IMC Based PID Controller Design for a Jacketed CSTR

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***Abstract*- This paper deals with the operation, mathematical modeling and controller design for the jacketed continuous stirred tank reactor. Ziegler Nichols tuned PID controller is poor in response for Non Linear processes. From the analysis of the process the IMC based controller is suits best for the CSTR.Controller is designed to control the reactant mixture temperature with in the reactor. The scheme is then design and tested using Matlab. The simulations are presented for PID and IMC controllers, and found IMC based Controller is best suits for the CSTR.**

***Key Terms:* Ziegler- Nichols tuning Method, PID Controller, IMC based Controller and CSTR**.

I. Introduction

The aim of this paper is to propose a scheme, using the IMC based controller, to control a CSTR process as shown in Fig (1) [1].The chemical reaction in CSTR is exothermic. In Exothermic reactions heat is evolved. The generated heat increases the temperature with in the reactor. In Continuous stirred tank reactor the several chemical reactants are combined to get the product. The stirrer is used to mix the reactants with in the reactor. The temperature with in the reactor is measured with temperature transducer. To protect the reactor from over temperatures a jacket is placed around the CSTR and the cooling agents i.e. coolants are injected into the jacket to flow around the reactor to decrease the temperature with in the reactor [2]. The temperature of the cooling agents is generally very low.

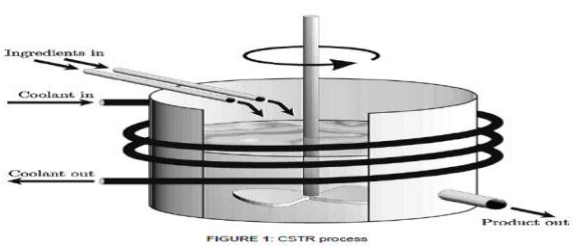

Figure (1). Jacketed Continuous Stirred Tank Reactor

Two types of controllers are designed to monitor and control the temperature with in the reactor i.e. controlled variable by adjusting the cooling agents flow rate i.e. manipulated variable. The Cooling agent flow rate increases the temperature with in the reactor decreases and as the cooling agents flow rate decreases the temperature with in the reactor increases.

This paper documents a CSTR process control simulation, which has been conducted first by using the PID controller and secondly by IMC based Controllers. All simulations were performed using Mat lab package.

II. Mathematical modeling of a Jacketed CSTR

It is a simple exothermic reaction occurred in the reactor, this is cooled by a coolant that flows in the jacket around the reactor. The dynamics of the reacting mixture depends on the Mass of the reactants and energy of reactants within the reactor. The reactor material balance equation [1] is

$$\frac{d}{dt}(V\rho) = F_{in}\rho_{in} - F_{out}\rho \qquad (1)$$

Where V is the reactor's volume, ρ is the density of reactants and ρ_{in} is the density of the reactant .F_{in} is flow rate of reactant and F_{out} is the flow rate of product. The flow rates are assumed constant. Consider a simple reaction A→B. The balance on component A is

$$V\frac{d}{dt}(C_A) = FC_{AF} - FC_A - V*r_a \qquad (2)$$

Where C_A is the concentration of reactant within the reactor and r_A is the rate of reaction per unit volume. The Arrhenius expression is used for the rate of reaction

$$r_A = k_0 \exp\left(\frac{-E}{RT}\right) C_A \quad (3)$$

Where k_o is the frequency factor, E is the activation energy, R is the ideal gas constant, and T is the reactor temperature.

The reactor energy balance [1], assume constant volume, heat capacity c_p and density and neglect the changes in the kinetic and potential energy is

$$V\rho c_p \frac{d}{dt}(T) = F\rho c_p (T_f - T) + (-\Delta H) V r_A - UA (T - T_j) \quad (4)$$

Where $-\Delta H$ is the heat of reaction, U is the heat transfer co efficient, A is the heat transfer area, T_f is the reactant temperature, and T_j is the coolant temperature in the jacket.

The steady state solution is obtained by equating the derivatives of reactant concentration and reactor temperature set equal to zero.

$$f_1(C_A, T) = \frac{d}{dt}(C_A) = 0 = \frac{F}{V}(C_{AF} - C_A) - r_A \quad (5)$$

$$f_2(C_A, T) = \frac{d}{dt}(T) = 0 = \frac{F}{V}(T_f - T) + \frac{-\Delta H}{\rho c_p} k_0 \exp\left(\frac{-E_a}{RT}\right) C_A - \frac{UA}{V\rho c_p}(T - T_j) \quad (6)$$

III. PID Controller Design

The PID controller transfer function, F (s), is:

$$F(s) = K_p\left(1 + \frac{1}{\tau_i} s + \tau_d s\right) \quad (7)$$

Where K_p the proportional gain of the controller is, $\frac{K_p}{\tau_i}$ is the integral gain of the controller, $K_p \tau_d$ is the differential gain of the controller.

Tuning of a PID Controller

Proportional-integral-derivative i.e. PID controller may be the most widely used controller in the industry for the past decades because of its simplicity and efficiency. How to appropriately tune the gains of PID controller is always an attractive problem. One of the most popular tuning methods is the Ziegler-Nichols method. The attempts to obtain better performance and robustness by improving tuning formulae lead to several achievements, such as Ziegler-Nichols formula. All the tuning formulae needs to know critical gain and critical period for tuning [3]-[4].

Ziegler-Nichols Rules for Tuning of PID Controller*:*

Ziegler-Nichols proposed rules for determining values of the proportional gain K_p, integral time τ_i, and derivative time τ_d based on the transient response characteristics of CSTR. The Ziegler-Nichols PID controller is

$$u(t) = K_p\, e(t) + K_i \int e(t)dt + K_d \frac{d}{dt}(e(t)) \quad (8)$$

In the Ziegler-Nichols method, first set τ_i as infinite and τ_d equal to zero. Use the proportional control action only, increase K_p from zero to a critical value K_u at which the output first exhibits sustained oscillation. Thus, the critical gain K_u and corresponding period i.e. critical period T_u are determined. By using Ziegler and Nichols the values of the parameters K_p, τ_i and τ_d according to the formula shown in Table.1 [4].

Table1 is the Ziegler-Nichols Tuning Rule Based on Critical K_u gain and Critical period T_u

TABLE (1). PID Controller Settings

Controller	K_p	τ_i	τ_d
P	$0.5K_u$	∞	0
PI	$0.45K_u$	$0.8T_u$	0
PID	$0.6\ K_u$	$0.5\ T_u$	$0.12T_u$

IV. IMC Controller

The Ziegler- Nichols method does not require a model of the process. The IMC uses a model based procedure, where a process model is embedded in the controller. For CSTR, a model of process decides heat flow that needs to be added to the

process to obtain a desired temperature trajectory, specified by the set point. A steady state energy balance provides the steady state heat flow is needed to obtain a new steady-state temperature. If the process is at steady state, and there are no disturbances, the inputs and outputs are zero. The IMC structure is shown in fig (2) [1]. Where u is the input variable and y is the output variable. The process model receives the same manipulated variable as the actual process, and subtracts the difference between the process output and process model output to determine the model error.

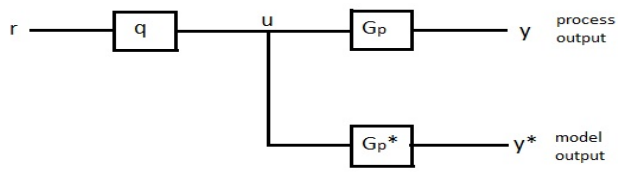

Figure (2). Process model in parallel with the actual process.

. In IMC structure disturbance effects the calculation of model uncertainty which includes unmeasured disturbances. This information can be used by the controller to compensate for the model uncertinity.IMC provides a transparency for control system designing and tuning.

V.IMC Based PID Controller

The IMC structure is rearranged to get a standard feedback control system so that open loop unstable system can be handled. This is done to improve the input disturbance rejection. The IMC based PID structure uses the process model as in IMC design. In the IMC procedure the controller $Q_c(s)$ is directly based on the invertible part of the process transfer function. The IMC results in only one tuning parameter which is filter tuning factor but the IMC based PID tuning parameters are the functions of this tuning factor. The selection of the filter parameter is directly related to the robustness. IMC based PID procedures uses an approximation for the dead time. And if the process has no time delays it gives the same performance as does the IMC. [1]

In ideal IMC structure the point of summation of the process and the model output is moved as shown in the figure(3) to form a standard feedback controller which is known as IMC based PID controller.

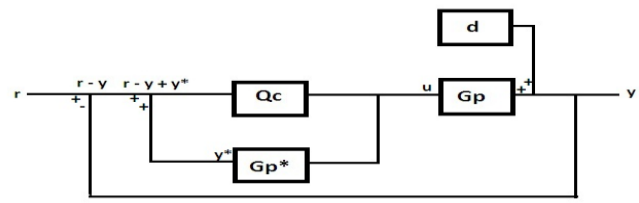

Figure (3).IMC Model Diagram.

Consider a process model $G_p^*(s)$ for an actual process or plant $G_p(s)$. The controller $Q(s)$ is used to control the process in which the disturbances $d(s)$ enter into the system.

The Equivalent standard feedback controller is obtained by rearranging the IMC [5], the standard feedback controller using:

$$G_c = \frac{Q_c}{1 - Q_c G^*_p} \tag{9}$$

Thus, output $y(s)$ is the output of series of $G_c(s)$ and $G_p(s)$ and the unity feedback system. The manipulated variable now is;

$$u(s) = \frac{[r(s) * G_c(s)]}{[1 + G_c(s) * G_p(s)]} \tag{10}$$

Output is; $$y(s) = \frac{[r(s) * G_c(s) * G_p(s)]}{[1 + G_c(s) * G_p(s)]} \tag{11}$$

Now compare with PID Controller transfer function for first order: $$G_c(s) = \frac{[K_c * (T_i * s + 1)]}{T_i * s} \tag{12}$$

PI tuning parameters are K_c is Proportional gain and T_i is the integral time are

$$K_c = \frac{T_p}{(lem * K_p)} \quad \textbf{and} \quad T_i = T_p \tag{13}$$

Similarly for second order system compare with the standard PID controller transfer function are

$$G_c(s) = K_c * \frac{(T_i T_d s^2 + T_i s + 1)}{T_i s} * \frac{1}{T_f s + 1} \tag{14}$$

Where $T = T_{au}$ (constant)

T_i = integral time constant

T_d = derivative time constant

T_f = filter tuning factor

K_c = proportional controller gain

The closed loop simulations for above procedure and adjust lem i.e. lemda considering a trade-off between performance and robustness i.e. sensitivity to model error. The IMC based PID design procedure for a first order system of CSTR process model is $G_p^*(s) = \dfrac{K_p^*}{[T_p^*(s)+1]}$ (15)

$$G_p^*(s) = G_{p+}^*(s) * G_{p-}^*(s) = \frac{K_p^*}{[T_p^*(s)+1]} \quad (16)$$

$$Q_c^*(s) = inv[G_{p-}^*(s)] = \frac{T_p^*(s)+1}{K_p^*} \quad (17)$$

$$Q_c(s) = Q_c^*(s) f(s) = \frac{[T_p^*(s)+1]}{[K_p^*(lem+1)]} \quad (18)$$

$$f(s) = \frac{1}{(lem * s+1)} \quad (19)$$

Equivalent feedback controller using transformation

$$G_c(s) = \frac{Q_c(s)}{(1-Q_c(s)G_p^*(s))} = \frac{[\{(T_p^*(s)+1)(T_p^*(s)+1)(T_p^*(s)+1)K_p^*(lem(s)+1)\}]}{[\{K_p^*(lem(s)+1)(1-K_p^*)\}]} \quad (20)$$

$$G_c(s) = \frac{\{T_p(s)+1\}}{K_p lem * s} \quad (21)$$

it is standard feedback controller for IMC

The transfer function for PI controller is

$$G_c(s) = \frac{[K_c(T_i s+1)]}{T_i s} \quad (22)$$

PI tuning parameters are $K_c = \dfrac{T_p}{(K_p^* lem)}$ and $T_i = T_p$

The IMC based PID design procedure for a second order system for the below process model of

$$G_p^*(s) = \frac{K_p^*}{[(T_{p1}^*(s)+1)(T_{p2}^*(s)+1)]} \quad (23)$$

$$G_p^*(s) = G_{p+}^*(s) G_{p-}^*(s) = \frac{K_p^*}{[T_p^*(s)+1]} \quad (24)$$

$$Q_c^*(s) = inv[G_{p-}^*(s)] = \frac{(T_p^*(s)+1)}{K_p^*} \quad (25)$$

$$Q_c(s) = Q_c^*(s) f(s) = \frac{[T_p^*(s)+1]}{[K_p^*(lem(s)+1)]} \quad (26)$$

$$f(s) = \frac{1}{(lem * s+1)} \quad (27)$$

Equivalent feedback controller using transformation

$$G_c(s) = \frac{Q_c(s)}{(1-Q_c(s)G_p^*(s))} = \frac{[(T_{p1}T_{p2}s^2 + (T_{p1}+T_{p2})s+1)]}{[K_p lem * s]} \quad (28)$$

It is the transfer function for the equivalent standard feedback controller

$$G_c(s) = \frac{[K_c\{(T_i T_d s^2 + T_i s+1)\}]}{[T_i s]} \quad (29)$$

The transfer function for ideal PID controller for second order PID tuning parameters on comparison

$$K_c = \frac{(T_{p1}+T_{p2})}{(K_p * lem)}$$

$$T_i = T_{p1} + T_{p2}$$

$$T_d = T_{p1}$$

VI. Results

The simulation results of conventional PID, IMC and IMC Based Controller are shown in figures (3-5) respectively. The simulation results show that the IMC based PID controller has no overshoot and fast response compared to the conventional PID. Moreover this method is feasible due to the effective communication realization between the CSTR process plant and MATLAB.

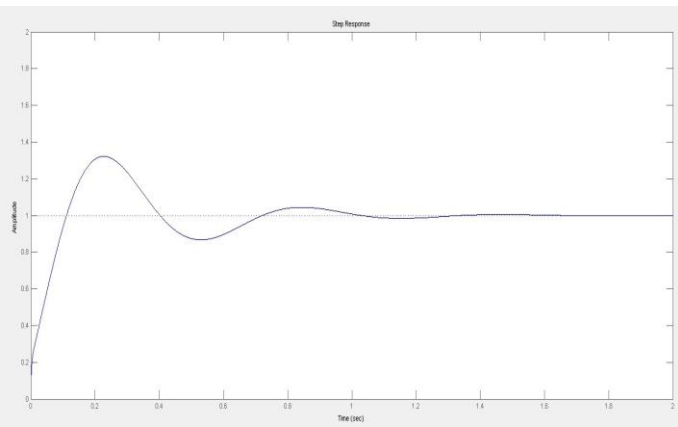

Figure (4): Step Response of PID Controller

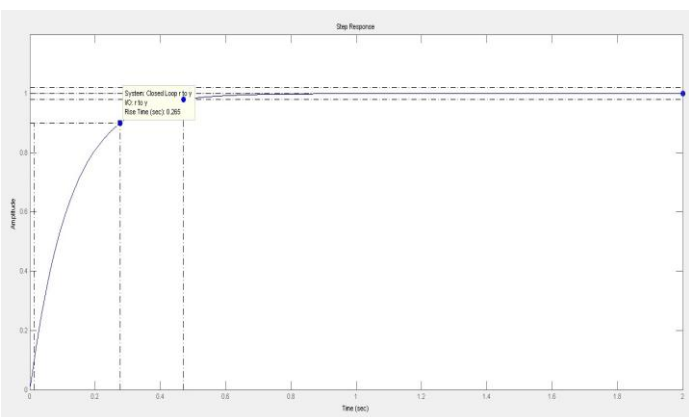

Figure (5): Step response of IMC Controller

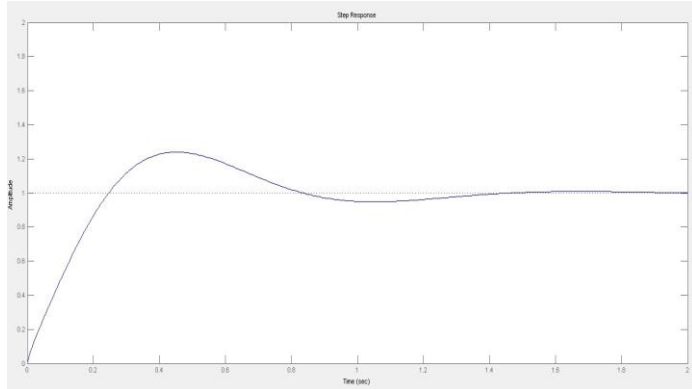

Figure (6): Step response of IMC based PID Controller

The simulation results for PID, IMC and IMC based PID Controller are shown in the Table (2). From the table IMC control suits best for controlling of CSTR

TABLE (2). Comparison Table

Specification	PID	IMC	IMC Based PID
Rise Time in Sec	0.0933	0.265	0.196
Peak Time in Sec	0.228	0.312	0.45
Settling Time in sec	0.962	0.471	1.31

VII. References

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