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Determination of Phase Matching Angle for Non-Resonant Total Internal Reflection Quasi-Phase Matching in a Parallel slab using MATLAB/SIMULINK Software

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***Abstract*—The simulated model calculates the phase matching angle for a parallel isotropic slab made of Zinc Telluride (ZnTe), Cadmium Telluride (CdTe), Gallium Arsenide (GaAs) and Zinc Selenide (ZnSe) having thickness 't' at a given wavelength of 10.6 µm by determining the wave-vector mismatch. The model also generates the status of fulfillment of Total Internal Reflection (TIR) inside the semiconductor slab. The simulated results indicate phase matching angle of 0.9034 rad for a slab thickness of 800 µm in case of ZnTe and 0.5457 rad for CdTe taking the slab thickness as 500 µm. The phase matching angle for ZnSe is 1.481 rad and that of GaAs is 0.6984 rad for a slab of thickness 800 µm for both the materials. For all the cases, the TIR condition is found satisfying.**

Keywords- Wave-vector mismatch, Phase matching, Quasi phase matching, Isotropic, Total Internal Reflection Quasi Phase Matching, Sellmeier's Equation.

I. INTRODUCTION

Mid-infrared tunable sources are of immense interest for applications starting from environmental monitoring to spectroscopy, medical diagnosis, thermography etc. Semiconductors of the technological mainstream are excellent candidates for optical parametric conversion in the mid-infrared region. They, indeed offer a number of advantages such as (1) are transparent in the near- and mid infrared regions, (2) possess high nonlinear second-order coefficients $\chi^{(2)}$, (3) benefit from a mature technology, and (4) can stand a high incident energy flux. These materials are, however, optically isotropic, so that no natural birefringence phase matching scenario is possible.

In their founding paper, Armstrong *et al.* [1] suggested the use of the relative phase change between the harmonics and the fundamental waves on total internal reflection in nonlinear materials. This principle was demonstrated by Boyd and Patel [2] as well as Komine *et al.* [3] in their seminal papers, by phase matching second-harmonic generation in isotropic semiconductors. Theoretical predictions were in good agreement with experimental observations and measurements. However, scarce efforts have been devoted to develop this technique and investigate its potentialities and limitations [4].

The following paper deals with the determination of the phase matching angle for TIR-QPM in a parallel slab (of thickness t and slab angle Ψ) for some optically important isotropic semiconductors such as GaAs, ZnSe, CdTe, ZnTe. The simulation has been done on matlab simulink platform due to the user friendly nature of the software. The user simply needs to input a no. which corresponds to a particular isotropic material from among the materials that have been considered for the analysis. The fundamental input wavelength, operating temp, slab thickness t and angle of the slanted slab surface w. r. t the vertical axis can be changed as per requirement of the user. On the basis of the input parameters, the phase-matching angle that needs to be subtended inside the parallel slab by the fundamental beam has been calculated. At the same time care has been taken that the subtended angle also satisfies the Total Internal Reflection (TIR) criteria. The results obtained from the simulink model have been verified with the help of an appropriate matlab program.

II. MATHEMATICAL ANALYSIS

A. Total Internal Reflection Quasi Phase Matching

It was first in 1962, when Armstrong *et al*. suggested that QPM can be obtained by Total Internal Reflection (TIR) in a plane parallel plate. This technique proposed a phase corrective scheme whereby the phase mismatch in a nonlinear optical process is periodically corrected by introducing a periodicity to the nonlinearity of the medium corresponding to the coherence length. The coherence

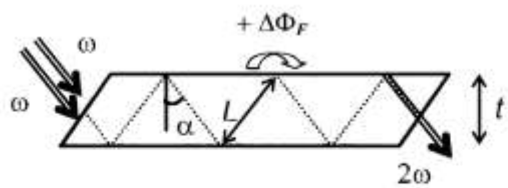

Figure 1. Phase matching technique using Total Internal Reflection in parallel semiconductor slab for second harmonic generation scheme.

length is defined as the distance taken by the fundamental and the harmonic light to become 180˚ out of phase. This TIR-QPM technique makes use of the differential Fresnel phase shifts experienced by the interacting waves as they undergo total internal reflection on the material-air interface. Hence this technique is also known as Fresnel Phase Matching. The schematic diagram of Fresnel Phase Matching for second harmonic generation is shown in fig.(1). Here L indicates the distance between successive bounces and t denotes the slab thickness; ω indicates the fundamental frequency, α is the angle of incidence inside the slab and $\Delta\phi_F$ denotes the differential Fresnel Phase shift.

In this scheme, two optical waves at same frequency ω are injected into the plate by one of the slanted face. The input optical waves and the resultant second harmonic ω_3 (= 2ω) generate wave vectors k, k and k_3 respectively in the semiconductor slab. The coherence length L_{coh} for SHG interaction is given as $L_{coh} = \pi/\Delta k$ where $\Delta k = k_3 - 2k$ is the wave vector mismatch.

1. Resonant Quasi Phase Matching

Boyd, Patel and Komine noted that Fresnel phase shift $\Delta\phi_F$ can virtually reach any value between 0 and 2π. This is basically different from the usual Quasi Phase Matching in which the phase shift from one domain to the other is bounded by π. The global parametric process efficiency is given by η α $\eta_1\eta_2$. At first there is a parametric conversion on each path L between two bounces so that during first term of conversion the efficiency $\eta_1\eta_2$ are maximized. The conversion yield η_1 can be written as [4]:

$$\eta_1 \infty [\sin(\Delta kL/2)/(\Delta kL/2)]^2 \qquad (1)$$

Then the individual fields generated along the path will interface with one another. This introduce the following term:

$$\eta_2 \infty [\sin(N\Delta\phi/2)/N\sin(\Delta\phi/2)]^2 \qquad (2)$$

where, N is the number of bounces inside the plate. Therefore, overall conversion efficiency is given as

$$\eta_2 \infty [\sin(\Delta kL/2)/(\Delta kL/2)]^2 [\sin(N\Delta\phi/2)/N\sin(\Delta\phi/2)]^2 \qquad (3)$$

Resonant is the situation in which ΔKL = π (mod 2π) i.e., the distance L between two successive bounce is exactly an odd number of coherence length L_c for conversion process (ΔKL = π). In that case the added phase shift $\Delta\phi_F + \delta\phi$ must be exactly π in order to get a quasi phase matched growth of conversion signal throughout the crystal.

2. Non-Resonant Quasi Phase Matching

When the distance between two successive bounces is not equal to an odd multiple of L_c then the situation is called non-resonant. The distance L between two successive bounces is not optimized for a one way conversion process. But still the quasi phase matching condition is assumed to be satisfied so that $\Delta\phi = \Delta KL + \Delta\phi_F + \delta\phi = 2\pi$ (mod 2π). The situation is possible only because the Fresnel phase shift $\Delta\phi_F$ added to δϕ can virtually compensate for any phase-mismatch, ΔKL. And ΔKL = (δ+1)π where 1 is odd integer. Fresnel phase shift $\Delta\phi_F$ combined with δϕ can virtually reach any value between 0 to 2π (mod 2π) thus greatly alleviating the quasi phase matching conditions. With suitable polarization configuration non-resonant phase matching plays an important role in such case [4].

B. Sellmeir's Equation

The **Sellmeier equation** is an empirical relationship between refractive index and wavelength for a particular transparent medium. The equation is used to determine the dispersion of light in the medium. It was first proposed in 1871 by W. Sellmeier, and was a development of the work of Augustin Cauchy on Cauchy's equation for modeling dispersion.

The usual form of the equation for glasses is

$$n^2(\lambda) = 1 + \frac{B_1\lambda^2}{\lambda^2 - C_1} + \frac{}{\lambda} \qquad (4)$$

where, n is the refractive index, λ is the wavelength, and B1,2,3 and C1,2,3 are experimentally determined *Sellmeier*

coefficients. These coefficients are usually quoted for λ in micrometres. Note that this λ is the vacuum wavelength; not that in the material itself, which is λ/ n(λ). A different form of the equation is sometimes used for certain types of materials, e.g. crystals.
In our analysis, for the isotropic materials, CdTe and ZnTe the temperature independent Sellmeier equation have been used while for GaAs and ZnSe temperature dependent Sellmeier equation have been used [5-9].

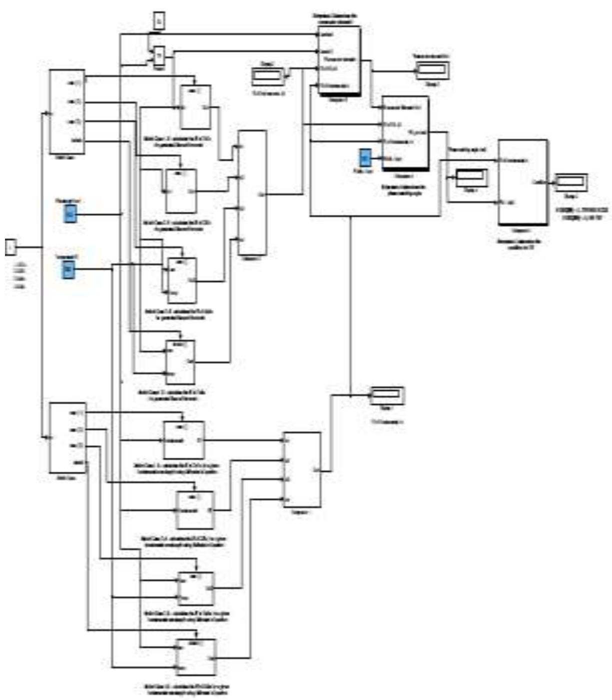

Figure 2. Simulated model for determining phasematching angle for non-resonant TIR-QPM in a parallel slab.

III. SIMULATED MODEL

The simulated model is used to find out the wave-vector mismatch, phase matching angle, and the status of TIR condition in a parallel plate of thickness t (fig.1) for a fixed fundamental wavelength of 10.6 µm at a temperature of 298 K. The simulated model is shown in the fig (2).

By selecting the materials by switch case (1 for ZnTe, 2 for CdTe, 3 for GaAs and 4 for ZnSe) the refractive index of the particular material at the fundamental wavelength and the generated Second Harmonic has been calculated by using Sellmeier's Equation in the switch case subsysytems.

The switch case subsystems are discussed below:

1. *Switch Case 1-A:*

It calculates the temperature independent refractive index of ZnTe at the fundamental wavelength by using Sellmeier's equation. The standard Sellemier's equation can be written as $n^2=A + [B\lambda^2/(\lambda^2-c^2)]$ where n is the refractive index and λ is the wavelength in microns. For ZnTe the best values of the parameters are A=4.27, B=3.01, and $c^2=0.142$ [5].

2. *Switch Case 2-A:*

It calculates the temperature independent refractive index of CdTe at the fundamental wavelength by using Sellmeier's equation. In this case, A=5.68, B=1.53, and $c^2=0.366$ [6].

3. *Switch Case 3-A:*

It calculates the temperature dependent refractive index of GaAs at the fundamental wavelength by using Sellmeier's equation as[7]:

$$n^2 = g_0 + \frac{g_1}{lam_1^{-2} - lam^{-2}} + \frac{g_2}{lam_2^{-2} - lam^{-2}} + \frac{g_3}{lam_3^{-2} - la} \quad (5)$$

where, g_0 = 5.372514, g_1 = 27.83972, g_2 = 0.031764 + (4.35 x 10^{-5} x Temp) + (4.664 x 10^{-7} x $Temp^2$), g_3 = 0.00143636, lam_1 = 0.4431307 + 5.0564 x 10^{-5} x Temp, lam_2 = 0.8746453 + 1.913 x 10^{-4} x Temp – 4.882 x 10^{-7} x $Temp^2$, lam_3 = 36.9166 – 0.011622 x Temp, Temp=Tempin-295; Tempin being the input temperature.

4. *Switch Case 4-A:*

It calculates the temperature dependent refractive index of ZnSe at the fundamental wavelength by using Sellmeier's equation as [8]:

$$n^2 = E_t + \frac{A_t}{lam^2 - lamu^2} + \frac{}{la} \quad (6)$$

where, E_t = 9.01536 + 1.4419 x 10^{-3} x Temp + 3.32973 x 10^{-7} x $Temp^2$ - 1.08159 x 10^{-9} x $Temp^3$ - 3.88394*10^{-12} x $Temp^4$, A_t=0.24482 + 2.77806 x 10^{-5} x Temp + 1.01703 x 10^{-8} x $Temp^2$ - 4.51746 x 10^{-11} x $Temp^3$ + 4.18509 x 10^{-13} x $Temp^4$, B_t=3.08889 + 1.13495 x 10^{-3} x Temp + 2.89063 x 10^{-7} x $Temp^2$ - 9.55657 x 10^{-10} x $Temp^3$ - 4.76123 x 10^{-12} x $Temp^4$, lamu=0.29934+1.004 x 10^{-4} x Temp, lami=48.38+6.29 x 10^{-3} x Temp, Temp=Tempin-293; Tempin being the input temperature.

Similarly, the refractive index of the materials at the Second harmonic has been calculated in the respective Switch Case 2-A, 2-B, 2-C and 2-D taking the half wavelength λ/2.

The parameters are calculated from the following subsystems as discussed below:

i) Subsystem 1& 2:

Subsystem 1 & 2 are used for selecting the calculated refractive index of the particular material for fundamental and generated second harmonic beam. These parameters are subsequently used to determine the wave-vector mismatch and phase matching angle.

ii) Subsystem 3:

This subsystem is used to calculate the wave-vector mismatch, given by $\Delta k = k_3 - 2k$, k and k3 being the wave-vectors of the fundamental and generated Second Harmonic wave respectively[4].

Here, $k=2\pi n/\lambda$, where n is the refractive index and λ being the given wavelength.

iii) Subsystem 4:

In subsystem4, the phase-matching angle is calculated depending on the width of the slab (which may be varied as per user's choice). The phase matching angle is calculated by using the Fresnel Phase Shift [10] :

$$\Delta\phi_F = -2\arctan\frac{(1-q)+qn^2_v}{n_c} \qquad (7)$$

iv) Subsystem 5:

This subsystem displays whether the TIR condition satisfies or not. If the phase matching angle which is the incident angle in this case is greater than the critical angle then the TIR condition is satisfied. The respective display shows 1 if TIR condition is satisfied else 0.

IV. RESULT AND DISCUSSION

In the simulated model, a parallel slab of defined thickness 't' has been considered and the simulation has been performed at a fundamental wavelength of 10.6 µm for SHG conversion. The simulated model analytically determines the wave-vector mismatch, phase matching angle and status of TIR condition for isotropic materials ZnTe, CdTe, GaAs and ZnSe. For GaAs and ZnSe,the temperature dependent Sellemier's Equation has been used, where the user can vary the temperature as per requirement. For ZnTe and CdTe, the Sellemier's Equation is independent of the temperature. Table I shows the obtained results of the different parameters for the chosen materials:

TABLE I. DETERMINATION OF PHASE MATCHING ANGLE FOR NON RESONANT TIR - QPM IN A PARALLEL SLAB

Material	***Input Temperature (K)***	***Thickness of slab (µm)***	***Wave vector mismatch (cm^{-1})***	***Phase Matching angle (rad)***	***TIR Condition***
ZnTe	0	800	3.198e+006	0.9034	1
CdTe	0	500	3.183e+006	0.5457	1
GaAs	298	800	3.951e+006	0.6984	1
ZnSe	298	800	2.846e+006	1.481	1

V. CONCLUSION

In this paper, a MATLAB simulink model is described which determines the phase matching angle of a given material as well as the wave vector mismatch showing the status of TIR condition for a parallel slab of defined thickness. This is a user friendly model whereby the phase matching angle can be obtained by choosing a material and its width at a fixed temperature. This model can also be used for determining the condition of TIR for a particular wavelength at a given temperature and thickness for a SHG converter.

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