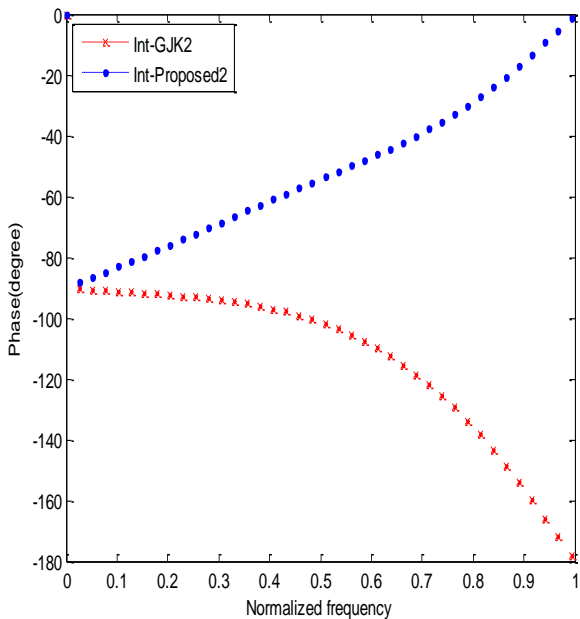


Figure 4. Phase responses of proposed  $H_{1mi2}(z)$  and the existing GJK  $H_{2mi2}(z)$  recursive digital integrators

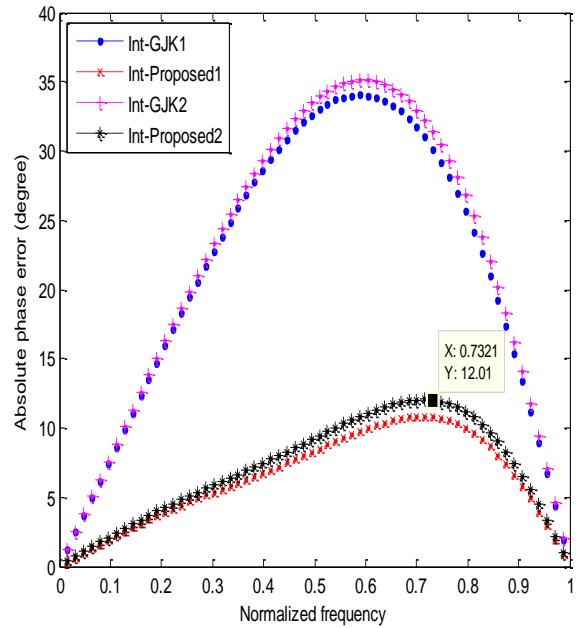


Figure 5. APEs of proposed  $H_{2mi1-2}(z)$  and the existing GJK  $H_{1mi1-2}(z)$  recursive digital integrators from the ideal linear phase responses

From Figure 5, it is observed that the proposed minimum phase recursive wideband digital integrators  $H_{2mi1-2}(z)$  have maximum phase deviations of nearly  $12^\circ$  from the ideal linear phase responses over the full Nyquist band, while the original or existing GJK integrator designs  $H_{1mi1-2}(z)$  have nearly  $35^\circ$  maximum phase deviations from the ideal linear phase responses over the full Nyquist band.

#### IV. CONCLUSIONS

Minimum phase designs of recursive wideband digital integrators are obtained by applying the zero-reflection approach over the existing non-minimum phase integrator designs. Minimum phase designs have the same magnitude responses or percentage relative errors (PREs) as of the original or existing designs, but having the nearly linear phase responses over wideband. The designed recursive digital integrators have not more than  $12^\circ$  absolute phase errors from the ideal linear phase responses over full Nyquist band.

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