Reduction of Higher Order Linear Dynamic SISO and MIMO Systems Using the Advantages of Improved Pole Clustering and PSO

G.Vasu[1]
Assistant professor, Dept. of Electrical Engineering,
S.V.P Engineering College
Visakhapatnam, A.P , India
E-mail: vasuganji_kvs@yahoo.com

P.Murari[2]
Assistant professor, Dept. of Electrical Engineering,
S.V.P Engineering College
Visakhapatnam, A.P , India
E-mail: murarinitt@gmail.com

R.Prasanna Kumar[3]
B.E. student, Dept of Electrical Engg,
S.V.P Engineering College
Visakhapatnam, A.P , India
E-mail: rapetiprasanna@gmail.com

Abstract—**The authors present a combined method for order reduction of higher order linear dynamic systems using the advantages of the improved pole clustering and error minimization by PSO. The denominator of the reduced model is obtained by the improved pole clustering and PSO is employed for determining numerator coefficients by minimizing the integral square error between the transient responses of the original and reduced order models, pertaining to unit step input. The reduction procedure is simple, efficient and computer oriented. The proposed algorithm has been extended for the reduction of linear multivariable system. The algorithm is illustrated with the help of two numerical examples to highlight the advantages of the approach and the results are compared with the other existing techniques.**

Keywords: **Order Reduction, Pole Clustering, PSO, Relative mapping Errors, Stability**

I.INTRODUCTION

In the analysis and design of complex systems, it is often necessary to simplify a high order system. The use of a reduced order model makes it easier to implement analysis, simulations and control system designs. Here we consider the system in the form of a transfer function. To establish a transfer function of lower order, numerous methods have been proposed [1]-[4].Further several methods have also been suggested by combining the features of two different methods [4]-[7]. In spite of the significant number of methods available, no approach always gives the best results for all systems. Almost all methods, however, aim at accurate reduced models for a low computational cost.

Further, numerous methods of order reduction are also available in the literature [8]-[11], which are based on the minimization of the integral square error (ISE) criterion. However, a common feature in these methods [10]-[11] is that the values of the denominator coefficients of the reduced order model (ROM) are chosen arbitrarily by some stability preserving methods such as dominant pole, Routh approximation methods, etc. and then the numerator coefficients of the ROM are determined by minimization of the ISE. In [12], Howitt and Luss suggested a technique, in which both the numerator and denominator coefficients are considered to be free parameters and are chosen to minimize the ISE in impulse or step responses. Recently, particle swarm optimization (PSO) technique appeared as a promising algorithm for handling the optimization problems. PSO is a population based stochastic optimization technique, inspired by social behaviour of bird flocking or fish schooling [13].In PSO, the potential solutions, called particles, fly through the problem space by following the current optimum particles.

In the present work, the authors present a combined method using the advantages of the improved pole clustering method[15] and error minimization by PSO for order reduction of Single and Multi variable linear dynamic systems. In this method the reduced denominator is obtained by the improved pole clustering method and the numerator is determined by minimizing the integral square error between the transient responses of original and reduced order systems using particle swarm optimization technique, pertaining to a unit step input. The comparison between the proposed and other well known existing order reduction techniques is also shown in the present work. In the following section the algorithm is described in detail with the help of two numerical examples.

II DESCRIPTION OF ALGORITHM

Let the transfer function of the original high order linear dynamic SISO system of order 'n' be:

$$G_n(s) = \frac{N(s)}{D(s)} = \frac{b_0 + b_1 s + \cdots + b_{n-1} s^{n-1}}{a_0 + a_1 s + \cdots + a_{n-1} s^{n-1} + a_n s^n} \quad \cdots (1)$$

and Let the corresponding r^{th} order reduced model is synthesized as:

$$G_r(s) = \frac{N_r(s)}{D_r(s)} = \frac{d_0 + d_1 s + \cdots + d_{r-1} s^{r-1}}{e_0 + e_1 s + \cdots + e_{r-1} s^{r-1} + s^r} \quad \cdots (2)$$

Further, the method consists of the following steps.

Step 1: *Determination of the reduced order denominator polynomial with an improved pole clustering technique[15]:*

Calculate the 'n' number of poles from the given higher order system denominator polynomial. The number of cluster centres to be calculated is equal to the order of the reduced system. The poles are distributed in to the cluster centre for the calculation such that none of the repeated poles present in the same cluster centre. Minimum number of poles distributed per each cluster centre is at least one. There is no limitation for the maximum number poles per cluster centre. Let 'k' number of poles available in a cluster centre: $P_1, P_2, P_3 \ldots P_k$. The poles are arranged in a manner such that $|P_1| < |P_2| < \cdots < |P_k|$. The cluster centre for the reduced order model can be obtained by using the following procedure.

1 Let 'k' number of poles available are $|P_1| < |P_2| < \cdots |P_k|$.

2 Set L=1; $\quad C_L = \left[\left(\frac{-1}{|P_1|} + \sum_{i=2}^{K} \frac{-1}{|P_i - P_1|} \right) \div K \right]^{-1}$

3 Find the pole cluster as ,

4 Check for L=K. If yes, then the final cluster centre is $C_C = C_l$ and terminates the process. Otherwise proceed on to next step.

5 Set L=L+1,

6 The improved cluster centre from

$$C_L = \left[\left(\frac{-1}{|P_1|} + \left(\frac{-1}{|C_L|} \right) \right) \div 2 \right]^{-1}$$

7 Check for L=K. If no, then go to the step (5). Otherwise go the next step.

8 Final cluster centre is $C_C = C_L$. On calculating the cluster centre values, we have following two cases as in.

Case I: *All the denominator poles are real*:
The corresponding reduced order denominator polynomial can be obtained as,

$$D_r(s) = (S + C_{C_1})(S + C_{C_2}) \cdots (S + C_{C_r}) \quad \cdots (3)$$

Where $C_{C_1}, C_{C_2} \cdots C_{C_r}$ are the improved cluster values required to obtain the reduced order denominator polynomial of order 'r'.

Case II: *All the poles are complex*:
Let $t = \left(\frac{K}{2} \right)$ pairs of complex conjugate poles in a Lth cluster be, $[(\sigma_1 \pm \omega_1), (\sigma_2 \pm \omega_2), (\sigma_3 \pm \omega_3) \cdots (\sigma_t \pm \omega_t)]$
Where, $|\sigma_1| < |\sigma_2| < \cdots |\sigma_L|$. apply the proposed algorithm individually for real and imaginary parts to obtain the respective improved cluster centres. The improved cluster centre is in the form of $\mu_j = A_j \pm B_j$. Where, A_j and B_j is the improved pole cluster values obtained for real and imaginary parts respectively.

The corresponding reduced order denominator polynomial can be obtained as,

$$D_r(s) = (S + |\mu_1|)(S + |\mu_2|) \cdots (S + |\mu_j|) \quad \cdots (4)$$

Where, j=r.

Case III: If some poles are real and some poles are complex in nature, applying an improved clustering algorithm separately for real and complex terms. Finally obtained improved cluster centres are combined together to get the reduced order denominator polynomial.

Step 2: *Determination of the numerator polynomial of the reduced order model by PSO technique:*

The PSO method is population based search algorithm where each individual solution [coefficients of reduced numerator] is referred to as one particle and Each particle flies through the bounded search space with an adaptable velocity that is dynamically modified according to its own flying experience and also the flying experience of the other particles. In PSO, each particle strives to improve itself by imitating traits from their successful peers. Further each particle has a memory and hence it is capable of remembering the best position in the search space ever visited by it. The position corresponding to the best fitness[ISE] of a particle is known as p_best(personal best) and overall best of all particles in the population is called g_best(global best).

In a d-dimensional search space, the best particles updates its velocity and positions with following equations:

$$\left. \begin{array}{l} v_{j,g}^{t+1} = W * v_{j,g}^t + c_{1*}\gamma_1() * (pbest_{j,g} - X_{j,g}^t) \\ \qquad\qquad + c_{2*}\gamma_2() * (gbest_g - X_{j,g}^t) \\ \\ X_{j,g}^{t+1} == X_{j,g}^t + v_{j,g}^{t+1} \end{array} \right\} \cdots (5)$$

with $j = 1,2,\cdots,n$ and $g = 1,2,\cdots,d$;
Where
n = no. Of particles in the swarm

d = Vector dimension of the particle X_j and its velocity v_j.

t = number of iteration.

W = inertia weight factor

$W = W_{max} - [W_{max} - W_{min}] * \frac{K-1}{N-1}$;
Where 'K' is the current iteration and 'N' is the maximum iteration.

C_1, C_2 =Cognitive and social acceleration factors respectively.
$\gamma_1(), \gamma_2()$ =Random numbers uniformly distributed in the range (0, 1).

The jth particle in the swarm is represented by d-dimensional vector $X_j = (X_{j1}, X_{j2}, \cdots, X_{jd})$ and its velocity is denoted by another d-dimensional vector $v_j = (v_{j1}, v_{j2}, \cdots, v_{jd})$. The best previous visited position of the jth particle is represented by

$pbest_j = (pbest_{j1}, pbest_{j2}, \cdots, pbest_{jd})$. The best particle among all of the particles in the swarm is represented by $gbest_g = (gbest_1, gbest_2, \cdots, gbest_d)$.

In PSO each particle moves in a search space with a velocity according to its owns previous best solution ($pbest_{j,g}$) and its groups previous best solution ($gbest_g$). The velocity update in the particle swarm consists of three parts; namely momentum, cognitive and social parts. The balance among these parts determines the performance of a PSO algorithm[14]. The parameter C_1 and C_2 determine the relative pull of pbest and gbest and parameters $\gamma_1(\ ), \gamma_2(\ )$ help in stochastically varying these pulls. The position and velocity updates of a particle in PSO for a two dimensional parameter space shown in fig 1.
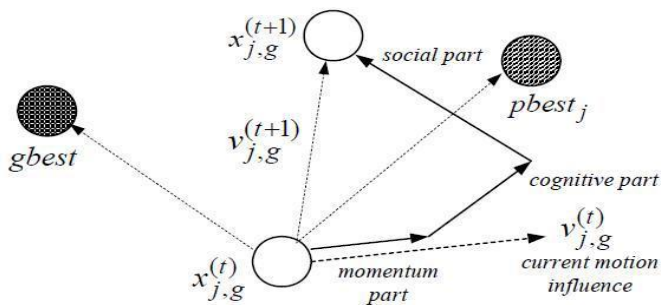

Fig. 1. Description of velocity and position updates in particle swarm Optimization for a two dimensional parameter space

In the present study, PSO is employed to minimize the objective function 'E' which is the integral square error in between the transient response of original and reduced model is given by[4]:

$$E = \int_0^\infty [y(t) - y_r(t)]^2 . dt \qquad \cdots (6)$$

Where $y(t)$ and $y_r(t)$ are the unit step responses of original $[G_n(s)]$ and reduced $[G_r(s)]$ order systems, and the parameters to be determined are the numerator coefficients of the reduced order model as given in Eq.(2). To eliminate any steady state error in the approximation, the condition is:

$$d_0 = \frac{b_0}{a_0} c_0 \qquad \cdots (7)$$

In Table I , the specified parameters of the PSO algorithm used in the present study are given. The computational flow chart of the proposed algorithm is shown in Figure.2.

TABLE I
PARAMETERS USED FOR PSO ALGORITHM

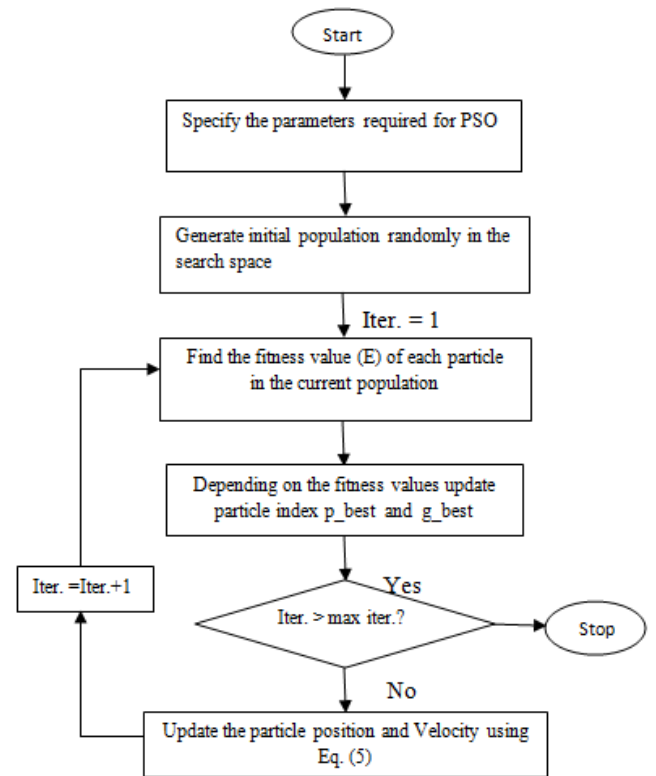| Parameters	Value
Swarm Size	20
Max. Iteration	50
C_1, C_2	2.0,2.0
W_{max}, W_{min}	0.9,0.4

Fig. 2 Flow chart of PSO algorithm

Extension to Multivariable System:
Let the transfer function matrix of the higher order '*n*' having '*p*' inputs and '*m*' outputs be

$$[G(s)] = \frac{1}{D_n(s)} \begin{bmatrix} b_{11}(s) & b_{12}(s) & \cdots & b_{1p}(s) \\ b_{21}(s) & b_{22}(s) & \cdots & b_{2p}(s) \\ \cdots & \cdots & \cdots & \cdots \\ b_{m1}(s) & b_{m2}(s) & \cdots & b_{mp}(s) \end{bmatrix} \quad \cdots (8)$$

Or $[G(s)] = [g_{ij}(s)], i = 1,2 \cdots, m; j = 1,2, \cdots, p;$ is a $m \times p$ transfer matrix.
The general form of $g_{ij}(s)$ of $[G(s)]$ is taken as

$$g_{ij}(s) = \frac{b_{ij}(s)}{D_n(s)} = \frac{b_0 + b_1 s + \cdots + b_{n-1} s^{n-1}}{a_0 + a_1 s + \cdots + a_{n-1} s^{n-1} + a_n s^n} \cdots (9)$$

Let the transfer function matrix of the reduced order '*r*' having same number of inputs and outputs to be synthesized as:

$$[G_r(s)] = \frac{1}{\tilde{D}_r(s)} \begin{bmatrix} a_{11}(s) & a_{12}(s) & \cdots & a_{1p}(s) \\ a_{21}(s) & a_{22}(s) & \cdots & a_{2p}(s) \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1}(s) & a_{m2}(s) & \cdots & a_{mp}(s) \end{bmatrix} \cdots (10)$$

Or $[G_r(s)] = [R_{ij}(s)], i = 1,2 \cdots, m; j = 1,2, \cdots, p;$ is a $m \times p$ transfer matrix.

The general form of $R_{ij}(s)$ of $[G_r(s)]$ is taken as

$$R_{ij}(s) = \frac{a_{ij}(s)}{\widetilde{D}_r(s)} = \frac{e_0 + e_1 s + \cdots + e_{r-1} s^{r-1}}{f_0 + f_1 s + \cdots + f_{r-1} s^{r-1} + f_r s^r} \quad \cdots (11)$$

For getting r^{th} order reduced transfer matrix $[G_r(s)]$, first of all common denominator $D(s)$ of Eq.(8) is reduced to $\widetilde{D}_r(s)$ using improved pole clustering technique and the coefficients of numerator $a_{ij}(s)$ of the reduced transfer matrix $R_{ij}(s)$ are successively determined by minimizing the error 'E', between the transient responses of original $[g_{ij}(s)]$ and reduced $[R_{ij}(s)]$ order models using PSO.

III. NUMERICAL EXAMPLES

Two numerical examples are chosen from the literature to show the flexibility and effectiveness of the proposed reduction algorithm than other existing methods, and the response of the original and reduced models are compared.

Example-1: Consider an Eighth order system transfer function taken from [16]:

$$G_8(s) = \frac{\begin{matrix}35s^7 + 1086s^6 + 13285s^5 + 82402s^4 \\ +278376s^3 + 511812s^2 + 482964s \\ +194480\end{matrix}}{\begin{matrix}s^8 + 21s^7 + 220s^6 + 1558s^5 + 7669s^4 \\ +24469s^3 + 46350s^2 + 45952s + 17760\end{matrix}} \quad \cdots (12)$$

Step-1: The poles are: $-1 \pm 6i, -1, -2, -3, -4, -4, -5$.

Let the 2nd order reduced model is required to be realized, Since poles have a combination of real and complex, for this purpose only one complex cluster is required. So apply an improved clustering algorithm separately for real and complex terms.

Let real cluster consists of poles $(-1, -2, -3, -4, -5)$ and imaginary cluster consists of (± 6). The improved cluster centres are computed as $\mu_1 = -1.0245468 \pm 6i$

Using Eq.(4), the denominator polynomial $D_2(s)$ is obtained as

$$D_2(s) = s^2 + 2.0490936s + 37.0496961$$

Step-2: By using PSO to minimize the objective function 'E', as described earlier, we have:

$$N(s) = 38.777313s + 405.710876$$

Therefore, finally $G_2(s)$ is given as:

$$G_2(s) = \frac{38.777313s + 405.710876}{s^2 + 2.0490936s + 37.0496961} \quad \cdots (13)$$

A comparison of the proposed algorithm with the other well known existing order reduction techniques for a second-order

reduced model is given in Table II. Figure 3(a) Presents diagram of step responses of $G_8(s)$ and $G_2(s)$ respectively.
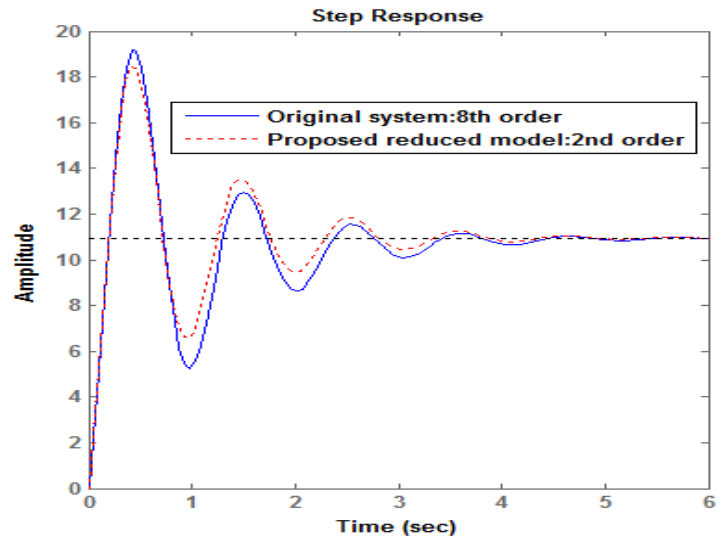

Fig. 3(a) Comparison of step responses of $G_8(s)$ and $G_2(s)$

TABLE II
COMPARISON OF REDUCED ORDER MODELS

Method of Reduction	Reduced Models: $G_2(s)$	ISE (E) = $\int_0^{\infty}[G_8(s) - G_2(s)]^2$
Proposed Algorithm	$\dfrac{38.7773s + 405.7108}{s^2 + 2.04909s + 37.0496}$	1.608666
S.N.Sivanandam, et al [16]	$\dfrac{35s + 401.21}{s^2 + 1.436s + 36.63}$	2.499506
C.S.Hsieh et al[17]	$\dfrac{1.6666s + 61.271473}{0.047619s^2 + s + 5.595338}$	33.662094
R.Prasad et al[6]	$\dfrac{8.690832s + 4.498007}{s^2 + 0.836381s + 0.41076}$	115.3929
Y.Shamash [18]	$\dfrac{13.09095s + 5.271465}{s^2 + 1.245549s + 0.481393}$	90.70348

Example- 2: Consider a 6th order two input two output system[19] described by the transfer function matrix:

$$[G(s)] = \begin{bmatrix} \dfrac{2(s+5)}{(s+1)(s+10)} & \dfrac{(s+4)}{(s+2)(s+5)} \\ \dfrac{(s+10)}{(s+1)(s+20)} & \dfrac{(s+6)}{(s+2)(s+3)} \end{bmatrix}$$

$$= \frac{1}{D(s)} \begin{bmatrix} b_{11}(s) & b_{12}(s) \\ b_{21}(s) & b_{22}(s) \end{bmatrix} \qquad \cdots (14)$$

Where, the common denominator $D(s)$ is given by :
$$D(s) = (s+1)(s+2)(s+3)(s+5)(s+10)(s+20)$$
$$= s^6 + 41s^5 + 571s^4 + 3491s^3 + 10060s^2 + 13100s + 6000$$

And
$$b_{11}(s) = 2s^5 + 70s^4 + 762s^3 + 3610s^2 + 7700s + 6000$$
$$b_{12}(s) = s^5 + 38s^4 + 459s^3 + 2182s^2 + 4160s + 2400$$
$$b_{21}(s) = s^5 + 30s^4 + 331s^3 + 1650s^2 + 3700s + 3000$$
$$b_{22}(s) = s^5 + 42s^4 + 601s^3 + 3660s^2 + 9100s + 6000$$

The proposed algorithm is successively applied to each element of the transfer function matrix of above multivariable system and the reduced order models $R_{ij}(s)$ of the low order system $[G_r(s)]$ are obtained. The general form of second order reduced transfer function matrix is taken as:

$$[G_2(s)] = \frac{1}{\widetilde{D}_2(s)} \begin{bmatrix} a_{11}(s) & a_{12}(s) \\ a_{21}(s) & a_{22}(s) \end{bmatrix} \qquad \cdots (15)$$

Let the first and second cluster contains the poles $-1, -2$ and $-3, -5, -10, -20$, respectively.
Where, the denominator polynomial for the second order reduced model is obtained as:

$$\widetilde{D}_2(s) = s^2 + 4.086345s + 3.0863435 \qquad \cdots (16)$$

and
$$a_{11}(s) = 1.323472s + 3.086344$$
$$a_{12}(s) = 1.048874s + 1.234537$$
$$a_{21}(s) = 0.579856s + 1.543172 \qquad \cdots (17)$$
$$a_{22}(s) = 1.81634s + 3.086344$$

Therefore, finally the second order reduced order model is obtained as:

$$[G_2(s)] = \frac{1}{s^2 + 4.086345s + 3.0863435}$$

$$\times \begin{bmatrix} 1.323472s + 3.086344 & 1.048874s + 1.234537 \\ 0.579856s + 1.543172 & 1.81634s + 3.086344 \end{bmatrix} \qquad \cdots (18)$$

The step responses of the reduced-order models $R_{ij}(s)$ are compared with the original system $g_{ij}(s)$ and are shown in Figure 3(b)-(e) and the proposed method is compared with the other well-known order-reduction techniques available in literature and are shown in the Table III by comparing the integral square error ISE in between the step responses of the original $g_{ij}(s)$ and reduced $R_{ij}(s)$ order systems. The ISE is given as

$$ISE(E) = \int_0^\infty \left[g_{ij}(t) - R_{ij}(t) \right]^2 \qquad \cdots (19)$$

Where $i = 1, 2$, $j = 1, 2$ and $g_{ij}(s)$, $R_{ij}(s)$ are the unit step responses of the original and reduced systems respectively for i th-output with j th-input.

TABLE III
Comparison of reduction methods

ISE for $R_{ij}(s)$ ($i = 1, 2$; $j = 1, 2$)

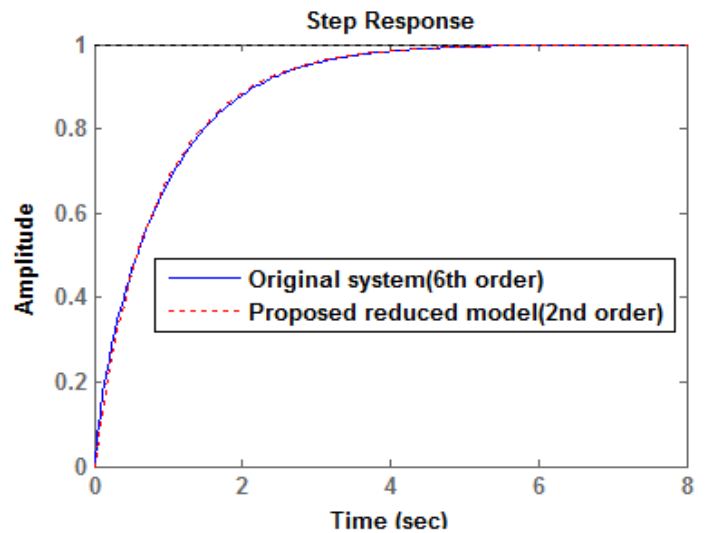| Reduction method	R_{11}	R_{12}	R_{21}	R_{22}
Proposed method	0.000398	0.000102	0.000045	0.006913
Diff and factor division [20]	0.04607	8.87×10^{-8}	0.0128	0.000397
Stabilty Eq and GA [21]	0.014498	0.008744	0.002538	0.015741
Prasad and Pal [22]	0.136484	0.002446	0.040291	0.067902
Safonov and chiang[23]	0.590617	0.037129	0.007328	1.066123
Prasad et al.[24]	0.030689	0.000256	0.261963	0.021683

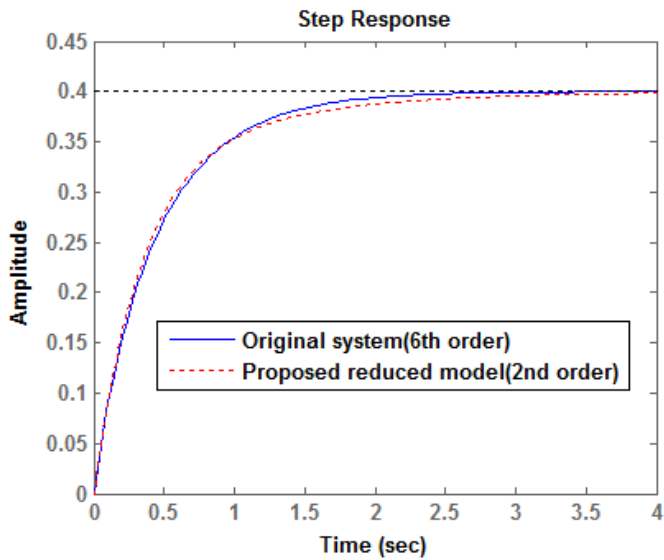Fig. 3(b) Comparison of step responses of $g_{11}(s)$ and $R_{11}(s)$

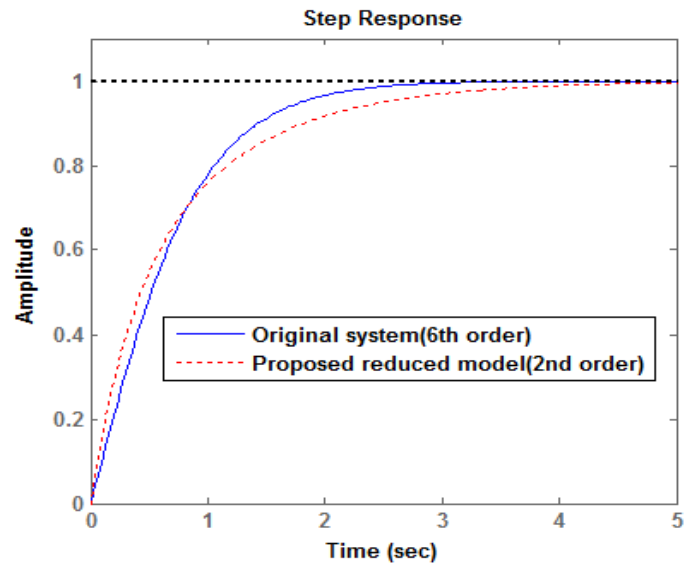Fig. 3(c) Comparison of step responses of $g_{12}(s)$ and $R_{12}(s)$

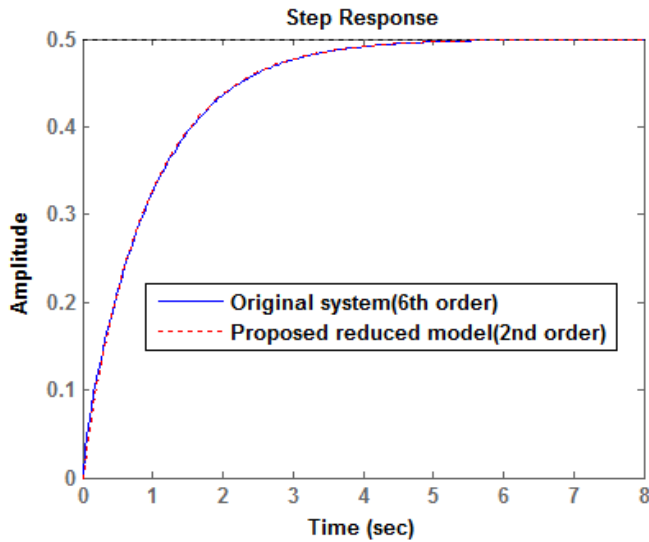Fig. 3(e) Comparison of step responses of $g_{22}(s)$ and $R_{22}(s)$

Fig. 3(d) Comparison of step responses of $g_{21}(s)$ and $R_{21}(s)$

IV CONCLUSION

The authors proposed a mixed algorithm for reducing the order of linear dynamic SISO and MIMO systems. In this algorithm, the concept of order reduction by an improved pole clustering method has been employed to determine the coefficients of reduced denominator while the coefficients of the reduced numerator are obtained by minimizing the integral square error between the transient responses of original and reduced models using PSO technique pertaining to unit step input. The algorithm is implemented in C-language. The matching of the unit step responses is assured reasonably well in the algorithm. The algorithm is simple, rugged and computer oriented and is compared with other well known existing order reduction techniques given in Table II and III, from which it is clear that the proposed algorithm compares well with the other techniques of model order reduction.

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