Comparision of Exponential Companding Transform and CB-ACE Algorithm for PAPR Reduction in OFDM Signal

Neelam Dewangan ,Suchita Chatterjee and Mangal Singh, Chhatrapati Shivaji Institute of Technology,Durg.
email- neelamdewangan@csitdurg.in,suchitachatterjee@csitdurg.in , mangalsingh@csitdurg.in

***Abstract*—One of the main disadvantages of Orthogonal Frequency Division Multiplexing (OFDM) is its high peak-to-average power ratio (PAPR). As the simplest approach to reducing the PAPR, Clipping based Active Contellation Extention (CB-ACE) exhibits good practicability, and the repeated clipping-and-filtering (RCF) algorithm proposed by Jean Armstrong provides a good performance in PAPR reduction and out-of-band power's filtering. However, its way of filtering in frequency-domain requires RCF operations to control the peak regrowth, which degrades the bit error rate (BER) performance and greatly increases the computational complexity. Therefore, this paper put forward comparision of two existing techniques namely Exponential Companding Transform and CB-ACE Algorithm. The simulation results show that, exponential Companding Transform gives better result for PAPR Reduction and provides low complexity in Algorithm.**

Keywords- CB-ACE, Exponential Companding Transform, OFDM, PAPR, RCF

I. INTRODUCTION

As a promising technique, OFDM has been widely used in many new and emerging broadband communication systems, such as digital audio broadcasting (DAB), high-definition television (HDTV), wireless local area network (IEEE 802.11a and HIPERLAN/2). However, as the OFDM signals are the sum of signals with random amplitude and phase, they are likely to have large PAPR that requires a linear high-power- amplifier (HPA) with an extremely high dynamic range, which is expensive and inefficient. Furthermore, any amplifier nonlinearity causes intermodulation products resulting in unwanted out-of-band power.

A number of approaches have been proposed to deal with the PAPR problem, including clipping, clipping-and-filtering (CF), coding, companding transform, active constellation extension (ACE), selected mapping (SLM), partial transmit sequence (PTS), and so on [1]. Compared with other methods, clipping is the simplest and of good practicality. In particular, Jean Armstrong has proposed a RCF Algorithm which is also called Clipping Based Active Constellation Extention, which dramatically reduces the PAPR and limits the out-of band power to a low level , but excessively increases the computational complexity as well. Based on Jean Armstrong's method, this paper describes an improved approach which can provide good performance and lower complexity.

II. DEFINITION OF OFDM SIGNALS AND PAPR

In OFDM, a block of N symbols, $\{X_k$, k=0, 1, … , N-1$\}$, is formed with each symbol modulating one of a set of subcarriers, $\{f_n, n = 0, 1, \ldots\ldots, N-1\}$ with equal frequency separation 1/T, where T is the original symbol period. An inverse discrete Fourier transform (IDFT) can efficiently generate the multicarrier symbols. The IDFT of vector $X[k] = [X_0, X_1, \ldots\ldots X_{N-1}]$ results in T/N spaced discrete time signal$x[n] = [x_0, x_1, \ldots\ldots x_{N-1}]^T$. Thus, the transmitted signal is

$$x_n = \frac{1}{\sqrt{N}} \int_{k=0}^{N-1} \exp j\frac{2\pi kn}{N} \qquad 0 \le k \le N-1 \qquad (1)$$

The PAPR of the transmitted signal can be written as

$$PAPR = \frac{\max\limits_{0\le n\le k-1} |x_n|^2}{E\{|x_n|^2\}} \qquad (2)$$

The complementary cumulative distribution function (CCDF) is one of the most frequently used performance measures for PAPR reduction techniques, which denotes the probability that the PAPR of a data block exceeds a given threshold z. The CCDF of the PAPR of a data block of N symbols with Nyquist rate sampling is derived as

$$P(PAPR > z) = 1 - P(PAPR \le z) = 1 - (1 - e^{-z})^N \qquad (3)$$

III. THE CB-ACE ALGORITHM

The basic principle of Clipping-Based Active Constellation Extension (CB-ACE) algorithm involves switching between the time domain and the frequency domain. Filtering and applying the ACE constraint in the frequency domain, after clipping in the time domain, both require iterative processing to suppress the subsequent regrowth of the peak power [3]

The CB-ACE algorithm is first used to clip the peak amplitude of the original Orthogonal Frequency Division Multiplexing (OFDM) signal. The clipping sample obtained after clipping the peak signals, denoted by $C_n^{(i)}$, is given by

$$C_n^{(i)} = \begin{matrix} (|x_n^{(i)}| - A)e^{j\theta_n}, |x_n^{(i)}| > A \\ 0, \qquad otherwise \end{matrix} \qquad (4)$$

where $C_n^{(i)}$ is the Clipping sample of ith iteration, $x_n^{(i)}$ is the oversampled OFDM signal, A is predetermined clipping level

The equation (4) says that the clipping sample is reduced to a value equal to zero when the peak amplitude of the original OFDM signal is less than or equal to the predetermined clipping level, A. If the peak amplitude of the original OFDM signal is greater than the predetermined clipping level, then the clipping sample is given by $\left(\left|x_n^{(i)}\right| - A\right)e^{j\theta_n}$, where the predetermined clipping level is subtracted from the oversampled OFDM signal an is then multiplied by an exponential value [3].

The predetermined clipping level, denoted by A, is related to the target clipping ratio, γ and is given by the equation 5 [3].

$$\gamma = \frac{A^2}{E\{|x_n|^2\}} \qquad (5)$$

Where, γ is the target clipping ratio and A is predetermined clipping level.The clipping of the peak signal results to distortion of the original OFDM signal, namely In-Band Distortion and Out-of-Band Distortion . [3], [4]. The in-band distortion results in the system performance degradation and cannot be reduced, while, the out-of-band distortion can be minimized by filtering the clipped signals. The signal obtained after filtering the clipped signal is given by [3].

$$x^{(i+1)} = x^i + \mu\tilde{c}^{(i)} \qquad (6)$$

where , μ is positive real number (μ varies from 0.1 to 1) and $\tilde{c}^{(i)}$ is the anti-peak signal at the ith iteration given by

$$\tilde{c}^{(i)} = T^{(i)}c^{(i)} \qquad (7)$$

where , $T^{(i)}$ is transfer matrix at the ith iteration which is given by

$$T^{(i)} = \hat{Q}^{*(i)}\hat{Q}^{(i)} \qquad (8)$$

where , $\hat{Q}^{*(i)}$ is conjugate of constellation order and $\hat{Q}^{(i)}$ is the constellation order

Though, the process of filtering completely eliminates the distortions caused by the clipping process, it introduces peak regrowth at some of the peak signals of the OFDM signal. The peak regrowth can be reduced by repeating the filtering process, which may again introduce some distortions. Therefore, the clipping and filtering processes are to be repeated until the peak signals are completely reduced. Hence, the Clipping-Based Active Constellation Extension (CB-ACE) Algorithm is also named as the Repeated Clipping and Filtering (RCF) process [3]

IV. THE EXPONENTIAL COMPANDING TRANSFORM ALGORITHM

The Exponential Companding Transform is also named as the Nonlinear Companding Transform. The idea of companding comes, from the use of companding in Speech Processing. Since, the Orthogonal Frequency Division Multiplexing (OFDM) signal is similar to that of the speech signal, in the sense that large signals occur very infrequently, the same companding technique can be used to improve the OFDM transmission performance .The key idea of the Exponential Companding Transform is to effectively reduce the Peak-to-Average Power Ratio (PAPR) of the transmitted or the companded Orthogonal Frequency Division Multiplexing (OFDM) signals by transforming the statistics of the amplitudes of these signals into uniform distribution. The uniform distribution of the signals can be obtained by compressing the peak signals and expanding the small signals. The process of companding enlarges the amplitudes of the small signals, while the peaks remain unchanged. Therefore, the average power is increased and thus the Peak-to-Average Power Ratio (PAPR) can be reduced .

The Exponential Companding Transform can also eliminate the Out-of-Band Interference (OBI), which is a type of distortion caused by clipping the original OFDM signals. The other advantage of the companding transform is that, it can maintain a constant average power level. The proposed scheme can reduce the PAPR for different modulation formats and sub-carrier sizes without increasing the system complexity and signal bandwidth. The Exponential Companding Transform also causes less spectrum side-lobes.

The original Orthogonal Frequency Division Multiplexing (OFDM) signal is converted into the companded signal by using the Exponential Companding Transform. The companded signal obtained by using the Exponential or Nonlinear Companding Transform is given by the equation 9

$$H(x) = sgn\,(x)\sqrt[d]{\alpha\left[1 - exp\left(-\frac{x^2}{\sigma^2}\right)\right]} \qquad (9)$$

Where, h(x) – Companded Signal obtained by Exponential Companding Transform, $sgn\,(x)$ -sign Function, $\alpha -$ Average Power of Output Signals, x-original OFDM signal.The average power of the output signals, denoted by α, is required in order to maintain the average amplitude of both the input and output signals at the same level. The average power of the output signals is given by the equation 10.

$$\alpha = \frac{E\left[|S_n|^2\right]}{E\left[\sqrt[d]{\left[1-exp\left(-\frac{|S_n|^2}{\sigma^2}\right)\right]^2}\right]} \qquad (10)$$

Where, α – Average Power of Output Signals

d – Power of the amplitude of the Companded Signal

V. SIMULATION RESULTS

The Peak-to-Average Power Ratio (PAPR) of the original Orthogonal Frequency Division Multiplexing (OFDM) signal i.e., the PAPR is to be calculated by using the equations (1), (2) and (3).

From the Figure 1, the Peak-to-Average Power Ratio (PAPR) of the original Orthogonal Frequency Division Multiplexing (OFDM) signal is equal to 11.8 dB with a Complimentary Cumulative Distribution Function (CCDF) of 10^{-2} or 0.01.The Peak-to-Average Power Ratio (PAPR) of the original Orthogonal Frequency Division Multiplexing (OFDM) signal is very high, which is evident from the Screen Shot 2.1. The high PAPR results to the increase in the complexity of the Analog-to-Digital Convertors (ADCs) and Digital-to-Analog Convertors (DACs), also reduces the efficiency of the power amplifiers.

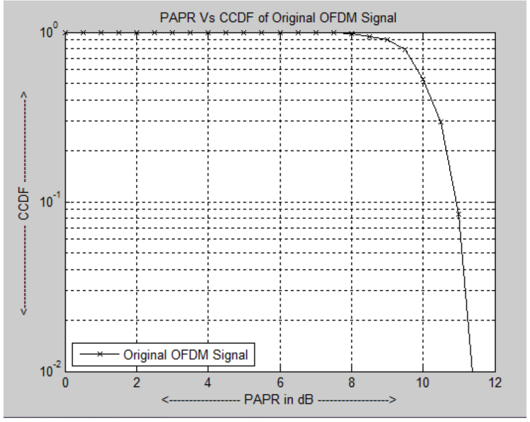

Figure1 – PAPR Vs CCDF of Original OFDM Signal

The Peak-to-Average Power Ratio (PAPR) by the Clipping-Based Active Constellation Extension (CB-ACE) algorithm is to be calculated for the Orthogonal Frequency Division Multiplexing (OFDM) signal which is obtained after filtering the clipped signal i.e., the PAPR is to be calculated for the equation (5) by using the equations (1), (2) and (3).

The Complimentary Cumulative Distribution Function (CCDF) by the Clipping-Based Active Constellation Extension (CB-ACE) algorithm is to be calculated for the Orthogonal Frequency Division Multiplexing (OFDM) signal which is obtained after filtering the clipped OFDM signal.

From the Figure 2, the Peak-to-Average Power Ratio (PAPR) of the Orthogonal Frequency Division Multiplexing (OFDM) signal obtained by using the Clipping-Based Active Constellation Extension (CB-ACE) algorithm is equal to 10 dB, 8.5 dB and 8.0 dB for the target clipping ratios of 0 dB, 2 dB and 4 dB respectively with a Complimentary Cumulative Distribution Function (CCDF) of 10^{-2} or 0.01.

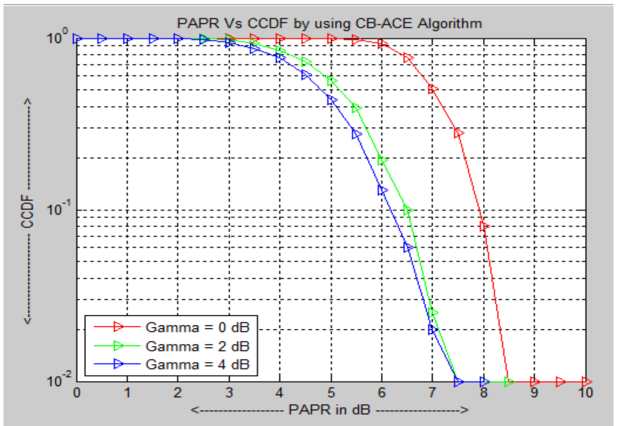

Figure 2 – PAPR Vs CCDF by using CB-ACE Algorithm *(For Different Target Clipping Ratios)*

The Peak-to-Average Power Ratios is increasing as the target clipping ratios is decreasing i.e., minimum PAPR cannot be achieved, when the target clipping level is set below an initially unknown optimum value, which results to low clipping ratio problem.

The other problems faced by the Clipping-Based Active Constellation Extension (CB-ACE) algorithm are Out-of-Band Interference (OBI) and peak regrowth. Here, the Out-of-Band Interference (OBI) is a form of noise or an unwanted signal, which is caused when the original Orthogonal Frequency Division Multiplexing (OFDM) signal is clipped for reducing the peak signals which are outside to the predetermined area and the peak regrowth is obtained after filtering the clipped signal. The peak regrowth results to, increase in the computational time and computational complexity.

From Figure 3, the original Orthogonal Frequency Division Multiplexing (OFDM) signal is companded i.e., the peak signals of the OFDM signal are compressed and the small signals of the OFDM signal are expanded by using the Exponential Companding Transform for different powers of the amplitude of the companded signals i.e., for d = 1, d = 2 and d = 3.

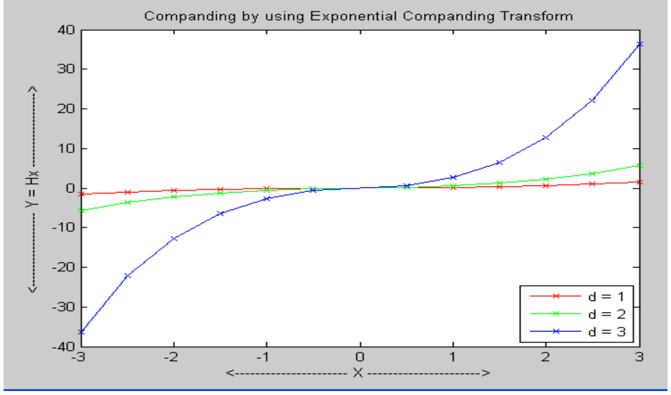

Figure 3 – Companding of OFDM Signal by using Exponential Companding Transform

The Peak-to-Average Power Ratio by using the Exponential Companding Transform is to be calculated for the Orthogonal Frequency Division Multiplexing (OFDM) signal which is obtained after compressing the peak signals and expanding the small signals i.e., PAPR is to be calculated for the equation (9) by using the equations (1), (2) and (3).

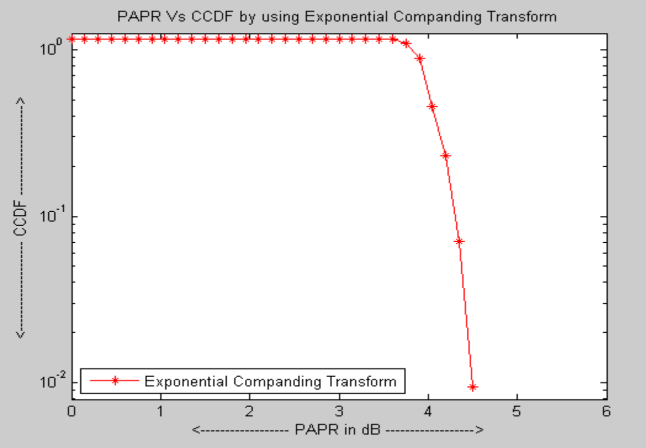

Figure 4 – PAPR Vs CCDF by using Exponential Companding Transform

From the Screen Shot 3.4, the Peak-to-Average Power Ratio (PAPR) of the Orthogonal Frequency Division Multiplexing (OFDM) signals obtained by using the Exponential Companding Transform is reduced to 4.5 dB with a Complimentary Cumulative Distribution Function (CCDF) of 10^{-2} or 0.01

Table 5.1 – Comparison of PAPR (in dB) and CCDF for different techniques

Different Techniques	PAPR (in dB)	CCDF
Original OFDM Signal	11.8	10^{-2} or 0.01
Clipping-Based Active Constellation Extension (CB-ACE) Algorithm	10.0 (For $\gamma = 0$ dB) 8.5 (For $\gamma = 2$ dB) 8.0 ((For $\gamma = 4$ dB)	10^{-2} or 0.01
Exponential Companding Transform	4.5	10^{-2} or 0.01

From the table 5.1, the Peak-to-Average Power Ratio of the Orthogonal Frequency Division Multiplexing systems is reduced or minimized by using the existing methods namely Clipping-Based Active Constellation Extension (CB-ACE) and the Exponential Companding Transform Algorithm at a Complimentary Cumulative Distribution Function of 10^{-2} or 0.01.

VI. CONCLUSIONS

The Clipping-Based Active Constellation Extension (CB-ACE) Algorithm reduces the high Peak-to-Average Power Ratio (PAPR) by clipping and filtering the original OFDM signal. The CB-ACE Algorithm results to peak regrowth, Out-of-Band Interference (OBI), low clipping ratio problem, increase in the Bit Error Rate (BER) and decrease in the Signal-to-Noise Ratio (SNR).

The Exponential Companding Transform improves the Bit Error Rate (BER) and minimizes the Out-of-Band Interference (OBI) in the process of reducing the Peak-to-Average Power Ratio (PAPR) effectively by compressing the peak signals and expanding the small signals. The improved BER transmits the data via a transmission channel with fewer errors, while the minimized OBI reduces the effects caused by clipping.

Hence, by reducing the Peak-to-Average Power Ratio (PAPR), the complexity of the Analog-to-Digital Converter (ADC) and Digital-to-Analog Converter (DAC) can be reduced. The reduced Peak-to-Average Power Ratio (PAPR) also increases the efficiency of the Power Amplifiers.

REFERENCES

[1] Seung Hee Han, Jae Hong Lee. An overview of peak-to-average power ratio reduction techniques for multicarrier transmission. Wireless Communications, IEEE, Vol.12, Issue 2, pp.56–65, April, 2005

[2] J. Armstrong. New OFDM Peak-to-Average Power Reduction Scheme. IEEE cnf vehicular technology, Vol.1, pp.756-760, May,2001

[3] J. Armstrong. Peak-to-average power reduction for OFDM by repeated clipping and frequency domain filtering. Electronics letters, Vol.38, No.5, pp.246-247, Feb.2002

[4] IEEE Standard, 802.11a. Part 11: Wireless LAN medium access control (MAC) and physical layer (PHY) specifications. IEEE, 1999