

Basic Components for Quantum Processor Working and their Result

Deepak Patel

Department of Computer Science and Engineering
Millennium Institute of Technology
Bhopal, India
deepaktest.patel@gmail.com

Fozia Z. Haque

Department of Physics
Maulana Azad National Institute of Technology
Bhopal, India
fozia.zia@rediffmail.com

Abstract—The ideas from classical information theory are carried over to quantum theory there emerges a revolution in our ability to process information. The very basic ways of expressing and manipulating information require physical states and processes. In quantum theory we know that the physical processes are fundamentally different than those of classical physics. Therefore manipulation of information based on quantum physical processes has to be also fundamentally different than their classical counterparts. It is this urge to understand what we can do with the new ways of expressing information, which has led to several surprising discoveries in last two decades or so. This research has discovered the possibility for an entirely new type of computer, one that operates according to the laws of quantum physics — a quantum computer. This paper include how qubit can replace bit, basic components for Quantum computer Processor they are quantum gates, there combination to make adder, subtractor. Each component is tested on QCAD simulator. Results of components test is showed in input and output tabular form.

Keywords— Adder, Bit, Quantum Gates, Qubit, Subtractor

1.1 INTRODUCTION

With the development of technology, tremendous change has been observed in computer architectures. In addition computers with different machine architectures can be observed at any given time. To make them useful mathematicians have to conceptualize and abstract all these superficial architectural differences away and also to focus on the factors that really composes computation. Moore's law, which predicts that the number of devices integrated on a chip will double every 18 months. Adherence to this exponential growth curve has been a monumental task requiring rapid improvements in all aspects of integrated circuit ~IC fabrication, to permit manufacturers to both shrink the size of devices and increase chip size while maintaining acceptable yields. quantum computing known to be an emerging technology and different computations have been performed during which operation based on quantum computations were executed on a small no. of qubits.

Quantum computer is a computation device that incorporates quantum mechanical phenomena distinctively to perform Operation on data in terms of superposition & entanglement. In contrast to traditional computer where data is represented by means of bits, Q.C makes use of qubits for data representation.

Qubit generalizes the concept of a bit since it may not only attain two values $|0\rangle$ and $|1\rangle$, say, but also “superpositions” of them, i.e., vector sums like $|0\rangle + |1\rangle$ or $|0\rangle - |1\rangle$. This is a radically new logical property, totally unknown in classical information processing. It enables new kinds of logical gates and an algorithm, working on quantum registers, ordered collections of several Qubit s. Bennett's theorem suggests that every future binary technology will have to use some kind of reversible gates in order to reduce power dissipation. This is also true for multiple-valued logic, which demonstrates several potential advantages over binary technology. Quantum technology is inherently reversible and is one of the most promising technologies for future computing systems

This paper is outlined as follows. *Section I* provides Qubit and superposition power. *Section II* provides the introduction to classical logic Gates there working and truth table. *Section III* provides different type of Quantum Gates there matrix Experimental Tool and Result obtained by QCAD. *Section IV* provides quantum gate reversibility, benefits and some drawback of quantum computer. *Section V* provides the conclusion of highlighted issues.

1.2 Qubit

The first papers on quantum models for computation were published in the 1980s. Similar to the research into reversible models, the motivation was mostly academic at the time, an exploration of the ultimate limits of computation. The real payoff for quantum computing did not come until 1994, when Shor announced his quantum algorithm for factoring large numbers with efficiency unparallel by any classical algorithm preceding. The factoring problem is used widely to encrypt messages in public key cryptography, which made the feasibility of quantum computing an urgent issue in the years to follow.

Memory of conventional computer is made up of bits and is used to hold either binary digit 1 or 0. The machine manipulates those bits by means of transferring from logic gates to memory and vice versa where as in Q.C, memory makes use of holding data in terms of qubits i.e. 1 or 0 or critically a superposition of these. Quantum Computing Notation Quantum Computing is best described in terms of linear algebra, where Dirac's bra-ket notation is usually used:

A. The primary objects of quantum computing are vectors and matrices of a Hilbert space (vector space with inner product) over the complex numbers. B: Vectors are written as bras such as $\langle \Phi |$ and kets such as $|\Psi\rangle$. C: $|\Psi\rangle$ corresponds to a normal (vertical) vector. D: $\langle \Phi |$ corresponds to transposed (horizontal) and complex conjugated vectors. The memory units of a classical discrete computer are usually taken as bits, two-valued classical variables, as we saw earlier. By analogy, we consider a two-valued quantum variable, called a Qubit, which may be simulated by a two-level quantum system. Suppose the levels, or eigenstates of the quantum variable, are labeled $|0\rangle$ and $|1\rangle$. This has a direct correspondence with the discrete states of a classical bit, 0 and 1. However, a Qubit is a quantum state, and as such can be in a superposition state also. That is, in addition to $|0\rangle$ and $|1\rangle$, a Qubit can exist more generally in the state, $c_0|0\rangle + c_1|1\rangle$, where c_0 and c_1 are complex coefficients normalized to 1. This is the main distinction between classical and quantum memory, in that even though a Qubit has discrete eigenstates, there is something analog about it also in the continuous range of superposition states that it can take on. In fact, there are really an infinite number of states possible to a Qubit, not just two, although only two of these are orthogonal or linearly independent.

1.3 Superposition Power

Consider conventional computing device operated on three bits register in which bits in the register at any given time pertains to definite state like 1-0-1 but in Q.C, qubits may be in a state of all the allowed classical states. In general, the register is described by the following wave function: $|\psi\rangle = a|000\rangle + b|001\rangle + c|010\rangle + d|100\rangle + e|110\rangle + f|011\rangle + g|101\rangle + h|111\rangle$ Where a, b, c are coefficients for complex numbers having probabilities expressed in terms of amplitudes squares for measuring each state qubits [2].

To record the register state having n qubits quantum register (Q.R), it requires 2^n complex numbers. For instance quantum register with 3 qubits requires $2^3=8$ numbers. It is concluded that no. of encoded classical states in Q.R exponentially grows with corresponding numbers of qubits.

2.1 Logic Gates

The output of a NOT gate is the opposite of its input. If the input is 1, then the output is 0, and if the input is 0, the output is 1 table (1). NOT gates can have only one input and one output. A NOT gate is also known as an inverter.

The output of an OR gate is 1 if any input is 1, otherwise the output is 0 table (2). An OR gate have two or more inputs and one output.

The output of an AND gate is 1 if ALL inputs are 1, otherwise the output is 0 table (3). An AND gate have two or more inputs and one output.

The *NAND gate* operates as an AND gate followed by a NOT gate. It acts in the manner of the logical operation AND followed by negation table (4). The output is 0 if both inputs are 1. Otherwise, the output is 1. Universal Classical gates are NAND, NOR because with the help of these gates we can design any gate.

Table 1
NOT GATE

Input	Output
a1	a3=Not(a1)
0	1
1	0

Table 2
OR GATE

Input	Output
a1 a2	a3= (a1 OR a2)
0 0	0
0 1	1
1 0	1
1 1	1

Table 3
AND GATE

Input	Output
a1 a2	a3= (a1 AND a2)
0 0	0
0 1	0
1 0	0
1 1	1

Table 4
NAND GATE

Input	Output
a1 a2	a3= (a1 NAND a2)
0 0	1
0 1	1
1 0	1
1 1	0

Table 5
XOR GATE

Input	Output
a1 a2	a3= (a1 Xor a2)
0 0	0
0 1	1
1 0	1
1 1	0

In above tables a1 and a2 act as input while a3 act as output.

3.1 Quantum Gates

Quantum gates are the basic building blocks of quantum circuits.

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Quantum gates are the basic building blocks of quantum circuits.

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (1)$$

Pauli X-Gate: It also performs operation on single Qubit like Not gate in classical computer Fig (1). By operating this gate on Qubit it make an opposite state table (6).

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad (2)$$

There is many other quantum gates such as Pauli Y-Gate, Pauli Z-Gate, Hadamard gate, Square root of Not gate, Mirror of Square root of Not gate [1].

All above gates are operate on single Qubit, now we have some more gates that need more than one Qubit [1]. Let we have two Qubit then representation of those Qubit in matrix of 4x1, gate also consist of matrix of 4x4.

Toffoli Gate: This gate is also know as controlled NOT gate, it consist of two Qubit fig. (2). One Qubit act as control Qubit and other act as target Qubit table(7). Let first Qubit act as control Qubit and its state is 0 then target qubit state not change.

$$\begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \quad (3)$$

Let first Qubit act as control Qubit and its state is 1 then target qubit state change.

$$\begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad (4)$$

Control-Control Not gate: This gate consists of two control Qubit and one target bit. In this case also if both control Qubit

Has state 1 than target value get change, otherwise we get same state of target Qubit table (8).

$$\begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} \quad (5)$$

3.2 .Experimental Tool and Result

For performing quantum gate operation we have use QCAD simulator. This software was developed at Nagoya University and the University of Tokyo [8].

First we perform simple operation of NOT gate on a Qubit which is perform by quantum Pauli-X gate then it give same truth table as of Classical NOT gate.



Fig.1. In above circuit we have represent Q1 as Qubit while |1> represent state of Qubit and X rectangle box represent pauli-x gate, which perform Not operation and change current state of Qubit .

TABLE 6
PAULI-X GATE (NOT)

Input	Output
Q1	Pauli-X(Q1)
0	1
1	0

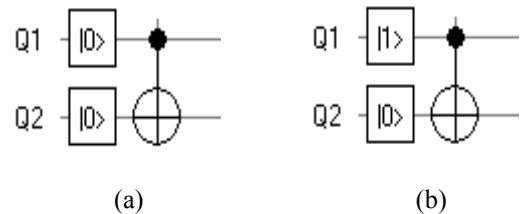


Fig.2. In above circuit Q1 Qubit act as Control Qubit and Q2 act as target Qubit, this is control by Q1.

TABLE 7
TOFFOLI GATE (C-NOT)

Input	Output
-------	--------

Q1	Q2	Q1	Q2
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	0

From above table we observe that when Q1 state is 0, state of Q2 not change while, when Q1 state is 1 than state of Q2 change.

Now we perform AND operation with the help of CC-NOT gate (Control Control Not gate) which consist of three Qubit, two Qubit act as control Qubit and third Qubit act as target Qubit .

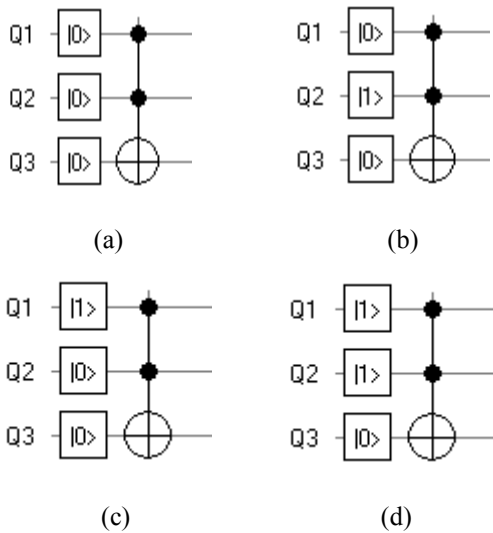


Fig.3. Here Q1, Q2 act as Control Qubit, while Q3 act as Target Qubit, which is control by Q1, Q2.

TABLE 8 CONTROL CONTROL NOT (AND)					
Input			Output		
Q1	Q2	Q3	Q1	Q2	Q3
0	0	0	0	0	0
0	1	0	0	1	0
1	0	0	1	0	0
1	1	0	1	1	1

From above table we observe that by adding one more control Qubit to C-NOT gate we obtain CC-NOT. If state of target Qubit is always set to 0, then CC-NOT act as AND gate of Classical computer.

Similar to classical OR gate we can perform OR operation by combining two Control Not gate and one Controlled-Controlled Not gate in Quantum computer fig(4).

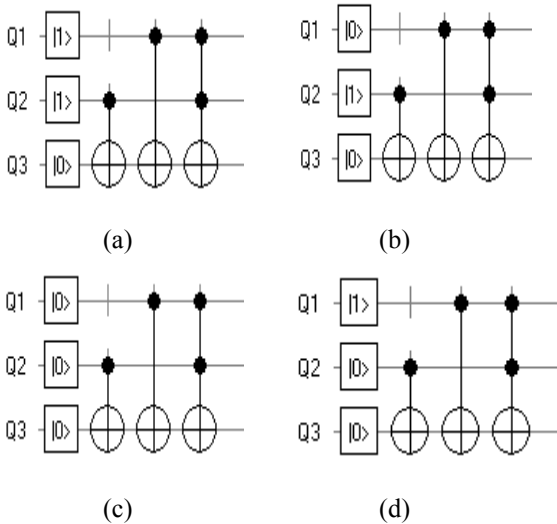


Fig.4. Here Q1, Q2 act as Control Qubit, while Q3 act as Target Qubit, which is control by Q1, Q2.

TABLE 9 (OR GATE)					
Input			Output		
Q1	Q2	Q3	Q1	Q2	Q3
0	0	0	0	0	0
0	1	0	0	1	1
1	0	0	1	0	1
1	1	0	1	1	1

From above table we observe that by combining two ControllNot and one Controlled-Controlled Not gate we can perform OR operation, similar to classical gate.

Similar to classical XOR gate we can perform XOR operation by combining two Control Not gate in Quantum computer fig.(5).

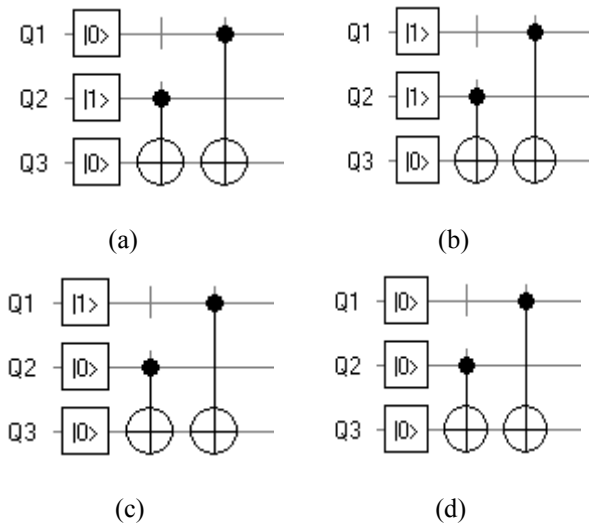


Fig.5. Here Q1, Q2 act as Control Qubit, while Q3 act as Target Qubit, which is control by Q1, Q2.

TABLE 10
(XOR GATE)

Input			Output		
Q1	Q2	Q3	Q1	Q2	Q3
0	0	0	0	0	0
0	1	0	0	1	1
1	0	0	1	0	1
1	1	0	1	1	0

From above table we observe that by combining two Controlled Not we can perform XOR operation, similar to classical gate.

Now let us make some more modification to above gate input parameter then we obtain Classical NAND gate. Circuit for this is as follow:

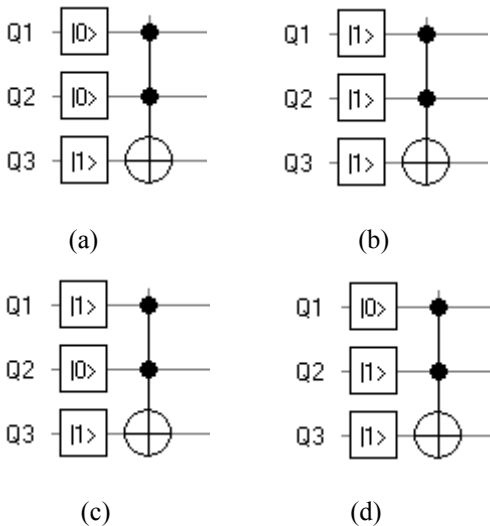


Fig.6. In above circuit Q1, Q2 act as control Qubit while Q3 act as target Qubit In circuit for different inputs we have notice that Q3 Qubit have remain in one state only, that is always $|1\rangle$ [8].

TABLE 11
CONTROL CONTROL NOT (NAND)

Input			Output		
Q1	Q2	Q3	Q1	Q2	Q3
0	0	1	0	0	1
0	1	1	0	1	1
1	0	1	1	0	1
1	1	1	1	1	0

From above table if we check only Q1, Q2 in input column then it act as input, while Q3 in output column act as output to Classical NAND gate.

We can make adder by using above quantum gates which can also produce result like classical gate[6]. Circuit given below use six Controlled Not gate, three Controlled-Controlled not gate, eight Qubit out of which two qubit act as input and one act as carry. While other Qubit is use to produce addition of input Qubit and their corresponding carry.

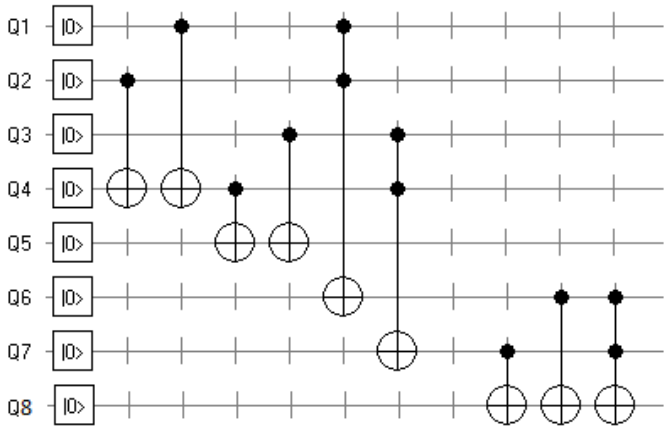


Fig.7. In above circuit Q1, Q2 act as input Qubit while Q3 act as carry Qubit which always set to $|0\rangle$. While Q4, Q5, Q6, Q7, Q8 is use to make this addition.

TABLE 12
(ADDER)

Input								Output				
Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q1	Q2	Q3	Q5	Q8
0	0	0	0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	1	0	1	0
1	0	0	0	0	0	0	0	1	0	0	1	0
1	1	0	0	0	0	0	0	1	1	0	1	1

Above circuit is act as Adder of two Qubit. We get sum at Q5 and carry at Q8. Q1, Q2 and Q3 remain unchanged during operation which are input and carry. Above unit is basic for two Qubit additions, for adding more Qubit we can repeat this unit, while forwarding carry to other unit.

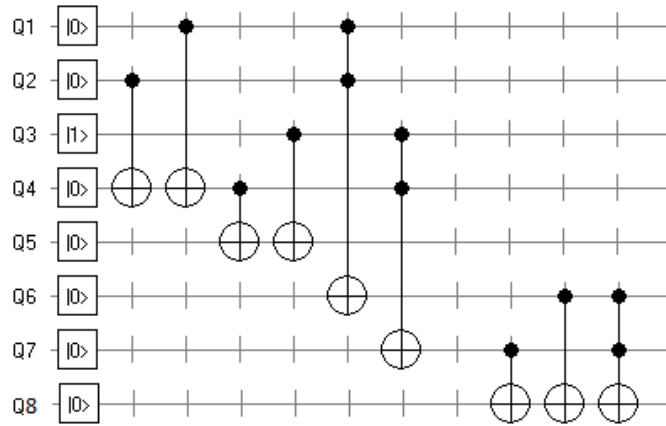


Fig.8. In above circuit Q1, Q2 act as input Qubit while Q3 act as borrow Qubit which always set to $|1\rangle$. While Q4, Q5, Q6, Q7, Q8 is use to make this addition.

TABLE 13
(SUBTRACTOR)

Input								Output				
Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q1	Q2	Q3	Q5	Q8
0	0	1	0	0	0	0	0	0	0	1	0	0
0	1	1	0	0	0	0	0	0	1	1	1	1
1	0	1	0	0	0	0	0	1	0	1	1	0
1	1	1	0	0	0	0	0	1	1	1	0	0

In case of subtractor we can use same unit the only difference is that initial unit always carry $|1\rangle$. We get difference at Q5 and borrow at Q8. Q1, Q2 and Q3 remain unchanged during operation which are input and borrow. Rest working is same as adder. So same unit act as adder as well as subtractor.

4. Benefits and Drawback

Benefits:

- 1). Quantum gates are much smaller in size then classical gates.
- 2). Classical gate need bit which can be store in FET (Field Effect Transistor) use semiconductor, but Qubit can be store in Quantum dot or photon [5].
- 3). At a time bit is in either 0 or 1 state. But in Qubit it can be in 0, 1, in-between (0, 1) state.
- 4). There calculation is faster due to superposition and Quantum physics.
- 5). As Quantum gates are reversible so the heat produce is also less.

Drawback:

The major Drawback of quantum computer is that we cannot read Qubit state while it is in superposition state this is entanglement [4], some error correcting algorithm for reading and correcting data has been developed.

5. Conclusion

Quantum gates can work as classical gates if we consider its two state $|0\rangle$ or $|1\rangle$. Showing NOT, AND, OR, NAND, XOR gate table by applying quantum gate. As NAND gate has been prepared which is universal gate, so we can make any classical circuit in quantum manner. With the help of Quantum gate we can make adder as well as subtractor in same unit by just changing carry Qubit state.

References

- [1] Andrzej Buller "Quantrix: Toward Automated Synthesis of Quantum Cascades" ATR Human Information Processing Laboratories, Kyoto 2003.03.15.
- [2] Ashok Muthukrishnan "An Introduction to Quantum Computing" Quantum Information Seminar Friday, Sep. 3, 1999.
- [3] G. L. Snider, A. O. Orlov, I. Amlani, X. Zuo, G. H. Bernstein, C. S. Lent, J. L. Merz, and W. Porod "Quantum-dot cellular automata:

Review and recent experiments .invited." journal of applied physics volume 85, number 8 15 APRIL 1999.

- [4] J. M. TAYLOR^{1*}, H.-A. ENGEL¹, W. DU², R.2, A. YACOBY³, C. M. MARCUS¹, P. ZOLLER² AND M. D. LUKIN¹ "Fault-tolerant architecture for quantum computation using electrically controlled semiconductor spins" Published online: 1 December 2005; doi: 10.1038/nphys174 .
- [5] Mark Friesen,^{1,2,*} Paul Rugheimer,² Donald E. Savage,² Max G. Lagally,^{1,2} Daniel W. van der Weide,³ Robert Joynt,¹ and Mark A. Eriksson^{1,†} "Practical design and simulation of silicon-based quantum-dot qubits" PHYSICAL REVIEW B 67, 121301(R) 2003.
- [6] *Majid Mohammadi, 1Mohammad Eshghi, 2Majid Haghparast and Abbas Bahrololoom* "Design and Optimization of Reversible BCD Adder/Subtractor Circuit for Quantum and Nanotechnology Based Systems". World Applied Sciences Journal 4 (6): 787-792, 2008 ISSN 1818-4952
- [7] S. Dasgupta, C. H. Papadimitriou, and U. V. Vazirani "Algorithms" July 18, 2006. Available: <http://www.mhhe.com/dasgupta>.
- [8] QCAD Software Project FY2002 (IPA, Japan). This software was developed at Nagoya University and the University of Tokyo.