Tunable OTA Low Pass Filter with the Fractional-Order step Technique

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***ABSTRACT*— This paper presents the tunable technique of the analog filter at high frequency fp = 1 MHz by used the Fractional-order $(n+\alpha)$ step, where n is an original of integer-order on circuit and α is an approximation order step $0<\alpha<1$, An approximation order is designed from the Fractance circuits, and also presents the approximation function of the Fractional-order Laplacian s^{α} design on the OTA low-pass filter circuit. This circuit used single active component. OTA and CFA components are in monolithic chip (IC LT1228).The CFA design with the Fractional-order elements to feedback control the OTA for relative stabilty gain of amplifier. The results shown that tunable filter by Fractional-order step technique provides the flat passband and reduced the peaking problem and improved slope of magnitude response of filter and result comparable with high-order filters.**

Keywords*– *Operation Transconductance; Fractional-order; Low Pass filter; Fractance circuits

I. Introduction

The filters are electronics circuit used in the signal processing for attenuation of unwanted frequency [1], and many research presented and designed the Fractional-order low pass filter by used the OP-AMP (Operation Amplifier) and not designed at high frequency. This filter has been designed by used the operation transconductance amplifier (OTA) designed with the Fractional-order calculus to improve the performance of filter. IC OTA LT1228 (Linear Technology) provides one of the largest bandwidth and available on the market today and this is the reason why it was selected to design and simulation. The structure of filter has designed by refer to the Butterworth model. Why we choose the Fractional-order theoretical to implement on the OTA low pass filter because we would like to use the positive effects of the Fractional-order behavior to increase relative stability in the frequency domain by the π/2 phase, increase gain with slope of 20 dB/dec and reducing the steady-state error.

The frequency response of the generalize filter. It is a ratio of two polynomials. It can be written in equation (1).

$$T_{(s)}=\frac{N_{(s)}}{D_{(s)}} \qquad (1)$$

The roots of the denominator polynomial N(s) are called poles and the roots of the numerator polynomial D(s) are refer to as zeros. The Fractional-order derivative is defined, according to the Riemann-Louville definition [2] in equation (2).

$$\frac{d^{\alpha}}{dt^{\alpha}}f(t)\equiv D^{\alpha}f(t)=\frac{1}{\Gamma(1-\alpha)}\frac{d}{dt}\int_{0}^{t}(t-\tau)^{-\alpha}d \qquad (2)$$

where 0 < α < 1, and α is the initial time instance, often assumed to be zero, i.e., α = 0.The differentiation is then denoted as $D^{\alpha}f(t)$.

Use the Laplace transform then yield is in equation (3)

$$L_0 d_t^{\alpha}=s^{\alpha}F_{(s)} \qquad (3)$$

The Riemann–Liouville definition is used definition in the Fractional-order calculus. The subscripts on both sides of D represent, respectively, the lower and upper bounds in the integration .The basic instruction of the filter came from [2].This filter can work in voltage and current modes and used only single operation transconductance amplifier to a second order design with the high frequency $f_p=1MH_z$ and improve the performance of filter circuit by Fractional-order step and design the fractance circuits to be the Fractional-order Laplacian.

In this paper we propose to optimize the performance of filter at high frequency by utilize the approximation order for the Fractional-order Laplacian. This paper shown that tunable obtained characteristics extends those of the Butterworth filter from approximation order step $(n+\alpha)$, α =0.1, 0.5 and 0.9.The PSPICE and MATLAB simulations and experimental results are depicted and correspond well with the theoretical.

II. Fractional Order In Frequency Domain

A. *Continuous-time of Fractional-order operator*

The Fractional-order operator [1], it is a Laplace and represented is s^{α} which exhibits straight line on both is magnitude and phase bode plot. It is useful to fit the frequency response over a frequency range of interest (ω_b,ω_h) .

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Continued fractional expansion (CFE) is use to ration-function approximation the Fractional-order operator. $G_{(s)} = S^{\alpha}$, the continued fraction expansion can be written in equation (4).

$$G(s) = \cfrac{b_0(s)}{a_0(s) + \cfrac{b_1(s)}{a_1(s) + \cfrac{b_2(s)}{a_2(s) + \text{K}}}}, \tag{4}$$

The Fractional-order integrator with S^{α} to second order. The rational function approximation using different continued fraction expansion is in equation (5) [1].

$$s^{\alpha} \cong \frac{(\alpha^2 + 3\alpha + 2)s^2 + (8 - 2\alpha^2)s + (\alpha^2 - 3\alpha + 2)}{(\alpha^2 - 3\alpha + 2)s^2 + (8 - 2\alpha^2)s + (\alpha^2 + 3\alpha + 2)} \tag{5}$$

The approximation of $S^{0.5}$, magnitude and phase bode plots response shown in Figure1.

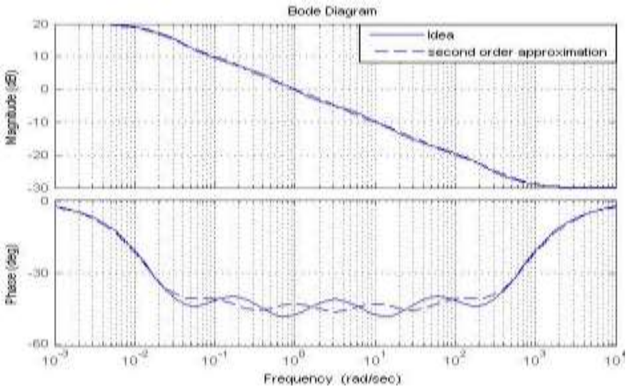

Figure 1. Magnitude and phase bode plots of $S^{0.5}$

The magnitude and phase bode plots are shown in Figure1.It seem that the Bode plots of filter is relatively close to the theoretical over the frequencies range of interested.

B. Fractance circuit for $S^{0.5}$ Fractional calculus

The Fractance circuit consists of resistor and capacitor describes by integer-order model. A tree-type infinite recursive formed by impedance Z_a and Z_b [3].It is possesses high self-similarity and shown in Figure 2.

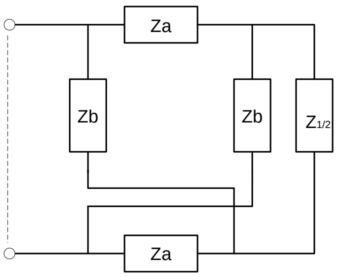

Figure 2. Equivalent circuit of $S^{0.5}$ net-grid-type analog Fractance circuit

According to linear algebra theoretical, so total impedances the yield is in equation (6)

$$Z_{1/2} = \frac{V_i}{i_a + i_b} = \frac{2Z_a Z_{b+} Z_{1/2}(Z_a + Z_b)}{2Z_{1/2} + Z_a + Z_b} \tag{6}$$

$$Z_{1/2} = (Z_a Z_b)^{\frac{1}{2}}$$

Then we have experimented $s^{0.5}$ order-differentiator with the rectangle-wave signal. The result shown in Figure 3.

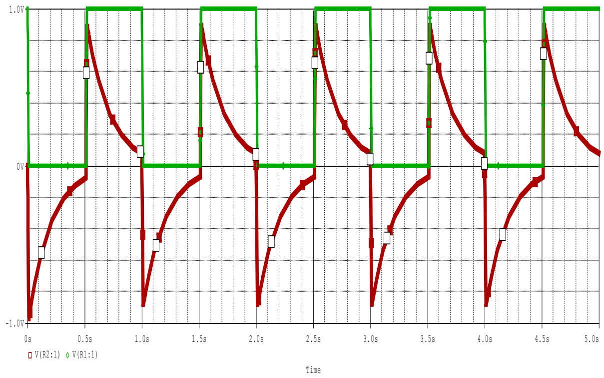

Figure 3. Waveform Input Vs Output of $S^{0.5}$ order-differentiator

In Figure 3, Green line is the input voltage of Fractance circuit in Figure 2, V1= 1V, V2= 0V, TD=10mS, TR=10mS, TF= 10mS, PW = 0.5S and PER=1S.We discovered the output (Red line) of circuit is beginning start up from the initial voltage level (0 volt) while the input is start up from the maximum voltage level, which is 1 volt in the simultaneity. That is fully proofed the circuit is properly worked and correspond to Fractional-order theoretical with we depicted to verify the $s^{0.5}$ order and used this technique to be a part in filter design.

III. THE FRACTIONAL-ORDER LOW PASS FILTERS

The Fractional of low pass filters classical transfer function [4] is in equation (7).

$$T_{FLPF}(s) = \frac{d}{s^{\alpha} + a} \tag{7}$$

The magnitude and phase of classical fractional transfer functions are describes in equation (8) and (9).

$$|T_{FLPF}(j\omega)| = \frac{d}{\sqrt{\omega^{2\alpha} + 2a\cos\left(\frac{\alpha\pi}{2}\right) + a^2}} \tag{8}$$

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$$\angle T_{FLPF}\left(j\omega\right) = -\tan^{-1}\frac{\omega^{\alpha}\sin\left(\frac{\alpha\pi}{2}\right)}{\omega^{\alpha}\cos\left(\frac{\alpha\pi}{2}\right)+a} \tag{9}$$

The important critical frequencies for this FLPF are found $\omega_m = \omega_o(-\cos(\alpha\pi/2))^{\frac{1}{\alpha}}$, $\omega_{rp} = \omega_o/(-\cos(\alpha\pi/2))^{\frac{1}{\alpha}}$, and $\omega_h = \omega_o(\sqrt{1+\cos^2(\alpha\pi/2)} - \cos(\alpha\pi/2))^{\frac{1}{\alpha}}$, From these expressions it is seen that both ω_m and ω_h exist only if $\alpha > 1$.

- ω_m is the frequency at which the magnitude response has a maximum or a minimum.
- ω_h is the half-power frequency at which the power drops to half the pass band power.
- ω_{rp} is the right-phase frequency at which the phase $\angle T_{FLPF}\left(j\omega_{rp}\right) = \pm\pi/2$.

IV. Realization Tunable OTA Low Pass Filter with the Fractional Order Step

A. *Circuit Design*

To realize the transfer function to correspond with the Fractional-order theoretical as shown in equation (7) that used single operation transconductance amplifier and CFA. The circuit for simulation and verify are shown in Figure 4.

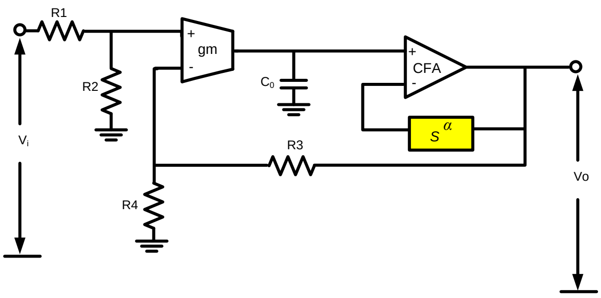


Figure 4. OTA LT1228 circuit used in the approximation to the Fractional low pass filter o order $(n+\alpha)$

The Low Pass Filter circuit in Figure 4, consists of transconductance amplifier and CFA (Current Feedback Amplifier).This filter used an integer order approximation of s^{α} and built an integer order filter that demonstrates the fractional step through the stop band**.** The CFA has designed to operate with the fractance circuit for finite gain bandwidth of filter and help for low output impedance capability. According to specification of IC the tranconductance can be configured by *Iset* .We replaced the fractance circuit on a negative feedback of CFA. The purpose of replacement to current feedback control gain of buffering and also backward feedback control to the input (Vin^{-}) of OTA which is proportional of control current output follows equation (10).

$$I_o = gm(Vin^{+} - Vin^{-}) \tag{10}$$

As the depiction if controllable the gain of amplifier and gain of buffer CFA by Fractional-order behavior that can be reduced error from gain amplifier, increasing gain with slope of 20 dB/dec and reduced the peaking of filter at the interval cut-off frequency.

B. *Simulation and Experimental results*

The transfer function in Figure 4, can be derived from KCL and node methods then yield approximate transfer function is.

$$T_{(S)} = \frac{Vo}{Vin} = \frac{\frac{gm}{C_0}\frac{R_2}{R_1+R_2}}{S^{(1+\alpha)}\left[\frac{R_3^2}{R_3+R_4}\right]+\frac{gm}{C_0}\frac{R_4}{R_3+R_4}S+1} \tag{11}$$

In the data sheet of IC the transconductance calculates from equation (12)

$$gm = h.Iset \tag{12}$$

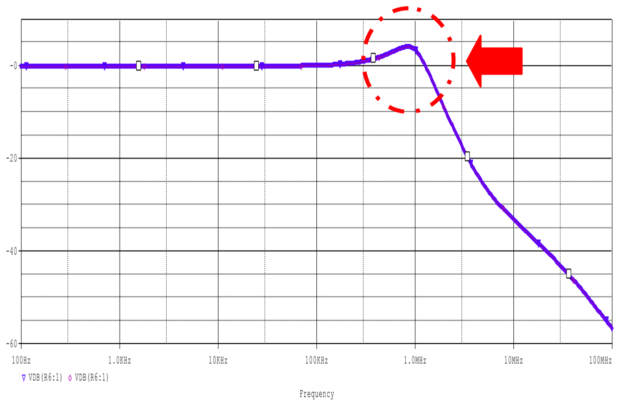
In equation (12) h = 8.8 because in the datasheet of IC LT1228 [5] the transconductance is specified h =10 for resistor load but this filter has designed with the Fractance load which is a virtual of capacitor, so h = 8.8.

The CFA can be used as the voltage follower and voltage of V_{C0} to supply the current input of the CFA then output had also feedback to the input of OTA (negative input) help for control gain amplifier of OTA and help for relative stability flat passband. The all of positive effects are from Fractance circuit operations. The value of *Iset* for filter can be configured as following

$$f_P \in 1\ \mathrm{MH_z}\ ,\quad Iset = 1.14\ \mathrm{mA}$$

C. *Simulation filter circuit without the Fractional-order step*

We have verified the filter before implement the Fractional-order step by used C_0 = 47pF, the CFA gain = 1, and *Iset* = 1.14 mA.



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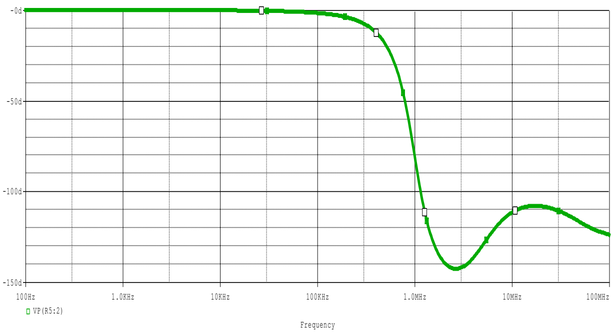

Figure 5. Magnitude and Phase Bode plots response

In Figure 5, we found on the magnitude response at the cut-off frequency of filter appears the peaking problem and found slope is not satisfied then analyzed and assume the peaking occurred from the steady-state error of amplifier that is the point why needed to improve the weakness.

D. Implement the Fractional-order step

We have implemented the Fractional order step $(1+\alpha)$ on the circuit in Figure 4, for the cases α =0.1, 0.5 and 0.9.The results in all pole frequency values and method of experimental same as item C.

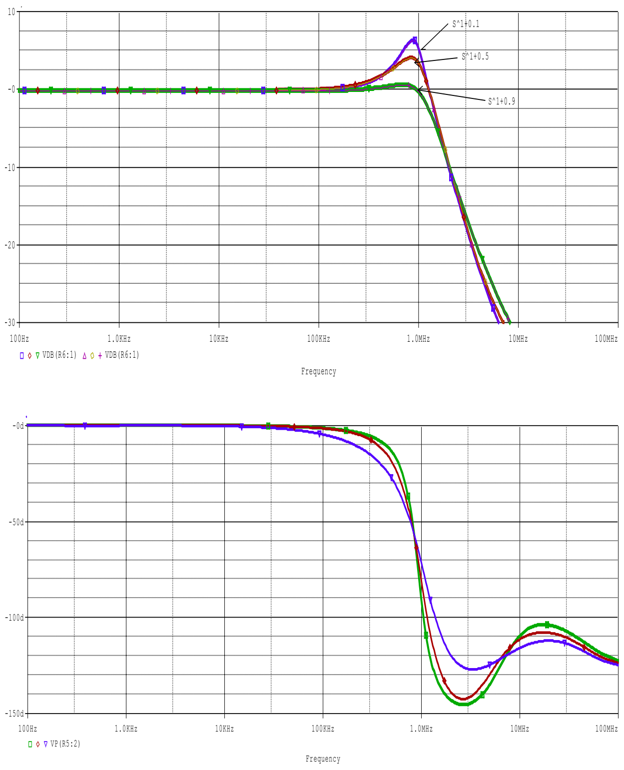

Figure 6. Magnitude and Phase response of filter after implemented the Fraction Order step, blue line is α =0.1, red line is α =0.5 and green line is α =0.9

In Figure 6, after the circuit has been implemented the Fractional-order step α =0.1, 0.5 and 0.9 to verify the suspicion .The step response of α =0.9 order. The result shown very stability and can be reduced the peaking at cut-off frequency.

Comparison Magnitude response between Figure 5, Vs Figure 6, found in the Figure 6, the slope of filter is satisfied. It comparable to high order filters which it is used more active components. The simulation results are proved that the integer order filter could accurately approximate the function and reduced the peaking problem.

V. CONCLUSION

The tunable OTA Low Pass Filter with the Fractional-order step is designed by used only single operation transconductance amplifier and Fractance circuits, The results of simulation that the tunable by Fractional-order step technique can be reduced the peaking problem at the cut-off frequency and maintaining the flat passband of magnitude response and during the experiment found some positive effect on slope of filter is stability than without uncontrolled (CFA gain =1). This work has designed for the high frequency and result of simulations are correspond to the theoretical anticipation. It fully proved the Fractional-order step can be applied to improve the performance of filters at high frequency. This tunable technique can be applied to the multifunction filters, phase lock loop circuit and instrument application circuit.

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